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SAFE BUILDING CONSTRUCTION

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SAFE BUILDING CONSTRUCTION

A TREATISE

GIVING IN SIMPLEST FORMS POSSIBLE PRACTICAL AND THEORETICAL RULES AND FORMULÆ USED IN CONSTRUCTION OF BUILDINGS AND GENERAL INSTRUCTION

BY

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Safe Building Construction.

PREFACE.

SAFE BUILDING CONSTRUCTION is the logical sequence of my former work on Construction, "Safe Building."

Methods of construction and building materials have changed so radically that this new book seems called for.

Much of my former work has been retained, though my object has been to *condense*—saving the student's time in these busy days—while still following the original plan which was:—"to furnish to any earnest student the opportunity to acquire, so far as books will teach, the knowledge necessary to erect *safely* any building. While of course, the work will be based strictly on the science of mechanics, all useless theory will be avoided. The object will be to make the articles simply practical. To follow any of the mathematical demonstrations, arithmetic, a rudimentary knowledge of algebra and plane geometry will be sufficient."

Special effort has been made to introduce in this new work a general survey and fair knowledge of *Concrete Construction,* including concrete foundations, grillage, piles, piers, and other subsoil as well as above-soil constructions, such as floors, columns, girders, walls, etc.

For the many flattering letters and comments received for "Safe Building" during the last two decades, some quite recently, I thank my professional friends, young and old, and hope this newer work—SAFE BUILDING CONSTRUCTION—may be equally acceptable.

CHAPTER I.

STRENGTH OF MATERIALS.

(German, *Festigkeit;* French, *Résistance des matériaux.*)

ALL solid bodies or materials are made up of an infinite number of atoms, fibres or molecules. These adhere to each other and resist separation with more or less tenacity, varying in different materials. This tenacity or tendency of the fibres to resume their former relation to each other after the strain is removed is called the elasticity of the material. It is when this elasticity is overcome that the fibres separate, and the material breaks and gives way.

There are to be considered in calculating strengths of materials two kinds of forces, viz., the external (or applied) forces and the internal (or resisting) forces. The external forces are any kind of forces applied to a material and tending to disrupt or force the fibres apart. Thus a load lying apparently perfectly tranquil on a beam is really a very active force; for the earth is constantly attracting the load, which tends to force its way downwards by gravitation and push aside the fibres of the beam under it. These latter, however, resist separation from each other, and the amount of the elasticity of all these fibres being greater than the attraction of the earth, the load is unable to force its way downwards and remains apparently at rest.

Strain. The amount of this tendency to disrupt the fibres (produced by the external forces) at any point is called the "strain" at that point.

Stress. The amount of the resistance against disruption of the fibres at such point is called the "stress" at that point.

External (or applied) forces, then, produce *strains.* Internal (or resisting) forces produce *stresses.*

This difference must be well understood and constantly borne in mind, as strains and stresses are the opposing forces in the battle of all materials against their destruction.

Ultimate Stress. When the strain at every point of the material just equals the stress, the material remains in equilibrium. The greatest stress, at any point of a material that it is capable of exerting is the ultimate stress (that is, the ultimate strength of resistance) at that point. Were the strain to exactly equal that ultimate stress, the material, though on the point of breaking, would still be

safe theoretically. But it is impossible for us to calculate so closely. Besides we can never determine accurately the actual ultimate stress, for different pieces of the same material vary in practice very greatly, as has been often proved by experiment. Therefore the actual ultimate stress might be very much less than that calculated.

Factor-of-Safety.

Again, it is impossible to calculate the exact strain that will always take place at a certain point; the applied forces or some other conditions might vary. Therefore, to provide for all possible emergencies, we must make our material strong enough to be surely safe; that is, we must calculate (allow) for a considerably greater ultimate stress at every point than there is ever likely to be strain at that point.

The amount of extra allowance of stress varies greatly, according to circumstances and material. The number of times that we calculate the ultimate stress to be greater than the strain is called the factor-of-safety (that is, the ratio between stress and strain).

If the elasticity of different pieces of a given material is practically uniform, and if we can calculate the strain very closely in a given case, and further, if this strain is not apt to ever vary greatly, or the material to decay or deteriorate, we can of course take a low or small factor-of-safety; that is, the ultimate stress need not exceed many times the probable greatest strain.

On the other hand, if the elasticity of different pieces of a given material is very apt to vary greatly, or if we cannot calculate the strain very closely, or if the strain is apt to vary greatly at times, or the material is apt to decay or to deteriorate, we must take a very high or large factor-of-safety, that is, the ultimate stress must exceed many times the probable greatest strain.

Factors-of-safety are entirely a matter of practice, experience, and circumstances. In general, we might use for stationary loads:

A factor of safety of 3 to 4 for wrought-iron and steel,
" " " 6 for cast-iron,
" " " 4 to 10 for wood,
" " " 10 for brick and stone.

For moving-loads, such as people dancing, machinery vibrating, dumping of heavy loads, etc., the factor-of-safety should be one-half larger, or if the shocks are often repeated and severe, at least double of the above amounts. Where the constants to be used in formulæ are of doubtful authority (as is the case with most of them for woods and stones), the factor-of-safety chosen should be the highest one.

In building-materials we meet with four kind of strains, and, of course, with the four corresponding stresses resisting them, viz.: —

STRAINS.

Compression, or crushing strains,
Tension, or pulling strains,
Shearing, or sliding strains, and
Transverse, or cross-breaking strains.

STRESSES.

The resistance to *Compression*, or crushing-stress,
The resistance to *Tension*, or pulling-stress,
The resistance to *Shearing*, or sliding-stress, and
The resistance to *Transverse* strains, or cross-breaking stress.

Materials yield to *Compression* in three different ways: —

1. By direct crushing or crumbling of the material, or
2. By gradual bending of the piece sideways and ultimate rupture, or
3. By buckling or wrinkling (corrugating) of the material lengthwise.

Materials yield to *Tension*,

1. By gradually elongating (stretching), thereby reducing the size of the cross-section, and then,
2. By direct tearing apart.

Materials yield to *Shearing* by the fibres sliding past each other in two different ways, either

1. Across the grain, or
2. Lengthwise of the grain.

Materials yield to *Transverse* strains,

1. By deflecting or bending down under the load, and (when this passes beyond the limit of elasticity),
2. By breaking across transversely.

In calculating strains and stresses, there are certain rules, expressions, and formulæ which it is necessary for the student to understand or know, and which will be here given without attempting elaborate explanations or proofs. For the sake of clearness and simplicity, it is essential that in *all* formulæ the same letters should *always* represent the same value or meaning; this will enable the student to *read* every formula off-hand, without the necessity of an explanatory key to each one. The writer has further made it a habit to express, in *all* cases, his formulæ in pounds and inches (rarely using tons or feet); this will frequently make the calculation a little more elabo-

rate, but it will be found to greatly simplify the formulæ, and to make their *understanding* and *retention* more easy.

In the following articles, then, a capital letter, if it were used, would invariably express a quantity (respectively), either in tons or feet, while a small letter *invariably* expresses a quantity (respectively), either in pounds or inches.

The following letters, *in all cases*, will be found to express the same meaning, *unless distinctly otherwise stated*, viz.: —

a signifies *area*, in square inches.
b " *breadth*, in inches.
c " constant for *ultimate resistance to compression*, in pounds, per square inch. (See Tables IV and V.)
d signifies *depth*, in inches.
e " constant for *modulus of elasticity*, in pounds-inch, that is, pounds per square inch. (See Table IV.)
f " *factor of safety*.
g " constant for *ultimate resistance to shearing*, per square inch, across the grain. (See Tables IV and V.)
g_1 " constant for *ultimate resistance to shearing*, per square inch, lengthwise of the grain. (See Table IV.)
h " *height*, in inches.
i " *moment of inertia*, in inches. (See Table I.)
k " *ultimate modulus of rupture*, in pounds, per square inch. (See Tables IV and V.)
l " *length*, in inches.
m " *moment* or *bending moment*, in pounds-inch. (See Table IX.)
n " *constant* in Rankine's formula for compression of long pillars. (See Table II.)
o " the *centre*.
p " the *amount* of the *left-hand re-action* (or support) of beams, in pounds.
q " the *amount* of the *right-hand re-action* (or support) of beams, in pounds.
r " *moment of resistance*, in inches. (See Table I.)
s " *strain*, in pounds.
t " constant for *ultimate resistance to tension*, in pounds, per square inch. (See Tables IV and V.)
u " *uniform load*, in pounds.
v " *stress*, in pounds.
w " *load at centre*, in pounds.
x, *y*, and *z* signify *unknown quantities*, either in pounds or inches.

δ signifies *total deflection*, in inches.

ρ^2 " *square of the radius of gyration*, in inches.

$\mathfrak{d}$ " *diameter*, in inches.

$\mathfrak{r}$ " *radius*, in inches.

$\pi = 3.14159$, or, say, $3\frac{1}{7}$ signifies the *ratio of the circumference and diameter of a circle.*

If there are more than one of each kind, the second, third, etc., are indicated with Roman numerals, as for instance, a, a_{I}, a_{II}, a_{III}, etc., or b, b_{I}, b_{II}, b_{III}, etc.

In taking moments, or bending moments, strains, stresses, etc., to signify at what point they are taken, the letter signifying that point is added, as for instance:—

m signifies moment or bending moment at *centre.*

m_A " " " " *point A.*

m_B " " " " *point B.*

m_X " " " " *point X.*

s " strain at *centre.*

s_B " " *point B.*

s_X " " " *X.*

v " stress at *centre.*

v_D " " *point D.*

v_X " " *X.*

w signifies load at *centre.*

w_A " " " *point A.*

CENTRE OF GRAVITY.

(German, *Schwerpunkt;* French, *Centre de gravité.*)

Centre of Gravity. The centre of gravity of a figure, or body, is that point upon which the figure, or body, will balance itself in whatever position the figure or body may be placed, provided no other force than gravity acts upon the figure or body.

To find the centre of gravity of a plane figure, find two neutral axes, in different directions, and their point of intersection will be the centre of gravity required.

NEUTRAL AXIS.

(German, *Neutrale Achse;* French, *Axe neutre.*)

Neutral Axis. The neutral axis of a body, or figure, is an imaginary line upon which the body, or figure, will always balance, provided the body, or figure, is acted on by no other force than gravity. The neutral axis always passes through the centre of gravity, and may run in any direction. In calculating transverse strains, the neutral axis

designates an imaginary line of the body, or of the cross-section of the body, at which the forces of compression and tension meet. The strain on the fibres at the neutral axis is always naught. **Extreme Fibres.** On the upper side of the neutral axis the fibres are compressed, while those on the lower side are elongated. The amount of compression or elongation of the fibres increases directly as their distance from the neutral axis; the greatest strain, therefore, being in the fibres along the upper and lower edges, these being farthest from the neutral axis, and therefore called the *extreme fibres*. It is necessary to calculate only the ultimate resistance of these extreme fibres, as, if they will stand the strain, certainly all the other fibres will, they all being nearer the neutral axis, and consequently less strained. Where the ultimate resistances to compression and tension of a material vary greatly, it is necessary to so design the cross-section of the body, that the "extreme fibres" (farthest edge) on the side offering the weakest resistance, shall be nearer to the neutral axis than the "extreme fibres" (farthest edge), on the side offering the greatest resistance, the distance of the "extreme fibres" from the neutral axis being on each side in direct proportion to their respective capacities for resistance. Thus, in cast-iron the resistance of the fibres to compression is about six times greater than their resistance to tension; we must therefore so design the cross-section, that the distance of the neutral axis from the top-edge will be six-sevenths of the total depth, and its distance from the lower edge one-seventh of the total depth.

To find the neutral axis of any plane-figure, some writers recommend cutting, in stiff card-board, **How to find Neutral Axis.** a duplicate of the figure (of which the neutral axis is sought), then to experiment until it balances on the edge of a knife, the line on which it balances being, of course, the neutral axis. This is an awkward and unscientific method of procedure, though there may be some cases where it will recommend itself as saving time and trouble.

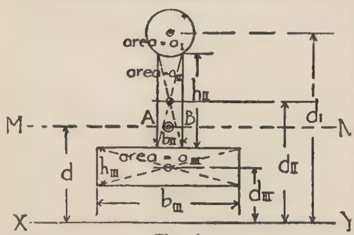


Fig. 1.

The following general formula, however, covers every case: To find the neutral axis *M–N* in any desired direction, draw a line *X–Y* at random, but parallel to the desired direction. Divide the figure into any number of *simple* figures, of which the areas and cen-

tres of gravity can be readily found, then the distance of the neutral axis $M-N$ from the line $X-Y$ will be equal to the *sum* of the *products* [1] of each of the small areas, multiplied by the distance of the centre of gravity of each area from $X-Y$, the whole to be divided by the entire area of the whole figure. An *Example* will make this more clear.

Find the horizontal neutral axis of the cross-section of a deck-beam, standing vertically on its bottom-flange.

Draw a line ($X-Y$) horizontally (Fig. 1), then let d_{I}, d_{II}, d_{III}, represent the respective distances from $X-Y$ of the centres of gravity of the small subdivided *simple* areas a_{I}, a_{II}, a_{III}, then let a stand for the whole area of section, that is:—

$$a_{\text{I}} + a_{\text{II}} + a_{\text{III}} = a,$$

then the required distance (d) of the neutral axis $M-N$ from $X-Y$, will be

$$d = \frac{a_{\text{I}}\,d_{\text{I}} + a_{\text{II}}\,d_{\text{II}} + a_{\text{III}}\,d_{\text{III}}}{a}.$$

To find the centre of gravity of the figure, we might find another neutral axis, but in a different direction, the point of intersection of the two being the required centre of gravity. But as the figure is uniform, we readily see that the centre of gravity of the whole figure must be half-way between points A and B.

Centres of Gravity. The centre of gravity of a circle is always its centre. The centre of gravity of a parallelogram is always the point of intersection of its two diagonals. The centre of gravity of a triangle is found by bi-secting two sides, and connecting these points each with its respective opposite apex of the triangle, the point of intersection of the two lines being the required centre of gravity, and which is always at a distance from each base equal to one-third of the respective height of the triangle. Any line drawn through either centre of gravity is a neutral axis.

MOMENT OF INERTIA.

(German, *Trägheitsmoment;* French, *Moment d'énertie*, or *Moment de giration.*)

Moment of Inertia. (See Table I.) The moment of inertia, sometimes called the moment of gyration, is the formula representing the inactivity (or state of rest) of any body rotating around any axis. The reason of the connection of this formula with the calculation of strains and the manner of obtaining it cannot be gone into here, as it would be quite beyond the scope of these articles. The moment of

[1] If line $X-Y$ is inside of (bisects) figure, take sum of products on one side only and *deduct* sum of products on other side.

inertia of any body or figure is the *sum* of the products of each particle of the body or figure multiplied by the *square* of its distance from the axis around which the body or figure is rotating.

A table of moments of inertia, of various sections, will be given later on and will be all the student will need for practical purposes.

THE CENTRE OF GYRATION AND RADIUS OF GYRATION.

(German, *Trägheitsmittelpunkt;* French *Centre de giration.*)

Square of Radius of Gyration. (See Table I.)

The centre of gyration "is that point at which, if the whole mass of a body rotating around an axis or point of suspension were collected, a given force applied would produce the same angular velocity as it would if applied at the same point to the body itself." The distance of this centre of gyration from the axis of rotation is called the *radius of gyration* (German, *Trägheitshalbmesser;* French, *Rayon de giration*); this latter is used in the calculation of strains, and is found by dividing the moment of inertia of the body by the area or mass of the body, and extracting the square root of the quotient, or,

$$\varrho = \sqrt{\frac{i}{a}}, \text{ or}$$

$$\varrho^2 = \frac{i}{a}.$$

A table will be given, later on, of the "squares of the radius of gyration" (German, *Quadrat des Trägheitshalbmessers;* French, *Carré du rayon de giration*).

THE MOMENT OF RESISTANCE.

(German, *Widerstandsmoment;* French, *Moment de résistance*).

Moment of Resistance. (See Table I.)

The moment of resistance of *any* fibre of a body, revolving around an axis, is equal to the moment of inertia of the whole body, divided by the distance of said fibre from the (neutral) axis, around which the body is revolving.

A table of moments of resistance will be given later on.

MODULUS OF ELASTICITY.

(German, *Elasticitäts modulus,* French, *Module d'élasticité*).

Modulus of Elasticity. (See Table IV.)

The modulus of elasticity of a given material is the force required to elongate a piece of the material (whose area of cross-section is equal to one square inch) through space a distance equal to its primary length. Thus, if a bar of iron, twelve inches long, and of one square inch area of cross-section, could be made so elastic as to stretch to

twice its length, the force required to stretch it until it were twenty-four inches long would be its modulus of elasticity in weight per square inch.

MODULUS OF RUPTURE.

(German *Bruchcoefficient;* French, *Module de rupture.*)

Modulus of Rupture. (See Tables IV and V.)

It has been found by actual tests that though the different fibres of materials under transverse strains are either in compression or tension, the ultimate resistance of the "extreme fibres" neither entirely agrees with their ultimate resistance to compression nor tension. Attempts have been made to account for this in many different ways; but the fact remains. It is usual, therefore, where the cross-section of the material is uniform above and below the neutral axis, to use a constant derived from actual tests of each material, and this constant (which should always be applied to the "extreme fibres," *i. e.*, those along upper or lower edge) is called the modulus of rupture, and is usually expressed in pounds per square inch.

TO FIND THE MOMENT OF INERTIA OF ANY CROSS-SECTION.

How to find moment of inertia of any cross-section.

Divide the cross-section into simple parts, and find the moment of inertia of each simple part around its own neutral axis (parallel to main neutral axis); then, if we call the moment of inertia of the whole cross-section i, and that of each part $i_{\prime}$, $i_{\prime\prime}$, $i_{\prime\prime\prime}$, $i_{\prime\prime\prime\prime}$, etc., and, further, if we call the area of each part $a_{\prime}$, $a_{\prime\prime}$, $a_{\prime\prime\prime}$, $a_{\prime\prime\prime\prime}$, etc., and the distance of the centre of gravity of each part from the main neutral axis of the whole cross-section, $d_{\prime}$, $d_{\prime\prime}$, $d_{\prime\prime\prime}$, $d_{\prime\prime\prime\prime}$, etc., we have:—

$$i=(d_{\prime}^2a_{\prime}+i_{\prime})+(d_{\prime\prime}^2a_{\prime\prime}+i_{\prime\prime})+(d_{\prime\prime\prime}^2a_{\prime\prime\prime}+i_{\prime\prime\prime})+(d_{\prime\prime\prime\prime}^2a_{\prime\prime\prime\prime}+i_{\prime\prime\prime\prime})+, \text{ etc.}$$

Referring back to Figure 1, we should have for Part I:—

$$i_{\prime}=\frac{11}{14}\,\mathrm{r}_{\prime}^4. \quad \text{(See Table I., column 8.)}$$

For Part II we should have:—

$$i_{\prime\prime}=\frac{b_{\prime\prime}h_{\prime\prime}^3}{12}$$

And for Part III:—

$$i_{\prime\prime\prime}=\frac{b_{\prime\prime\prime}h_{\prime\prime\prime}^3}{12}$$

For the distances of individual centres of gravity from main centre of gravity we should have for Part I: $d_{\prime}-d$.

For Part II: $d_{\prime\prime}-d$.

And for Part III: $d-d_{\prime\prime\prime}$.

Therefore the moment of inertia, i, of the whole deck-beam would be:—

$$i = \left\{ (d_{\text{I}} - d)^2 . a_{\text{I}} + \frac{11}{14} \mathfrak{r}_{\text{I}}^4 \right\} + \left\{ (d_{\text{II}} - d)^2 . a_{\text{II}} + \frac{b_{\text{II}} h_{\text{II}}^3}{12} \right\} +$$
$$\left\{ (d - d_{\text{III}})^2 . a_{\text{III}} + \frac{b_{\text{III}} h_{\text{III}}^3}{12} \right\}$$

But $a_{\text{I}} = \frac{22}{7} \mathfrak{r}_{\text{I}}^2$

Further, $a_{\text{II}} = b_{\text{II}}\, h_{\text{II}}$,

And $a_{\text{III}} = b_{\text{III}}\, h_{\text{III}}$, which, inserted above, gives for

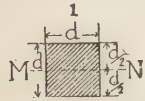
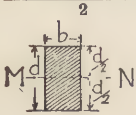
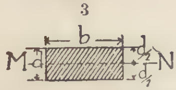
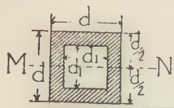
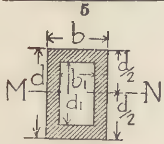
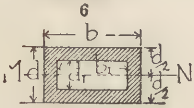
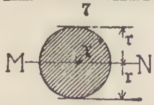
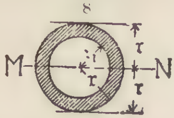
$$i = \frac{\mathfrak{r}_{\text{I}}^2}{14} \left\{ 11 \mathfrak{r}_{\text{I}}^2 + 44 (d_{\text{I}} - d)^2 \right\} + \frac{b_{\text{II}} h_{\text{II}}}{12} \left\{ h_{\text{II}}^2 + 12 (d_{\text{II}} - d)^2 \right\} +$$
$$\frac{b_{\text{III}} h_{\text{III}}}{12} \left\{ h_{\text{III}}^2 + 12 (d - d_{\text{III}})^2 \right\}$$

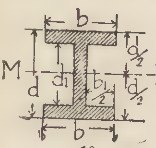
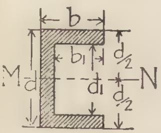
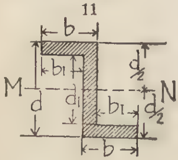
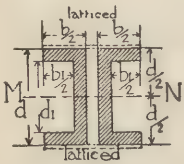
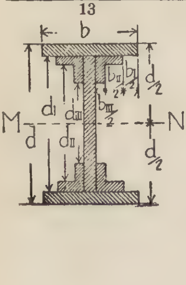
The following table (I) gives the values for the moment of inertia (i), moment of resistance (r), area (a), square of radius of gyration (ϱ^2), etc., for nearly every cross-section likely to be used in building. Those not given can be found from Table I by dividing the cross-section into several simpler parts, for which examples can be found in the table. Note, that it makes a great difference whether the neutral axis is located through the centre of gravity (of the part), or elsewhere. When making calculations we must, of course, insert in the different formulæ in place of i, r, a, ϱ^2, their values (for the respective cross-section), as given in Table I.

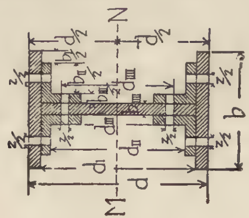
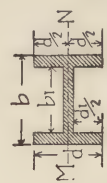
Note.—Throughout this work the writer uses the foreign punctuation in formulæ, viz: a period or point denotes the sign of multiplication; while a comma in connection with numerals denotes the decimal sign.

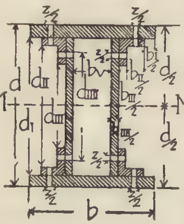
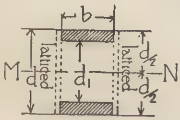
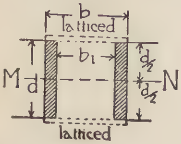
TABLE I.

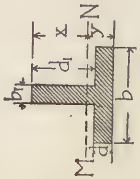
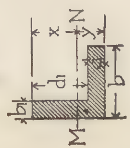
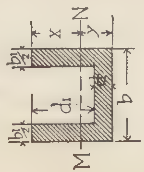
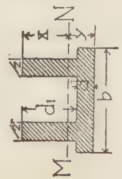
DISTANCE OF EXTREME FIBRES, MOMENTS OF INERTIA AND RESISTANCE, SQUARE OF RADIUS OF GYRATION, AND AREAS OF DIFFERENT SHAPES OF CROSS-SECTIONS.

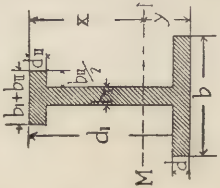
Number and Form of Section.	Distance of Neutral axis M......N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r	Area. a.	Square of Radius of Gyration. ρ^2
1 	$\frac{d}{2}$	$\frac{d^4}{12}$	$\frac{d^3}{6}$	d^2	$\frac{d^2}{12}$
2  3 	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	bd	$\frac{d^2}{12}$
4 	$\frac{d}{2}$	$\frac{d^4-d_1^4}{12}$	$\frac{d^4-d_1^4}{6d}$	$d^2-d_1^2$	$\frac{d^2+d_1^2}{12}$
5  6 	$\frac{d}{2}$	$\frac{bd^3-b_1d_1^3}{12}$	$\frac{bd^3-b_1d_1^3}{6d}$	$bd-b_1d_1$	$\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}$
7 	r	$\frac{11}{14}r^4$	$\frac{11}{14}r^3$	$\frac{22}{7}r^2$	$\frac{r^2}{4}$
8 	r	$\frac{11}{14}(r^4-r_1^4)$	$\frac{11}{14}\frac{(r^4-r_1^4)}{r}$	$\frac{22}{7}(r^2-r_1^2)$	$\frac{r^2+r_1^2}{4}$

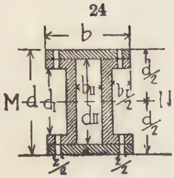
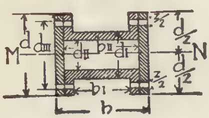
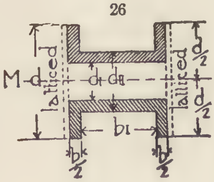
Number and Form of Section.	Distance of Neutral axis M......N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
9  10  11  12 	$\frac{d}{2}$	$\frac{bd^3-b_1d_1^3}{12}$	$\frac{bd^3-b_1d_1^3}{6d}$	$bd-b_1d_1$	$\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}$
13 	$\frac{d}{2}$	$\frac{1}{12}(bd^3-b_1d_1^3-b_{11}d_{11}^3-b_{111}d_{111}^3)$	$\frac{bd^3-b_1d_1^3-b_{11}d_{11}^3-b_{111}d_{111}^3}{6d}$	$bd-b_1d_1-b_{11}d_{11}-b_{111}d_{111}$	$\frac{1}{12}\cdot\frac{bd^3-b_1d_1^3-b_{11}d_{11}^3-b_{111}d_{111}^3}{bd-b_1d_1-b_{11}d_{11}-b_{111}d_{111}}$

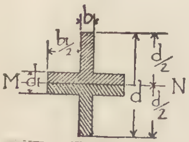
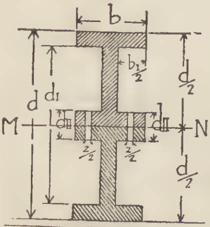
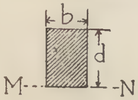
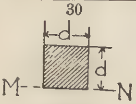
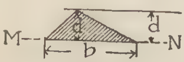
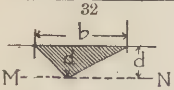
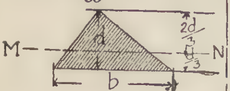
Number and Form of Section.	14 	15 
Distance of Neutral axis M......N from extreme fibres.	$\frac{d}{2}$	$\frac{d}{2}$
Moment of Inertia i.	$\frac{1}{12}\left\{(b-z)d^3-b_Id_I^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{IIII})(d_{IIII}^3-(d_{IIII}-z)^3)\right\}$	$\frac{d^3(b-b_I)+b_I(d-d_I)^3}{12}$
Moment of Resistance. r.	$\frac{(b-z)d^3-b_Id_I^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{IIII}).(d_{IIII}^3-(d_{IIII}-z)^3)}{6d}$	$\frac{d^3(b-b_I)+b_I(d-d_I)^3}{6d}$
Area. a.	$bd-b_Id_I-b_{II}d_{II}-b_{III}d_{III}-z(d-d_{II})-z(b_{III}+b_{IIII})$	$d(b-b_I)+b_I(d-d_I)$
Square of Radius of Gyration. ρ^2	$\frac{(b-z)d^3-b_Id_I^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{IIII})(d_{IIII}^3-(d_{IIII}-z)^3)}{12\left\{bd-b_Id_I-b_{II}d_{II}-b_{III}d_{III}-z(d-d_{II})-z(b_{III}+b_{IIII})\right\}}$	$\frac{d^3(b-b_I)+b_I(d-d_I)^3}{12\left\{d(b-b_I)+b_I(d-d_I)\right\}}$

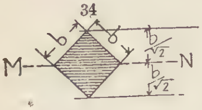
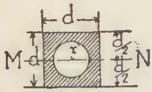
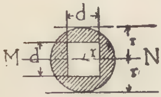
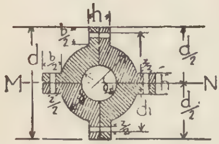
Number and Form of Section.	Distance of Neutral axis M......N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
16 	$\frac{d}{2}$	$\frac{1}{12}\left\{(b-z)d^3-(b_I+b_v)d_I^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{IIII})(\overline{d_{IIII}^3-(d_{IIII}-z)^3})\right\}$	$\frac{(b-z)d^3-(b_I+b_v)d_I^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{IIII})(\overline{d_{IIII}^3-(d_{IIII}-z)^3})}{6d}$	$bd-(b_I+b_v)d_I-b_{II}d_{II}-b_{III}d_{III}-z(d-d_{II})-z(b_{III}+b_{IIII})$	$\frac{(b-z)d^3-(b_I+b_v)d_I^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{IIII}).(\overline{d_{IIII}^3-(d_{IIII}-z)^3})}{12\left\{bd-(b_I+b_v)d_I-b_{II}d_{II}-b_{III}d_{III}-z(d-d_{II})-z(b_{III}+b_{IIII})\right\}}$
17 	$\frac{d}{2}$	$\frac{b}{12}(d^3-d_I^3)$	$\frac{b}{6d}(d^3-d_I^3)$	$b(d-d_I)$	$\frac{d^3-d_I^3}{12(d-d_I)}$
18 	$\frac{d}{2}$	$\frac{d^3}{12}(b-b_I)$	$\frac{d^2}{6}(b-b_I)$	$d(b-b_I)$	$\frac{d^2}{12}$

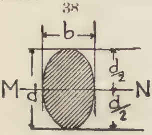
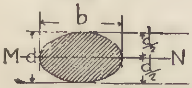
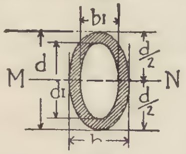
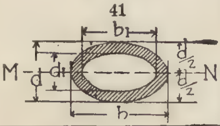
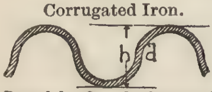
Square of Radius of Gyration. ρ^2	$\dfrac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3(bd+b_1d_1)}$		
Area. a.	$bd+b_1d_1$		
Moment of Resistance. r.	Lower Fibres. $\dfrac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3y}$	Upper Fibres. $\dfrac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3x}$	
Moment of Inertia i.	$\dfrac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3}$		
Distance of Neutral axis M......N from extreme fibres.	Lower Fibres $y=\dfrac{\frac{bd^2}{2}+b_1d_1\left(d+\frac{d_1}{2}\right)}{bd+b_1d_1}$	Upper Fibers. $x=\dfrac{\frac{b_1d_1^2}{2}+bd(d_1+\frac{d}{2})}{bd+b_1d_1}$	$x+y$ should be $=d+d_1$ and $x:y=c:t$, where c = ultimate resistance to compression, t = ultimate resistance to tension, per square inch.
Number and Form of Section.	19  20 	21 	22 

Square of Radius of Gyration. ρ^2	$\dfrac{b_1x^3+b_{11}\left\{x^3-(x-d_{11})^3\right\}+by^3-(b-b_1).(y-d)^3}{3(bd+b_1d_1+b_{11}d_{11})}$
Area. a.	$bd+b_1d_1+b_{11}d_{11}$
Moment of Resistance. r.	Lower Fibres. $\dfrac{b_1x^3+b_{11}\left\{x^3-(x-d_{11})^3\right\}+by^3-(b-b_1).(y-d)^3}{3y}$ Upper Fibres. $\dfrac{b_1x^3+b_{11}\left\{x^3-(x-d_{11})^3\right\}+by^3-(b-b_1).(y-d)^3}{3x}$
Moment of Inertia i.	$\dfrac{b_1x^3+b_{11}\left\{x^3-(x-d_{11})^3\right\}+by^3-(b-b_1).(y-d)^3}{3}$
Distance of Neutral axis M......N from extreme fibres.	Lower Fibres. $y=\dfrac{\frac{bd^2}{2}+b_1d_1(d+\frac{d_1}{2})+b_{11}d_{11}(d+d_1-\frac{d_{11}}{2})}{bd+b_1d_1+b_{11}d_{11}}$ Upper Fibres. $x=\dfrac{\frac{b_{11}d_{11}^2}{2}+b_1\frac{d_1^2}{2}+bd(d_1+\frac{d}{2})}{bd+b_1d_1+b_{11}d_{11}}$ where $x+y=d+d_1$ and $x:y=c:t$.
Number and Form of Section.	23 

Number and Form of Section.	Distance of Neutral axis M......N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
24 	$\frac{d}{2}$	$\frac{(b-z)d^3-(b_I-z)d_I^3-b_{II}d_{II}^3}{12}$	$\frac{(b-z)d^3-(b_I-z)d_I^3-b_{II}d_{II}^3}{6d}$	$(b-z)d-(b_I-z)d_I-b_{II}d_{II}$	$\frac{(b-z)d^3-(b_I-z)d_I^3-b_{II}d_{II}^3}{12\left\{(b-z)d-(b_I-z)d_I-b_{II}d_{II}\right\}}$
25 	$\frac{d}{2}$	$\frac{(b-b_I)\left\{d^3-d_{III}^3+(d_{III}-z)^3\right\}+b_Id_I^3-b_{II}d_{II}^3}{12}$	$\frac{(b-b_I)\left\{d^3-d_{III}^3+(d_{III}-z)^3\right\}+b_Id_I^3-b_{II}d_{II}^3}{6d}$	$(b-b_I)(d-z)+b_Id_I-b_{II}d_{II}$	$\frac{(b-b_I)\left\{d^3-d_{III}^3+(d_{III}-z)^3\right\}+b_Id_I^3-b_{II}d_{II}^3}{12(b-b_I)(d-z)+b_Id_I-b_{II}d_{II}}$
26 	$\frac{d}{2}$	$\frac{b(d^3-d_I^3)+b_I(d_{II}^3-d_I^3)}{12}$	$\frac{b(d^3-d_I^3)+b_I(d_{II}^3-d_I^3)}{6d}$	$b(d-d_I)+b_I(d_{II}-d_I)$	$\frac{b(d^3-d_I^3)+b_I(d_{II}^3-d_I^3)}{12\left\{b(d-d_I)+b_I(d_{II}-d_I)\right\}}$

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
27 	$\frac{d}{2}$	$\frac{bd^3+b_1d_1^3}{12}$	$\frac{bd^3+b_1d_1^3}{6d}$	$bd+b_1d_1$	$\frac{bd^3+b_1d_1^3}{12(bd+b_1d_1)}$
28 	$\frac{d}{2}$	$\frac{bd^3-b_1(d_1^3-d_{11}^3)-zd_{11}^3}{12}$	$\frac{bd^3-b_1(d_1^3-d_{11}^3)-zd_{11}^3}{6d}$	$bd-b_1(d_1-d_{11})-zd_{11}$	$\frac{bd^3-b_1(d_1^3-d_{11}^3)-zd_{11}^3}{12(bd-b_1(d_1-d_{11})-zd_{11})}$
29 	d	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	bd	$\frac{d^2}{3}$
30 	d	$\frac{d^4}{3}$	$\frac{d^3}{3}$	d^2	$\frac{d^2}{3}$
31 	d	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{bd}{2}$	$\frac{d^2}{6}$
32 	d	$\frac{bd^3}{4}$	$\frac{bd^2}{4}$	$\frac{bd}{2}$	$\frac{d^2}{2}$
33 	Lower Fibres. $\frac{d}{3}$ Upper Fibres. $\frac{2d}{3}$	$\frac{bd^3}{36}$	Lower Fibres. $\frac{bd^2}{12}$ Upper Fibres. $\frac{bd^2}{24}$	$\frac{bd}{2}$	$\frac{d^2}{18}$

Number and Form of Section.	Distance of Neutral axis M......N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
34 	$\frac{b}{\sqrt{2}}=\frac{b}{1.4142}$	$\frac{b^4}{12}$	$0.1179\ b^3$	b^2	$\frac{b^2}{12}$
35 	$\frac{d}{2}$	$\frac{d^4}{12}-\frac{11}{14}r^4$	$\frac{d^3}{6}-\frac{11}{7d}r^4$	$d^2-\frac{22}{7}r^2$	$\frac{7d^4-66r^4}{12(7d^2-22r^2)}$
36 	r	$\frac{11}{14}r^4-\frac{d^4}{12}$	$\frac{11}{14}r^3-\frac{d^4}{12r}$	$\frac{22}{7}r^2-d^2$	$\frac{66r^4-7d^4}{12(22r^2-7d^2)}$
37 	$\frac{d}{2}$	$\frac{\vartheta^4-\vartheta_1^4}{20}+\frac{h(bh^2+d^3-\vartheta^3)}{12}-\frac{h\left\{d_1^3-(d_1-z)^3-zh^2\right\}}{12}$	$\frac{\vartheta^4-\vartheta_1^4}{10d}+\frac{h(bh^2+d^3-\vartheta^3)}{6d}-\frac{h\left\{d_1^3-(d_1-z)^3-zh^2\right\}}{6d}$	$\frac{11}{14}(\vartheta^2-\vartheta_1^2)+2h(b-z)$	$\frac{7}{30}\left\{\frac{3(\vartheta^4-\vartheta_1^4)+5h(bh^2+d^3-\vartheta^3)-5h\left\{d_1^3-(d_1-z)^3-zh^2\right\}}{11(\vartheta^2-\vartheta_1^2)+28h(b-z)}\right\}$

Number and Form of Section.	Distance of Neutral axis M - - - N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
38  39 	$\frac{d}{2}$	$\frac{11}{224}bd^3$	$\frac{11}{112}bd^2$	$\frac{11}{14}bd$	$\frac{d^2}{16}$
40  41 	$\frac{d}{2}$	$\frac{11}{224}(bd^3-b_1d_1^3)$	$\frac{11}{112d}(bd^3-b_1d_1^3)$	$\frac{11}{14}(bd-b_1d_1)$	$\frac{bd^3-b_1d_1^3}{16(bd-b_1d_1)}$
42 Corrugated Iron.  Breadth of each sheet $= b$.			$\frac{4}{15}dbh$		

CALCULATION OF STRAINS AND STRESSES.

As we have already noticed, the stress should exceed the strain as many times as the adopted factor-of-safety, or :—

$$\frac{\text{Stress}}{\text{Strain}} = \text{factor-of-safety.}$$

Or, stress = strain $\times$ factor-of-safety.

This holds good for all calculations, and can be expressed by the following simple fundamental formula :—

Fundamental Formula.

$$v = s.f \qquad (1)$$

Where v = the ultimate stress in pounds.

" s = " strain in pounds.

And where f = the factor-of-safety.

COMPRESSION.

Compression, short columns. In pieces under compression the load is *directly* applied to the material. In short pieces, therefore, which cannot give sideways, the strain will just equal the load, or we have: —

$$s = w.$$

Where s = the strain in pounds.

And where w = the load in pounds.

The stress will be equal to the area of cross-section of the piece being compressed, multiplied by the amount of resistance to compression its fibres are capable of.[1] This amount of resistance to compression which its fibres are capable of is found by tests, and is given for each square inch cross-section of a material. A table of constants for the resistance to crushing of different materials will be given later on. (See table IV.)

In all the formulæ these constants are represented by the letter c.

We have, then, for the stress of short pieces under compression:—

$$v = a.c$$

Where v is the ultimate stress in pounds.

Where a is the area of cross-section of the piece in inches.

And where c is the ultimate resistance to compression in pounds per square inch.

Inserting this value for v, and w for s in the fundamental formula (1), we have for short pieces under compression, which cannot yield sideways: —

$$a.\,c = w.f, \text{ or:} —$$

$$w = a.\left(\frac{c}{f}\right). \qquad (2)$$

Short Columns. Where w = the *safe* total load in pounds.

Where a = the area of cross-section in inches.

And where $\left(\frac{c}{f}\right)$ = the *safe* resistance to crushing per square inch.

Example.

What is the safe load which the granite cap of a 12″ x 12″ pier will carry, the cap being twelve inches thick?

The cap being only twelve inches high, and as wide and broad as

---

[1] This is not theoretically correct, as there is in every case a tendency for the material under compression to spread; but it is near enough for all practical purposes.

high, is evidently a short piece under compression, therefore the above formula (2) applies.

The area is, of course: $a = 12.12 = 144$ square inches.

The ultimate resistance of granite to crushing per square inch is, say, fifteen thousand pounds, and using a factor-of-safety of ten, we have for the safe resistance: —

$$\frac{c}{f} = \frac{15000}{10} = 1500 \text{ lbs.}$$

Therefore the safe load w on the block would be: —

$$w = 144.\ 1500 = 216000 \text{ pounds.}$$

Compression, Long Columns.

Where long pieces (pillars) are under compression, and are not secured against yielding sideways, it is evident they would be liable to bend before breaking. To ascertain the *exact* strain in such pieces is probably one of the most difficult calculations in strains, and in consequence many authors have advanced different theories and formulæ. The writer has always preferred to use Rankine's formula, as in his opinion it is the most reliable. According to this, the greatest strain would be at the centre of the length of the pillar, and would be equal to the load, plus an amount equal to the load multiplied by the square of the length in inches, and again multiplied by a certain constant, n, the whole divided by the "square of the radius of gyration" of the cross-section of the pillar. We have therefore for the total strain at the centre of long pillars: —

$$s = w + \frac{w.l^2 n}{\rho^2}$$

Where s = the strain in pounds.
" w = the total load in pounds.
" l = the length in inches.
" ρ^2 = the square of the radius of gyration of the cross-section.
" n = a constant, as follows: —

TABLE II.

VALUE OF n IN FORMULA FOR COMPRESSION.

Material of pillar.	Both ends of pillar smooth (turned or planed.)	One end smooth (turned or planed) other end a pin end.	Both ends pin ends.
Cast-iron	0.0003	0.0004	0.00057
Wrought-iron	0.000025	0.000033	0.00005
Steel	0.00002	0.000025	0.000033
Wood	0.00033	0.00044	0.00067
Stone	0.002		
Brick	0.0033		

The stress of course will be as before: —

$$v = a.\, c.$$

Where $v =$ the ultimate stress in pounds.

" $a =$ the area of cross-section in inches.

" $c =$ the ultimate resistance to crushing per square inch.

Inserting the values for strain, s, and stress, v, in the fundamental formula (1) we have: —

$$c.a = \left(w + \frac{w.l^2.n}{\rho^2}\right).f$$

or: —

$$a.\left(\frac{c}{f}\right) = w\left(1 + \frac{l^2 n}{\rho^2}\right)$$

or: —

Long Columns.

$$w = \frac{a.\left(\frac{c}{f}\right)}{1 + \frac{l^2.n}{\rho^2}} \qquad (8)$$

Where $w =$ the *safe* total load on the pillar.

" $a =$ the area of cross-section in inches.

" $\rho^2 =$ the square of the radius of gyration of the cross-section.

" $l =$ the length in inches.

" $\frac{c}{f} =$ the *safe* resistance to crushing per square inch.

" $n =$ a constant as given in Table II.

Example.

What safe load will a 12″ x 12″ brick pier carry, if the pier is ten feet long, and of good masonry?

The area of cross-section will be: —

$$a = 12.12 = 144 \text{ square inches.}$$

The square of the radius of gyration according to Section No. 1 in Table I would be: —

$$\frac{d^2}{12}, \text{ and as } d = 12, \text{ we have } \rho^2 = \frac{12.12}{12} = 12$$

For the safe resistance to crushing per square inch, we have, using a factor-of-safety of ten, and considering the ultimate resistance to be 2,000 pounds per square inch,

$$\left(\frac{c}{f}\right) = \frac{2000}{10} = 200 \text{ lbs.}$$

The length will be ten feet, or one hundred and twenty inches; therefore: —

$$l^2 = 14400$$

For n we must use (according to Table II), for brickwork: —

$$n = 0{,}0033;$$

Therefore the safe total load on the pier would be:—

$$w = \frac{144.\ 200}{1 + \frac{14400.\ 0{,}0033}{12}} = \frac{28800}{1+3{,}96} = 5806 \text{ lbs.}$$

In all formulæ where constants and factors of safety are used, it will be found simpler and avoiding confusion to immediately reduce the constant by dividing it by the factor-of-safety, and then using only the reduced or *safe* constant.

Thus if $c = 48000$ pounds, and if $f = 4$, do not write into your formula for $\left(\frac{c}{f}\right) = \frac{48000}{4}$, but use at once for $\left(\frac{c}{f}\right) = 12000$.

Materials in compression that have an even bearing on all parts of the bed will stand very much more compression to the square inch than materials with rough, uneven or rounded beds, or where the bearing is on part of the cross-section only, as in the case of pins (in trusses) bearing on eye-bars. It is usual in calculating to make allowance for this. Columns with perfectly even bearing on all parts of the bed (planed or turned iron or dressed stone) will stand the largest amount of compression. Columns with rough, rounded or uneven ends are calculated the same as for pin-ends of eye-bars. In the table (II) giving the values for n of Rankine's formula for compression, the different values for smooth and also for pin ends are given.

WRINKLING STRAINS.

Thin pieces of wrought-iron under compression endwise may neither crush nor deflect (bend), but give way by ***wrinkling,*** that is, buckling or corrugating, provided there are no stiffening-ribs lengthwise.

Fig. 2.

Thus a square, tubular column, if the sides are very thin might give way, as shown in Figure 2, which is called wrinkling. Or, in a similar way, the top plate of a boxed girder, if very thin, might wrinkle, as shown in Figure 3, under heavy compressive strains. To calculate this strain use the following formula:

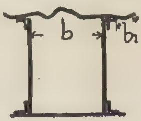


Fig. 3.

$$b = d. \left(\frac{w_r}{w}\right)^2 \qquad (4)$$

Where $w =$ the amount of ultimate compression in pounds per square inch, which will wrinkle the material.

$w_r =$ a constant, as given in Table III,

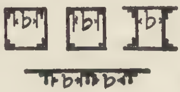
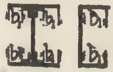
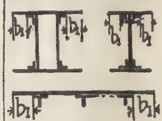
d = the thickness of plate in inches,

b = the unstiffened breadth of plate in inches.

If a plate has stiffening ribs along both edges, use for b the actual breadth between the stiffening ribs; if the plate is stiffened along one edge only, use $4b_1$ in place of b. Thus, in the case of the boxed girder, Figure 3, if we were considering the part of top plate between the webs, we should use for b in the formula, the actual breadth of b in inches; while, if we were considering the overhanging part b_1 of top plate, we should use $4b_1$ in place of b in formula. For rectangular columns use 160,000 pounds for w_r; for tubular beams, top plates of girders, and single plates use 200,000 pounds for w_r. With a factor-of-safety of 3, we should have $\frac{160000}{3} = 53000$ pounds for rectangular columns, and $\frac{200000}{3} = 66000$ pounds for tubular beams, top plates of riveted girders and single plates.

For w we shall use, of course, $\frac{36000}{3} = 12000$ pounds, which is the safe allowable compressive strain. This would give the following table for safe unstiffened breadth of wrought-iron plates, to prevent wrinkling of plates.

TABLE III.

Thickness of Plate in inches.	Safe breadth in inches of Plate stiffened along both edges. (use b.)		Safe breadth in inches of Plate stiffened along one edge only. (use $4b_1$)	
	Rectangular Columns.	Tubular Beams, riveted Girders, and single Plates.	Rectangular Columns.	Riveted Girders and single Plates.
				
$\frac{1}{8}$	$2\frac{7}{16}$	$3\frac{3}{4}$	$\frac{5}{8}$	$\frac{15}{16}$
$\frac{1}{4}$	$4\frac{7}{8}$	$7\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{7}{8}$
$\frac{3}{8}$	$7\frac{5}{16}$	$11\frac{3}{8}$	$1\frac{7}{8}$	$2\frac{13}{16}$
$\frac{1}{2}$	$9\frac{3}{4}$	$15\frac{1}{8}$	$2\frac{7}{16}$	$3\frac{3}{4}$
$\frac{5}{8}$	$12\frac{3}{16}$	$18\frac{7}{8}$	3	$4\frac{11}{16}$
$\frac{3}{4}$	$14\frac{5}{8}$	$22\frac{11}{16}$	$3\frac{11}{16}$	$5\frac{5}{8}$
$\frac{7}{8}$	$17\frac{1}{16}$	$26\frac{1}{2}$	$4\frac{1}{4}$	$6\frac{9}{16}$
1	$19\frac{1}{2}$	$30\frac{1}{4}$	$4\frac{7}{8}$	$7\frac{5}{16}$
$1\frac{1}{4}$	$24\frac{3}{8}$	$37\frac{13}{16}$	$6\frac{1}{8}$	$9\frac{7}{16}$
$1\frac{1}{2}$	$29\frac{1}{4}$	$45\frac{3}{8}$	$7\frac{5}{16}$	$11\frac{5}{16}$
$1\frac{3}{4}$	$34\frac{1}{8}$	$52\frac{15}{16}$	$8\frac{9}{16}$	$13\frac{3}{16}$
2	39	$60\frac{1}{2}$	$9\frac{3}{4}$	$15\frac{1}{8}$

The above table will cover every case likely to arise in buildings.

Two facts should be noticed in connection with wrinkling:

1. That the length of plate does not in any way affect the resistance to wrinkling, which is dependent only on the breadth and thickness of the part of plate unstiffened, and

2. That the resistance of plates to wrinkling being dependent on their breadth and thickness only, to obtain equal resistance to wrinkling at all points (in rectangular columns with uneven sides), the thickness of each side should be in proportion to its breadth.

Thus, if we have a rectangular column $30'' \times 15''$ in cross section and the $30''$ side is $1''$ thick, we should make the $15''$ side but $\frac{1}{2}''$ thick, for as $30'' : 1'' :: 15'' : \frac{1}{2}''$.

Of course, we must also calculate the column for direct crushing and flexure, and in the case of beams for rupture and deflection, as well as for wrinkling.

Example of Wrinkling.

It is desired to make the top plate of a boxed girder as wide as possible, the top flange is to be $1\frac{1}{4}''$ *thick, and is to be subjected to the full amount of the safe compressive strain, viz:* 12,000 *pounds per square inch; how wide apart should the webs be placed, and how much can the plate overhang the angles without danger of wrinkling? Each web to be* $\frac{1}{2}''$ *thick, and the angles* $4'' \times 4''$ *each?*

For the distance between webs we use b in Formula (4).

$$b = 1\tfrac{1}{4} \cdot \left(\frac{66000}{12000}\right)^2 = 1\tfrac{1}{4} \cdot 5\tfrac{1}{2}^2 = 37\tfrac{13}{16}'',$$

which is the safe width *between* webs to avoid wrinkling

For the overhanging part of top plate we must use $4b_1$ in place of b in Formula (4).

$$4b_1 = 1\tfrac{1}{4}\left(\frac{66000}{12000}\right)^2 = 37\tfrac{13}{16}, \text{ therefore,}$$

$$b_1 = \frac{37\frac{13}{16}}{4} = 9{,}453, \text{ or say, } \quad b_1 = 9\tfrac{7}{16}''.$$

The total width of top plate will be, therefore, including $1''$ for two webs and $8''$ for the two angles, or $9''$, and remembering that there is an overhanging part, b_1, each side,

$$9'' + b + b_1 + b_1$$
$$= 9 + 37\tfrac{13}{16} + 9\tfrac{7}{16} + 9\tfrac{7}{16}$$
$$= 65\tfrac{11}{16}''.$$

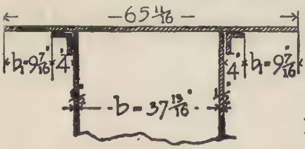


Fig. 4

By referring to Table III, we should have obtained the same result, without the necessity of any calculation.

Figure 4 will make the above still more clear.

LATERAL FLEXURE IN TOP FLANGES OF BEAMS, GIRDERS, OR TRUSSES, DUE TO COMPRESSION.

The usual formulæ for rupture and deflection assume the beam, girder or truss to be supported against possible lateral flexure (bending sideways). Now, if the top chord of a truss or beam is comparatively narrow and not supported sideways, the heavy, compressive strains caused in same may bend it sideways. To calculate this lateral flexure, use the formula given for long columns in compression, but in place of l use only two-thirds of the span of the beam, girder or truss, that is $\frac{2}{3}l$, and for w use one-third of the greatest compressive strain in top chord, which is usually at the centre.

Inserting this in Formula (3) we have:

$$\frac{w}{3}=\frac{a\left(\frac{c}{f}\right)}{1+\frac{4l^2n}{9\rho^2}} \text{ transposing, we have, } w=\frac{3a\left(\frac{c}{f}\right)}{1+\frac{4l^2n}{9\rho^2}} \quad (5)$$

where a the area of the cross-section of the top chord in inches,

ρ^2 is the square of the radius of gyration of the top chord around its *vertical axis;* we must therefore reverse the usual positions of b and d, that is the breadth of top chord, becomes the depth or d, and the depth of top chord becomes the thickness, or b (both in formulæ given in last column of Table I.)

$\frac{w}{3}$ is the greatest allowable compressive strain in pounds at any point to resist lateral flexure safely at that point.

$\left(\frac{c}{f}\right)$ is the safe resistance of the material to compression per square inch in pounds.

l is the total length of span in inches.

n is given in Table II.

Example.

A trussed girder is 60′ *long between bearings, and not supported sideways; the top chord consists of two plates each* 22″ *deep and* 1″ *thick; the plates are* 2″ *apart, as per Figure* 5. *The greatest compressive strain on top chord has previously been ascertained to be on the central panel, and to be* 525000 *pounds. Is there danger of the girder bending sideways?*

The girder is safe against lateral flexure so long as the strain at centre does not exceed $\frac{w}{3}$ in Formula (5).

Now, the area $a = 2.1.22 = 44$.

Using 48000 pounds per square inch for ultimate resistance to compression of wrought-iron, and a factor-of-safety of 4, we have

$$\left(\frac{c}{f}\right) = \frac{48000}{4} = 12000$$

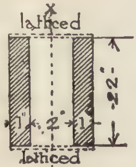


Fig. 5.

The length is 60′, or 720″, therefore

$$l^2 = 518400.$$

From Table II we have

$$n = 0,000025.$$

And from Table I, section Number 18, we have for the above cross-section,

$$\varrho^2 = \frac{d^3 - d_1^3}{12(d-d_1)}$$

As we are considering the section for bending sideways, we must, of course, take the neutral axis x---y vertically, therefore d becomes 4″ and d_1 becomes 2″. This supposes the plates to be stiffly latticed or bolted together, with separators between. We have then

$$\varrho^2 = \frac{4^3 - 2^3}{12(4-2)} = 2\tfrac{1}{3}$$

Then for w we have,

$$w = \frac{3.44.12000}{1 + \frac{4.518400}{9.\ 2\frac{1}{3}}\,0.000025.}$$

$$= \frac{1584000}{1+2,47} = \frac{1584000}{3,47} = 456\ 484 \text{ lbs.}$$

Or, we find that there is danger of the girder bending sideways long before the actual compressive strain of 525000 pounds has been reached. It will, therefore, be necessary to re-design the top chord, so that it will be stiffer sideways. This subject will be more fully treated when considering trusses.

TENSION.

In tension the load is applied *directly* to the material, and it is, therefore, evident that no matter of what shape the material may be, the strain will always be the same. This strain, of course, will be just equal to the load, and we have, therefore:—

$$s = w.$$

Where $s =$ the amount of strain.

Where $w =$ the amount of load.

The weakest point of the piece under tension will, of course, be where it has the smallest area of cross-section; and the stress at

such point will be equal to the area of cross-section, multiplied by the amount of resistance its fibres are capable of.[1]

The amount of resistance to tension the fibres of a material are capable of is found by experiments and tests, and is given for each material per square inch of cross-section. A table of constants for the ultimate and safe resistances to tension of different materials will be given later; in all the formulæ these constants are represented by the letter t.

We have, then, for the stress: —

$$v = a . t$$

Where v = the amount of ultimate stress.

Where a = the area of cross-section.

Where t = the ultimate resistance to tension, per square inch of the material.

Therefore, the fundamental formula (1), viz.: $v = s . f$, becomes for pieces under tension: —

$$a . t = w . f, \text{ or}: -$$

$$w = a . \left(\frac{t}{f}\right) \qquad (6)$$

Where w = the safe load or amount of tension the piece will stand.

Where a = the area of cross-section at the weakest point (in square inches).

Where $\left(\frac{t}{f}\right)$ = the safe resistance to tension per square inch of the material.

Example.

A weight is hung at the lower end of a vertical wrought-iron rod, which is firmly secured at the other end. The rod is 3″ at one end and tapers to 2″ at the other end. How much weight will the rod safely carry?

The smallest cross-section of the rod, where it would be likely to break, would be somewhere very close to the 2″ end, or, say, 2″ in diameter. Its area of cross-section at this point will be: —

$$a = \frac{22}{7} . \frac{2^2}{4} = 3\tfrac{1}{7} \text{ square inches.}$$

The ultimate resistance to tension of wrought-iron per square inch is, from forty-eight thousand pounds to sixty thousand pounds. We do not know the exact quality, and, therefore, take the lower figure;

---

[1] This, again, is not theoretically correct, as a piece under tension is apt to stretch and so reduce the area of its cross-section; but the above is sufficiently correct for all practical purposes.

using a factor-of-safety of four, we have for the safe resistance to tension per square inch:—

$$\left(\frac{t}{f}\right)=\frac{48\,000}{4}=12\,000 \text{ pounds.}$$

Therefore, the safe load will be:—

$$w=3\tfrac{1}{7}.\ \ 12\,000\ \ =37\,714 \text{ pounds.}$$

SHEARING.

In compression the fibres are shortened by squeezing; in tension they are elongated by pulling. In shearing, however, the fibres are not disturbed in their individualities, but slide past each other.

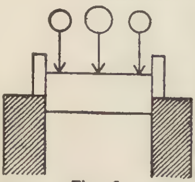


Fig. 6.

When this sliding takes place across the grain of the fibres, the action of shearing is more like cutting across. When this sliding takes place along the grain, the action of shearing is more like splitting. Thus, if a very deep, but thin, beam is of short span and heavily loaded, it might not break transversely, nor deflect excessively, but shear off at the supports, as shown in Figure 6, the action of the loads and supports being like a large cutting-machine, the weights cutting off the central part of beam and forcing it downwards past the support. This would be shearing across the grain.

If the foot of a main rafter is toed-in to the end of a tie-beam, and the foot forces its way outwardly, pushing away the block or part of tie-beam resisting it (splitting it out as it were), this would be shearing along the grain.

In most cases (*except in transverse strains*) the load is directly applied to the point being sheared off; the strain will, therefore, just equal the load, and we have:—

$$s=w.$$

Where $s=$ the amount of the shearing strain.
" $w=$ " " load.

The stress will be equal to the area of cross-section (affected by the shearing strain) multiplied by the amount of resistance to separation from each other that its fibres are capable of.

This amount of resistance is found by tests and experiments, and is given for each material per square inch of cross-section. A table of constants for resistance to shearing of different materials will be given later; in the formulæ these constants are represented by the letter g for shearing across the grain, and g_1 for shearing along the grain.

We have, then, for the stress: —

$$v = a.\ g.$$

Where $v =$ the amount of ultimate stress.

Where $a =$ the area of cross-section in square inches.

Where $g =$ the ultimate resistance to shearing across the grain per square inch.

Therefore, the fundamental formula (1) $v = s.\ f$, becomes for pieces under shearing strains across the grain: —

$$a.\ g = w.\ f, \text{ or: —}$$

$$w = a. \left(\frac{g}{f}\right) \tag{7}$$

And similarly, of course, we shall find: —

$$w = a. \left(\frac{g_1}{f}\right) \tag{8}$$

Where $w =$ the safe-load.

Where $a =$ the area of cross-section in square inches, at the point where there is danger of shearing.

Where $\left(\frac{g}{f}\right) =$ the safe-resistance to shearing *across* the fibres per square inch.

Where $\left(\frac{g_1}{f}\right) =$ the safe-resistance to shearing *along* the fibres per square inch.

Example.

At the lower end of a vertical wrought-iron flat bar is suspended a load of eight thousand pounds. The bar is in two lengths, riveted together with one rivet. What diameter should the rivet be?

The strain on the rivet will, of course, be a shearing strain across the grain, and will be equal to the amount of tension on the bar, which we know is equal to the load. We use Formula (7), and have: —

$$w = 8000 \text{ pounds.}$$

The safe shearing for wrought-iron is about ten thousand pounds per square inch; inserting this in formula, we have: —

$$8000 = a.\ 10000, \text{ or } a. = \frac{8000}{10000} = \tfrac{4}{5}.$$

The area of rivet must, therefore, be four-fifths of a square inch. To obtain diameter, we know that: —

$$d = \sqrt{\tfrac{14}{11}.a} = \sqrt{\tfrac{14}{11}.\tfrac{4}{5}} = \sqrt{\tfrac{56}{55}} = \sqrt{1,01818}$$

This is, practically, equal to one; therefore, the diameter of rivet should be 1″.

In transverse strains the (vertical) cross-shearing is generally not equal to the load, but varies at different points of the beam or cantilever. The manner of calculating transverse strains, however, allows for straining only the edges (extreme fibres) up to the maximum; so that the intermediate fibres, not being so severely tested, generally have a sufficient margin of unstrained strength left to more than offset the shearing strain. In solid beams it can, therefore, as a rule, be overlooked, except at the points of support. (In plate-girders it must be calculated at the different points where weights are applied.) The amount of the shearing at each support is equal to the amount of load coming on or carried by the support.

We must, therefore, substitute for w in Formula (7) either p or q, as the case may be, and have at the left-hand end of beam for the safe resistance to shearing:—

$$p = a.\left(\frac{g}{f}\right) \tag{9}$$

And at the right-hand end of beam:—

$$q = a.\left(\frac{g}{f}\right) \tag{10}$$

Where $p =$ the amount of load, in pounds, carried on the left-hand support.

Where $q =$ the amount of load, in pounds, carried on the right-hand support.

Where $a =$ the area of cross-section, in inches, at the respective support.

Where $\left(\frac{g}{f}\right) =$ the safe resistance, per square inch, to cross-shearing.

Example.

A spruce beam of 5′ clear span is 24″ deep and 3″ wide; how much uniform load will it carry safely to avoid the danger of shearing off at either point of support?

The beam being uniformly loaded, the supports will each carry one-half of the load; if, therefore, we find the safe resistance to shearing at either support, we need only double it to get the safe load (instead of calculating for the other support, too, and adding the results).

Let us take the left-hand support. From Formula (9) we have:—

$$p = a.\left(\frac{g}{f}\right)$$

Now, we know that $a = 24.3 = 72$ square inches.

The ultimate resistance of spruce to cross-shearing is about thirty-six hundred pounds per square inch; using a factor-of-safety of ten, we have for the safe resistance per square inch: —

$$\left(\frac{g}{f}\right) = \frac{3600}{10} = 360 \text{ pounds.}$$

We have, now: —

$$p = 72.360 = 25920 \text{ pounds.}$$

Similarly, we should have found for the right-hand support: —

$$q = 25920 \text{ pounds.} \quad \text{And as:} —$$

$$u = p + q = 51840 \text{ pounds,}$$

that will, of course, be the safe uniform load, so far as danger of shearing is concerned.

The beam must also be calculated for transverse strength, deflection and lateral flexure, before we can consider it entirely safe. These will be taken up later on.

Should it be desired to find the amount of vertical shearing strain x at any point of a beam, other than at the points of support, use: —

$$x = \left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\} - \Sigma w \qquad (11)$$

Where x = the amount of vertical shearing strain, in pounds, at any point of a beam.

Where $\left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\}$ = the reaction, in pounds, (that is, the share of the total loads carried) at the *nearer* support to the point.

Where Σw = the sum of all loads, in pounds, between said *nearer* support and the point.

When x is found, insert it in place of w, in Formula (7), in order to calculate the strength of beam necessary at that point to resist the shearing.

Example.

A spruce beam, 20′ long, and 8″ deep, carries a uniform load of one hundred pounds per running foot. What should be the thickness of beam 5′ from either support, to resist safely vertical shearing?

Each support will carry one-half the total load; that is, one thousand pounds; so that we have for Formula (11): —

$$\left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\} = 1000 \text{ pounds.}$$

The sum of all loads between the nearer support and a point 5′ from support will be: —

$$\Sigma w = 5.\ 100 = 500 \text{ pounds.}$$

Therefore, the amount of shearing at the point 5′ from support will be: —

$$x = 1000 - 500 = 500 \text{ pounds.}$$

Inserting this in Formula (7) we have: —

$$500 = a. \left(\frac{g}{f}\right), \text{ or, } a = \frac{500}{\left(\frac{g}{f}\right)}$$

We have just found that for spruce,

$$\left(\frac{g}{f}\right) = 360 \text{ pounds.}$$

Therefore, $a = \frac{500}{360} = 1{,}39$ square inches.

And, as $b.\, d = a$, or $b = \frac{a}{d}$, we have, $b = \frac{1{,}39}{8} = \frac{1''}{6}$

This is such a small amount that it can be entirely neglected in an 8″ wooden beam.

To find the amount of vertical shearing at any point of a canti-lever, other than at the point where it is built in, use: —

$$x = \Sigma\, w \qquad (12)$$

Where x the amount of vertical shearing strain, in pounds, at any point of canti-lever.

Where $\Sigma\, w$ the sum of all loads between the *free* end and said point.

To find the strength of beam at said point necessary to resist the shearing, insert x for w in Formula (7).

In transverse strains there is also a *horizontal* shearing along the entire neutral axis of the piece. This stands to reason, as the fibres above the neutral axis are in compression, while those below are in tension, and, of course, the result along the neutral line is a tendency of the fibres just above and just below it, to slide past each other or to *shear* off along the grain.

We can calculate the *intensity* (not amount) of this horizontal shearing at any point of the piece under transverse strain.

If x represents the amount of vertical shearing at the point, then the intensity of horizontal shearing at the point is $= \frac{3}{2} \frac{x}{a}$.

If this intensity of shearing does not exceed the safe-constant $\left(\frac{g_1}{f}\right)$ for shearing along the fibres, the piece is safe, or: —

$$\frac{3}{2} \cdot \frac{x}{a} = \left(\frac{g_1}{f}\right) \qquad (13)$$

Where x is found by formulæ (11) or (12) for any point of beam or,

Where $x = \left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\} =$ the amount of supporting force, in pounds, for either point of support.

Where $a =$ the area of cross-section in square inches.

Where $\left(\frac{g_1}{f}\right) =$ the amount of safe resistance, per square inch, to shearing along fibres.

Example.

Take the same beam as before. The amount of vertical shearing 5′ *from support we found to be five hundred pounds, or:* —

$$x = 500.$$

The area was 8″ multiplied by thickness of beam, or: —

$$a = 8\,b.$$

The ultimate shearing along the fibres of spruce is about four hundred pounds per square inch, and with a factor-of-safety of ten, we should have: —

$$\left(\frac{g_1}{f}\right) = \frac{400}{10} = 40.$$

Inserting this in Formula (13) $\frac{3}{2} \cdot \frac{500}{8b} = 40$

or $b = \frac{1500}{16.40} = 2,''34.$

The beam should, therefore, be at least $2\frac{1}{2}''$ thick, to avoid danger of longitudinal shearing at this point. At either point of support the vertical shearing will be equal to the amount supported there; that is, one-half the load, or one thousand pounds. Substituting this for x in Formula (13), we have: —

$$\frac{3}{2} \cdot \frac{1000}{8b} = 40, \text{ or } b = \frac{3000}{16.40} = 4,''68.$$

The beam would, therefore, have to be $4\frac{2}{3}''$ thick at the points of support, to avoid danger of longitudinal shearing. The beam, as it is, is much too shallow for one of such span, a fact we would soon discover, if calculating the transverse strength or deflection of beam, which will be taken up later on. It will also be found that the greater the depth of the beam, the smaller will be the danger from longitudinal shearing, and, consequently, to use thinner beams, it would be necessary to make them deeper.

TABLE IV

STRENGTH OF MATERIALS PER SQUARE INCH.

WOODS.	Compression. Ultimate c	Compression. Safe $\frac{c}{f}$	Tension. Ultimate t	Tension. Safe $\frac{t}{f}$	Shearing. Ultimate g or g_1	Shearing. Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Modulus of Rupture. Ultimate k	Modulus of Rupture. Safe $\frac{k}{f}$	Modulus of Elasticity. e	Weight per Cubic ft.
Acacia	—	—	11000	1100	—	—	7000	1400	1150000	48
Alder	6900	700	9500	1000	—	—	4000	800	800000	38
Apple	—	—	13000	1300	—	—	4500	900	—	50
Ash, English	8000	800	10500	1000	—	—	7800	1300	1200000	48
" American	8000	800	11000	1100	$g_1 = 670$	65	8000	1350	1250000	40
" " across	2060	200	—	—	$g = 6000$	600	—	—	—	—
" Canadian	5000	500	6500	650	$g_1 = 1400$	140	5000	850	1000000	38
Bamboo	—	—	6000	600	—	—	—	—	—	25
Baywood	7200	720	10500	1000	—	—	—	—	—	—
Beech	8000	800	8000	800	$g = 5000$	500	6000	1000	1350000	48
Birch, English	5000	500	10000	1000	—	—	6000	1000	1250000	47
" American	8500	850	10000	1000	$g_1 = 670$	65	5800	950	1300000	43
" " across	1800	180	—	—	$g = 5600$	560	—	—	—	—
Box	10000	1000	15000	1500	—	—	7500	1250	2000000	62
Cedar	6000	600	8000	800	—	—	7000	800	500000	41
" across	1950	200	—	—	$g = 1400$	140	—	—	—	—
Cherry	8000	800	—	—	—	—	4500	900	1200000	40
" across	2400	240	—	—	$g = 2900$	290	—	—	—	—
Chestnut, American	5300	530	8500	850	$g_1 = 700$	70	5400	900	750000	41
" " across	840	84	—	—	$g = 1500$	150	—	—	—	—
" English	—	—	10000	1000	—	—	8000	800	1000000	38
Crab	6800	680	—	—	—	—	4500	750	—	—
Deal	6000	600	8500	850	$g_1 = 625$	60	4800	800	—	30
Ebony	18000	1800	18000	1800	$g = 7500$	750	18000	2000	—	76
Elder	8700	870	10000	1000	—	—	4500	900	—	43
Elm, English	8000	800	8000	800	$g_1 = 1200$	120	5000	800	700000	35
" " across	—	—	—	—	$g = 3000$	300	—	—	—	—
" Canadian	8500	850	9000	900	$g_1 = 1400$	140	9000	900	2000000	47
" " across	—	—	—	—	$g = 5000$	500	—	—	—	—
Fir, Riga	6000	600	7500	750	—	—	7000	700	870000	40
" New England	6700	670	7500	750	$g_1 = 690$	70	4000	800	1000000	40
" " across	1200	120	690	70	—	—	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

WOODS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Greenheart	13000	1300	8000	800	—	—	9500	950	1500000	65
Gum	7000	700	10500	1000	5900	600	6000	900	1400000	62
Hazel	—	—	12000	1200	—	—	—	—	—	—
Hawthorn	—	—	7500	750	—	—	—	—	—	—
Hemlock	4500	450	7000	700	$g_1 = 500$	50	4500	750	800000	26
" across	500	50	550	55	$g = 2750$	275	—	—	—	—
Hickory, American	9500	1000	10000	1000	—	—	8000	1000	1100000	50
" " across	3000	300	—	—	$g = 6500$	650	—	—	—	—
Holly	5250	525	10500	1000	—	—	4500	900	—	—
Hornbeam	6600	660	14000	1400	—	—	6000	1000	750000	47
Jugeb	—	—	12500	1250	—	—	—	—	—	—
Lancewood	—	—	18000	1800	—	—	—	—	—	52
Larch	4500	450	7000	700	$g_1 = 1300$	130	5500	700	1000000	35
" across	1300	130	1300	130	$g = 3000$	300	—	—	—	—
Lignum-vitæ	9900	1000	10000	1000	—	—	8000	1000	1080000	62
Locust, American	10000	1000	12000	1200	$g_1 = 1150$	120	10000	1100	1500000	46
" " across	3000	300	—	—	$g = 7200$	720	—	—	—	—
Mahogany	8200	820	11000	1100	—	—	6000	1000	1200000	43
" across	2400	240	—	—	—	—	—	—	—	—
Maple	6500	650	8000	800	$g_1 = 450$	45	7000	700	877000	45
" across	1875	200	—	—	$g = 6000$	600	—	—	—	—
Oak, American, Red	6000	600	8000	800	$g_1 = 750$	75	6500	800	1200000	52
" " White	7200	720	11000	1100	$g_1 = 800$	80	7200	1100	900000	52
" " " across	2400	240	2300	230	$g = 4400$	440	—	—	—	—
" Black bog	—	—	7700	770	—	—	—	—	—	—
" Canadian	6000	600	7500	750	—	—	7000	750	—	54
" Dantzic	7500	750	6000	600	$g = 2300$	230	6000	800	1165000	47
" English	8300	830	12000	1200	$g_1 = 2316$	230	6800	1100	873000	54
" " across	1800	200	2400	240	$g = 4000$	400	—	—	—	—
" Live	6850	700	11000	1100	$g_1 = 700$	70	7300	1100	—	69
" " across	4500	450	—	—	$g = 8500$	850	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

WOODS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Pear	7500	750	8000	800	—	—	3400	700	800000	42
Pine, American yellow	5400	540	9000	900	—	—	6000	800	1100000	34
" " " across	—	—	—	—	$g = 4800$	500	—	—	—	—
" " red	6500	650	9000	900	$g_1 = 650$	65	6100	800	900000	36
" " white	5500	550	9000	900	$g_1 = 450$	45	4000	900	850000	28
" " " across	700	70	550	60	$g = 2500$	250	—	—	—	—
" Canadian yellow	5300	530	6000	600	$g_1 = 350$	40	4800	600	1000000	33
" " " across.	1425	150	—	—	$g = 4300$	430	—	—	—	—
Pine, Georgia yellow	7400	750	12000	1200	$g_1 = 500$	50	7200	1200	1200000	48
" **Georgia Yel'w across**	1850	200	—	—	$g = 5700$	570	—	—	—	—
" Pitch	8900	900	10000	1000	$g_1 = 510$	50	6600	1000	1225000	65
" " across	—	—	—	—	$g = 5000$	500	—	—	—	—
Plum	9000	900	8000	800	—	—	7000	900	—	45
Poon	—	—	9000	900	—	—	8000	900	1600000	—
Poplar (white wood)	5000	500	5500	550	$g_1 = 400$	40	4500	600	762000	27
" across	1000	100	1800	180	$g = 4400$	440	—	—	—	—
Redwood, California	6000	600	7300	750	—	—	4500	750	—	23
Spruce	6500	650	10000	1000	$g_1 = 400$	40	4400	1000	850000	30
" **across**	750	75	550	60	$g = 3600$	360	—	—	—	—
Sycamore	8100	810	10000	1000	—	—	6000	1000	1000000	40
Teak	12000	1200	14000	1400	—	—	8000	1300	1400000	46
Walnut	7000	700	8000	800	—	—	5500	800	900000	39
" across	2100	210	—	—	$g = 4000$	400	—	—	—	39
Willow	6000	600	8500	850	—	—	4500	800	566000	25
Yew	—	—	6000	600	—	—	7000	700	—	50

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

METALS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Aluminium bronze..........	130000	25000	73000	12000	—	—	—	—	—	—
Antimony, cast..............	—	—	1100	200	—	—	—	—	—	418
Bismuth....................	—	—	3200	500	—	—	—	—	—	614
Brass, yellow, cast..........	80000	8000	28000	4500	37000	6000	20000	5000	9000000	506
" wire, unannealed.......	—	—	80000	8000	—	—	—	—	14000000	530
" " annealed.........	—	—	50000	5000	—	—	—	—	—	—
Bronze, gun-metal............	120000	25000	32000	8000	—	—	53000	12500	10000000	529
" wire, unannealed.....	—	—	150000	30000	—	—	—	—	10000000	520
" " annealed........	—	—	63000	13000	—	—	—	—	—	—
Copper, pure cast............	117000	25000	30000	10000	33000	11000	22000	7000	10000000	543
" cast..................	—	—	25000	8000	—	—	—	—	—	—
" sheet wrought.........	103000	21000	34000	11000	—	—	—	—	15000000	556
" wire, unannealed.....	—	—	63000	20000	—	—	—	—	18000000	—
" " annealed........	—	—	36000	12000	—	—	—	—	—	—
" bolts..................	—	—	32000	11000	—	—	—	—	—	556
Gold, cast.....................	—	—	20000	5000	—	—	—	—	—	1204
" wire, unannealed........	—	—	30000	7500	—	—	—	—	—	1217
" " annealed..........	—	—	5000	1250	—	—	—	—	—	—
Iron, cast, Am., 1″ thick...	85000	15000	16500	2500	18000	2500	40000	5000	15000000	454
" " English...........	70000	13000	13000	2000	14000	2000	36000	4000	12000000	450
" **wrought, American**..	47000	12000	50000	12000	40000	8000	43000	12000	27000000	483
" " English........	35000	9000	45000	11000	36000	7000	34000	10000	26000000	480
" corrugated..............	—	—	—	—	—	—	40000	10000	—	—
" wire ropes, unannealed..	—	—	80000	20000	—	—	—	—	25300000	480
" " " annealed.....	—	—	50000	12000	—	—	—	—	15000000	—
" chains..................	—	—	35000	9000	—	—	—	—	—	—
" welded with steel, 5 ply..	—	—	55500	14000	—	—	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

METALS.	Compression. Ultimate c	Compression. Safe $\frac{c}{f}$	Tension. Ultimate t	Tension. Safe $\frac{t}{f}$	Shearing. Ultimate g or g_1	Shearing. Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Modulus of Rupture. Ultimate k	Modulus of Rupture. Safe $\frac{k}{f}$	Modulus of Elasticity. e	Weight per Cubic ft.
Lead, sheet	—	—	2500	500	—	—	—	—	—	—
" cast	7700	1200	2000	400	—	—	—	—	720000	712
" drawn pipe	—	—	1700	350	—	—	—	—	—	709
" wire	—	—	1400	300	—	—	—	—	—	—
Mercury at 32°	—	—	—	—	—	—	—	—	—	851
Nickel	—	—	—	—	—	—	—	—	—	488
Palladium, wire	—	—	50000	10000	—	—	—	—	—	—
" drawn	—	—	26000	5200	—	—	—	—	—	—
" annealed	—	—	7100	1500	—	—	—	—	—	—
Pewter	—	—	—	—	—	—	—	—	—	453
Platinum, wire, unannealed	—	—	53000	11000	—	—	—	—	—	1350
" " annealed	—	—	25000	5000	—	—	—	—	—	1379
Plumbago	—	—	—	—	—	—	—	—	—	142
Silver, cast	—	—	41000	8000	—	—	—	—	—	655
" annealed	—	—	4300	900	—	—	—	—	—	658
" wire, unannealed	—	—	38000	7600	—	—	—	—	—	—
Solder, soft	—	—	7500	1200	—	—	—	—	—	—
Steel, American, wrought	90000	15000	108000	18000	75000	12000	100000	15000	29000000	488
" English	—	—	97000	15000	65000	10000	80000	13000	28000000	490
" wire, unannealed	—	—	160000	25000	—	—	—	—	—	490
" " annealed	—	—	80000	13000	—	—	—	—	—	—
" rope, galvanized	—	—	6700	1200	—	—	—	—	—	—
" chains	—	—	42000	8500	—	—	—	—	—	—
" cast	150000	25000	85000	15000	57000	10000	125000	15000	30000000	—
" welded hard & soft, 5 ply	—	—	63300	15000	—	—	—	—	—	—
Tin, cast	15500	2500	5500	1000	—	—	4000	1000	4000000	462
" wire	—	—	7000	1200	—	—	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

METALS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	E	
Zinc, sheet	—	—	12000	2000	—	—	—	—	13500000	449
" wire	—	—	20000	3300	—	—	—	—	—	—
" cast	40000	7000	3300	600	—	—	7500	1200	—	438
MISCELLANEOUS.										
Bone	—	—	5000	500	—	—	—	—	—	—
Cement, Portland	3000	300	500	50	—	—	900	90	800000	81
Glass, common green	2800	1500	2850	300	—	—	4000	400	8000000	173
" crystal	17000	1000	2413	240	—	—	—	—	5800000	157
" flint	25000	1400	2300	230	—	—	—	—	—	—
" white crown	30000	1600	2500	250	—	—	—	—	—	—
" plate	—	—	5000	700	—	—	—	—	—	—
Ice, 32°	2[illegible]0	20	—	—	—	—	—	—	6000000	57
Ivory	—	—	16000	1600	—	—	—	—	—	114
Limestone	7000	700	1000	100	—	—	1500	150	3300000	146
Marble, Italian	12000	1200	700	70	—	—	2080	200	2500000	168
Mortar, lime	400	100	50	5	—	—	125	12	1285000	98
" Portland cement	2000	200	300	30	—	—	700	70	600000	109
Ox leather	—	—	4200	400	—	—	—	—	240000	—
Portland stone	3500	350	900	90	—	—	1530	150	1600000	146
Silk fibre	—	—	5200	520	—	—	—	—	1300000	—
Skin, ox undressed	—	—	6000	600	—	—	—	—	—	—
Slate	11000	1100	10000	800	—	—	5000	500	12000000	160
Ropes, hempen	—	—	8000	800	—	—	—	—	—	—
Whalebone	—	—	7700	770	—	—	—	—	820000	81
Yarn, flaxen	—	—	25000	1500	—	—	—	—	—	—

TABLE V.

STRENGTH OF STONES, BRICKS, AND CEMENTS PER SQUARE INCH.

STONES.	Compression on bed.		Compression on edge.		Tension.		Modulus of Rupture.		Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate c_1	Safe $\frac{c_1}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate k	Safe $\frac{k}{f}$	
Bluestone	13500	1350	—	—	1400	140	2300	230	160
Granite, average	15000	1500	—	—	580	50	1860	180	170
" Aberdeen (Scotch)	10700	1000	—	—	—	—	—	—	165
" Blue, Staten Island, N. Y	22250	2000	—	—	—	—	1300	130	179
" Canadian	12000	1200	—	—	—	—	—	—	162
" Connecticut	12000	1200	13000	1300	—	—	—	—	166
" Cornish (English)	6300	630	—	—	—	—	—	—	183
" Keene, N. H	14500	1450	—	—	—	—	1500	150	—
" Kingston (Dublin), Ireland	10450	1050	—	—	—	—	—	—	171
" Maine	15000	1500	15000	1500	—	—	—	—	168
" Massachusetts	16000	1600	9250	925	—	—	1870	180	165
" Millstone Point	15700	1570	—	—	—	—	1900	190	—
" Mt. Sorrel	12900	1300	—	—	—	—	—	—	—
" New Jersey	22000	2200	—	—	—	—	—	—	189
" New York State	15000	1500	15000	1500	—	—	—	—	168
" Peterhead (Scotch)	8300	830	—	—	—	—	—	—	166
" Virginia	14000	1400	—	—	—	—	—	—	164
" Greenstone, Giants' Causeway, Ireland	18000	1800	—	—	—	—	—	—	—
Limestone, average	7000	700	6000	600	1000	100	1500	150	146
" Anglesea, English	7600	760	—	—	—	—	—	—	—
" Bath, English	1450	150	—	—	—	—	370	37	—
" Caen, French	3550	350	—	—	150	15	600	60	119
" Chalk, average	1000	100	—	—	118	10	670	50	145
" Chilmark, English	3240	325	—	—	500	50	—	—	153
" Canton, Mo., drab	7200	720	—	—	—	—	—	—	146
" Marquette, Mich., drab	7900	790	7800	780	—	—	1500	150	146
" Dublin, Irish	17000	1700	—	—	—	—	—	—	—
" English, compact	8300	830	—	—	—	—	800	80	160
" Glen Falls, N. Y	11500	1100	11750	1000	—	—	—	—	169
" Lake Champlain, N. Y	25000	2000	21500	1800	—	—	—	—	172
" North River, N. Y	17000	1500	14150	1400	—	—	—	—	166
" Purbeck (hard bed), English	8000	800	—	—	—	—	—	—	153

TABLE V. (Continued.)

STRENGTH OF STONES, BRICKS, AND CEMENTS PER SQUARE INCH.

STONES.	Compression on bed. Ultimate c	Compression on bed. Safe $\frac{c}{f}$	Compression on edge. Ultimate c_1	Compression on edge. Safe $\frac{c_1}{f}$	Tension. Ultimate t	Tension. Safe $\frac{t}{f}$	Modulus of Rupture. Ultimate k	Modulus of Rupture. Safe $\frac{k}{f}$	Weight per Cubic ft.
Limestone, Portland, English	3500	350	—	—	900	90	1530	150	146
" Joliet, Ill., white	13000	1300	7050	700	—	—	—	—	159
" Marblehead, Ohio, white	11500	1150	—	—	—	—	1500	150	152
Marble, average	8000	800	8000	800	700	70	1200	120	160
" Devonshire red (English)	7300	730	—	—	—	—	—	—	163
" Eastchester	13000	1300	—	—	1600	160	1900	190	179
" Illinois	9700	970	9800	980	1000	100	1200	120	159
" Italian	12000	1200	—	—	700	70	2080	200	168
" Vermont	7600	760	8670	870	675	67	1200	120	166
Rock, average	12000	1000	11000	1000	—	—	—	—	165
" quartz (American)	22000	1500	13000	1000	—	—	—	—	165
" Holyhead (English)	25000	1500	13700	1000	—	—	—	—	165
" hard, New York City	11250	1000	12500	1000	—	—	—	—	182
Sandstone, average	5000	500	—	—	150	15	1000	100	144
" Berlin, Ohio	14250	1000	12000	1000	—	—	—	—	—
" Belville, N. J. (brown)	11700	1000	10250	800	—	—	650	50	141
" Corsehill (red Scotch)	7760	700	—	—	—	—	—	—	154
" Gatelawbridge (red Scotch)	8500	750	—	—	—	—	—	—	152
" Little Falls, N. Y. (brown)	9850	980	9150	900	—	—	1730	170	140
" Craigleith (Scotch)	5500	550	—	—	450	45	770	77	144
" Longmeadow, (Worcester) brown	10300	1000	—	—	—	—	—	—	142
" Longmeadow, (Kibbe) red	12700	1200	—	—	—	—	—	—	149
" Middletown, Conn. (brown)	6950	600	5550	500	590	50	1000	100	148
" Amherst, Ohio (brown-gray)	6000	600	5450	500	105	10	600	.60	135
" Bramley Falls, English	5900	550	—	—	—	—	—	—	142
" Dorchester, Nova Scotia (freestone)	9200	900	—	—	100	10	800	80	150
" " (olive)	4250	400	6050	600	—	—	—	—	150
" Drab Berea (Ohio)	8750	800	—	—	105	10	700	70	134
" Runcorn red, (English)	2100	200	—	—	—	—	—	—	129
" Yorkshire paving (English)	5600	500	—	—	—	—	750	75	166
Slate	11000	1100	—	—	10000	800	5000	500	160
Whinstone (Scotch)	8100	800	—	—	1450	140	—	—	—
Stonework	with	level	dressed	beds	allow	4-10 of	strength	of	stone.
Rubblework	500	100	—	—	—	—	—	—	150

TABLE V. (*Continued.*)

STRENGTH OF STONES, BRICKS, AND CEMENTS PER SQUARE INCH.

BRICKS.—CEMENTS, ETC.—SOILS.	Compression on bed.		Compression on edge.		Tension.		Modulus of Rupture.		Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate c_1	Safe $\frac{c_1}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate k	Safe $\frac{k}{f}$	
BRICKS.									
Bricks, light red	640	65	—	—	40	4	—	—	—
" good common	10000	800	—	—	200	20	600	60	100
" best hard	12000	1000	—	—	400	40	750	75	107
" Philadelphia, pressed	6000	600	—	—	—	—	650	65	105
" brown glazed (English)	1300	130	—	—	—	—	—	—	—
" fire (English)	1700	170	—	—	—	—	—	—	138
Brickwork, common	1000	100	—	—	50	5	—	—	110
" good	1500	150	—	—	100	10	—	—	111
" best, in cement mortar	2000	200	—	—	300	30	—	—	112
Terra-cotta	5000	400	—	—	—	—	—	—	110
Terra-cotta work	2000	200	—	—	—	—	—	—	112
CEMENTS, ETC.									
Cement, Rosendale	1500	150	—	—	290	29	400	40	56
" Portland	3000	300	—	—	500	50	900	90	81
" lime	600	60	—	—	—	—	200	20	59
" plaster-of-Paris	600	60	—	—	70	7	—	—	78
Mortar, lime	400	100	—	—	50	5	125	12	98
" lime and Rosendale	550	125	—	—	75	8	200	20	100
" Rosendale cement	700	150	—	—	145	15	300	30	102
" Portland cement	2000	200	—	—	300	30	700	70	109
" plastering	400	50	—	—	—	—	—	—	86
Concrete, Portland cement	1500	200	—	—	427	40	162	16	150
" Rosendale cement	400	100	—	—	100	10	66	8	140
SOILS.									
Clay, horizontal layers	—	40	—	—	—	—	—	—	120
Firm earth	—	20	—	—	—	—	—	—	115
Gravel, firm with earth	—	100	—	—	—	—	—	—	125
Loam, soft	—	10	—	—	—	—	—	—	110
Sand, solid dry	—	7	—	—	—	—	—	—	112

The amounts given in Table IV for compression, tension and shearing are *along* fibres, except where marked *across*.

It will also be noticed that the factors-of-safety chosen are very different; the reason being that where figures seemed reliable the factor chosen was low, and became higher in proportion to the unreliability of the figures. The tables, as they are, are extremely unsatisfactory and unreliable, though the writer has spent much time in their construction. Any one, who will devote to the subject even the slightest research, will find that there are hardly any two original experimenters who agree, and in most cases, the experiments are so carelessly made or recorded that they are of but little value.

TABLE VI.

WEIGHT PER CUBIC FOOT OF MATERIALS.

(Not included in Tables IV and V.)

Material.	Weight.	Material.	Weight.
Ashes	59	Peat	85
Asphalt	150	Petrified wood	145
Butter	60	Pitch	75
Camphor	63	Plumbago	131
Charcoal	23	Pumice-stone	56
Coal, solid	93	Resin	68
" loose	54	Rock crystal	172
Coke	50	Rubber	62
Cork	15	Salt	134
Cotton in bales	20	Saltpetre	130
Fat	58	Snow, fresh fallen	6
Gunpowder	56	" solid	20
Hay in bales	17	Sugar	82
Isinglass	70	Sulphur	125
Lead, red	560	Tiles	115
Paper	55	Water	63

TRANSVERSE STRENGTH. — RUPTURE.

If a beam is supported at two ends, and loads are applied to the beam, it is evident: —

1st, that the beam will bend under the load, or *deflect*.

2d, that if the loading continues, the beam will eventually break, or be *ruptured*.

Deflection when non-important. The methods of calculating deflection and rupture differ very greatly. In some cases, where deflection in a beam would do no damage—such as cracking plaster, lowering a column, making a floor too uneven for machinery, etc.,—or where it would not look unsightly, we can leave deflection out of the question, and calculate for rupture only. Where, however, it is important to guard against deflection, we must calculate for both.

REACTION OF SUPPORTS.

If we imagine the loaded beam supported at both ends by two giants, it is evident that each giant would have to exert a certain amount of force upwards to keep his end of the beam from tipping.

Amount of Reaction. We can therefore imagine in all cases the supports to be resisting or *reacting* with force sufficient to uphold their respective ends. The amount of this reaction for either support is equal to the load multiplied by its distance from the further support, the whole divided by the length, or (see Fig. 7)

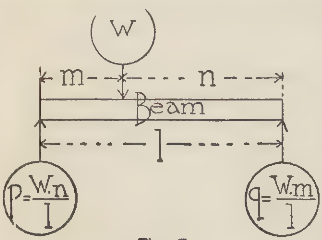


Fig. 7.

$$p = \frac{w.\,n}{l} \qquad (14)$$

Where $p =$ the amount of the *left* hand reaction or supporting force.

$$\text{and } q = \frac{w.\,m}{l} \qquad (15)$$

Where $q =$ the amount of the *right* hand reaction or supporting force.

If there are several loads the same law holds good for each, the reaction being the sum of the products, or (see Fig. 8)

$$p = \frac{w_{I}\, n}{l} + \frac{w_{II}\, s}{l} \qquad (16)$$

$$\text{and } q = \frac{w_{I}\, m}{l} + \frac{w_{II}\, r}{l} \qquad (17)$$

As a check add the two reactions together and their sum must equal the whole load, that is, $p + q = w_{I} + w_{II}$

Example.

A beam 9′ 2″ long between bearings carries two loads, one of 200 lbs. 4′ 2″ from the left-hand support, and the other of 300 lbs. 3′ 4″ from the right-hand support. What are the right-hand and left-hand reactions?

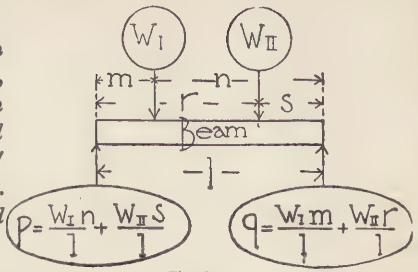


Fig. 8.

Referring to Figure 8 we should have $w_{\text{I}} = 200$ lbs., and $w_{\text{II}} = 300$ lbs., further $l = 110''$; $m = 50''$; $n = 60''$; $s = 40''$, and $r = 70''$, therefore the left-hand reaction would be: —

$$p = \frac{200.\ 60}{110} + \frac{300.\ 40}{110} = 218\tfrac{2}{11} \text{ pounds.}$$

and the right-hand reaction would be: —

$$q = \frac{200.\ 50}{110} + \frac{300.\ 70}{110} = 281\tfrac{9}{11} \text{ pounds.}$$

As a check add p and q together, and they should equal the whole load of 500 lbs., and we have in effect: —

$$p + q = 218\tfrac{2}{11} + 281\tfrac{9}{11} = 500 \text{ pounds.}$$

If the load on a beam is uniformly distributed, or is concentrated at the centre of the beam, or is concentrated at several points along the beam, each half of beam being loaded similarly, then each support will react just one half of the total load.

THE PRINCIPLE OF MOMENTS.

Law of Lever. The law of the lever is well known. The distance of a force from its fulcrum or point where it takes effect is called its *leverage*. The *effect* of the force at such point is equal to the amount of the force multiplied by its leverage.

Moment of a force. The *effect* of a force (or load) at any point of a beam is called the *moment* of the force (or load) at said point, and is equal to the amount of the force (or load) multiplied by the distance of the force (or load) from said point, *the distance measured at right angles to the line of the force.* If therefore we find the moments — for all of the forces acting on a beam — at any single point of the beam we know the total moment at said point, and this is called the *bending-moment* at said point. Of course, forces

Bending moment. acting in opposite directions will give opposite moments, and will counteract each other; to find the bending-moment, therefore, for any single point of a beam take the difference between the sums of the opposing moments of all forces acting at that point of the beam.

Now on any loaded beam we have two kinds of forces, the loads which are pressing downwards, and the supports which are resisting upwards (theoretically *forcing* upwards). Again, if we imagine that the beam will break at any certain point, and imagine one side of the beam to be *rigid*, while the other side is tending to break away from the rigid side, it is evident that the effect at the point of rupture will be from one side only; therefore we must take the forces on *one side* of the point *only*. It will be found in practice that no matter for what point of a beam the bending moment is sought, the bending moment will be found to be the same, whether we take the forces to the right side or left side of the point. This gives an excellent check on all calculations, as we can calculate the bending moment from the forces on each side, and the results of course should be the same.

Now to find the actual strain on the fibres of any cross-section of the beam, we must find the bending moment at the point where the cross-section is taken, and divide it by the moment of resistance of the fibre, or,

$$\frac{m}{r} = s$$

Where $m =$ the bending moment in lbs. inch.

Where $r =$ the moment of resistance of the fibre in inches.

Where $s =$ the strain.

The stress, of course, will be equal to the resistance to cross-breaking the fibres are capable of. In the case of beams which are of uniform cross-section above and below the neutral axis, this resistance is called the Modulus of Rupture (k). It is found by experiments and tests for each material, and will be found in Tables IV and V. We have, then, for *uniform cross-sections:* —

$$v = k$$

Where $v =$ the ultimate stress per square inch.

Where $k =$ the modulus of rupture per square inch. Inserting this and the above in the fundamental formula (1), viz.: $v = s.f.$, we have: —

$$k = \frac{m}{r} . f, \text{ or}$$

Transverse strength uniform cross-section.

$$\frac{m}{\left(\frac{k}{f}\right)} = r \qquad (18)$$

Where m = the bending moment in lbs. inch at a given point of beam.

Where r = the moment of resistance in inches of the fibres at said point.

Where $\left(\frac{k}{f}\right)$ = the safe modulus of rupture of the material, per square inch.

Transverse strength section not uniform.

If the *cross-section is not uniform* above and below the neutral axis, we must make two distinct calculations, one for the fibres above the neutral axis, the other for the fibres below; in the former case the fibres would be under compression, in the latter under tension. Therefore, for the fibres *above the neutral axis*, the ultimate stress would be equal to the ultimate resistance of the fibres to compression, or $v = c$.

Inserting this in the fundamental formula (1), we have:—

$$c = \frac{m}{r} \cdot f, \text{ or}$$

Upper fibres.

$$\frac{m}{\left(\frac{c}{f}\right)} = r \qquad (19)$$

Where m = the bending moment in lbs. inch, at a given point of beam.

Where r = the moment of resistance in inches of the fibres at said point.

Where $\left(\frac{c}{f}\right)$ = the safe resistance to crushing of the material, per square inch.

For the *fibres below the neutral axis*, the ultimate stress would be equal to the ultimate resistance of the fibres to tension, or, $v = t$.

Inserting this in the fundamental formula (1) we have:—

$$t = \frac{m}{r} \cdot f, \text{ or}$$

Lower fibres.

$$\frac{m}{\left(\frac{t}{f}\right)} = r \qquad (20)$$

Where m = the bending moment in lbs. inch at a given point of beam.

Where r = the moment of resistance in inches of the fibres at said point.

Where $\left(\frac{t}{f}\right)$ = the safe resistance to tension of the material, per square inch.

The same formulæ apply to cantilevers as well as beams.

The moment of resistance r of any fibre is equal to the moment of inertia of the whole cross-section, divided by the distance of the fibre from the neutral axis of the cross-section.

Greatest strains on extreme fibres.

The greatest strains are along the upper and lower edges of the beam (the extreme fibres); we, therefore, only need to calculate their resistances, as all the intermediate fibres are nearer to the neutral axis, and, consequently, *less* strained. The distance of fibres chosen in calculating the moment of resistance is, therefore, the distance from the neutral axis of either the upper or lower edges, as the case may be. The moments of resistance given in the fourth column, of Table I, are for the upper and lower edges (the extreme fibres), and should be inserted in place of r, in all the above formulæ.

Point of greatest bending moment.

To find at what point of a beam the greatest bending moment takes place (and, consequently, the greatest fibre strains, also), begin at either support and move along the beam towards the other support, passing by load after load, until the amount of loads that have been passed is equal to the amount of the reaction of the support (point of start); the point of the beam where this amount is reached is the point of greatest bending moment.

In cantilevers (beams built in solidly at one end and free at the other end), the point of greatest bending moment is *always* at the point of the support (where the beam is built in).

In light beams and short spans the weight of the beam itself can be neglected, but in heavy or long beams the weight of the beam should be considered as an independent uniform load.

RULES FOR CALCULATING TRANSVERSE STRAINS.

Summary of Rules.

1. *Find Reaction of each Support.*

If the loads on a girder are uniformly or symmetrically distributed, each support carries or reacts with a force equal to one-half of the total load. If the weights are unevenly distributed, each support carries, or the reaction of each support is equal to, the *sum* of the products of *each* load into its

distance from the *other* support, divided by the whole length of span. See Formulæ (14), (15), (16), and (17).

2. Find Point of Greatest Bending Moment.

The greatest bending moment of a uniformly or symmetrically distributed load is always at the centre. To find the *point* of greatest bending moment, when the loads are unevenly distributed, begin at either support and pass over load after load until an amount of loads has been passed *equal* to the amount of reaction at the support from which the start was made, and this is the desired point. In a cantilever the point of greatest bending moment is always at the wall.

3. Find the Amount of the Greatest Bending Moment.

In a beam (supported at both ends) the greatest bending moment is at the centre of the beam, provided the load is uniform, and this moment is equal to the product of the whole load into one-eighth of the length of span, or

$$m = \frac{u.l}{8} \tag{21}$$

Where $m =$ the greatest bending moment (at centre), in lbs. inch, of a uniformly-loaded beam supported at both ends.

Where $u =$ the total amount of uniform load in pounds.

Where $l =$ the length of span in inches.

If the above beam carried a central load, in place of a uniform load, the greatest bending moment would still be at the centre, but would be equal to the product of the load into one-quarter of the length of span, or

$$m = \frac{w.l}{4} \tag{22}$$

Where $m =$ the greatest bending moment (at centre), in lbs. inch, of a beam with concentrated load at centre, and supported at both ends

Where $w =$ the amount of load in pounds.

Where $l =$ the length of span in inches.

To find the greatest bending moment of a beam, supported at both ends, with loads unevenly distributed, imagine the girder cut at the point (previously found) where the greatest bending moment is known to exist; then the *amount* of the bending moment at that point will be equal to the product of the reaction (of either support) into its distance from said point, less the *sum* of the products of *all* the loads on the *same* side into their respective distances from said point, *i. e.*, the point where the beam is supposed to be cut. To check the whole calculation, try the reaction and loads of the discarded side of the beam, and the same result should be obtained.

To put the above in a formula, we should have:—

Amount of greatest bending moment.

$$m_A = p.x - \Sigma\,(w_I\, x_I + w_{II}\, x_{II} + w_{III}\, x_{III}, \text{ etc.}) \quad (23)$$

And as a *check* to above:

$$m_A = q.(l-x) - \Sigma\,(w_{IIII}\, x_{IIII} + w_V\, x_V + w_{VI}\, x_{VI}) \text{ etc.} \quad (24)$$

Where $A =$ is the point of greatest bending moment.

Where $m_A =$ is the amount of bending moment, in lbs. inches, at A

Where $p =$ is the left-hand reaction, in pounds.

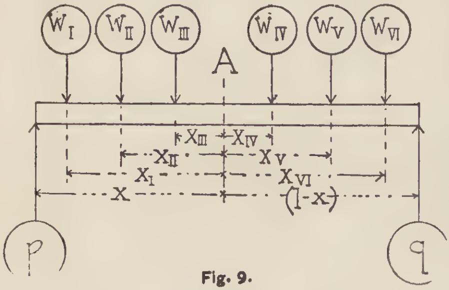


Fig. 9.

Where $q =$ is the right-hand reaction, in pounds.

Where x and $(l-x)$ = the respective distances in inches, of the left and right reactions from A.

Where x_I, x_{II}, x_{III}, etc., = the respective distances, in inches, from A, of the loads w_I, w_{II}, w_{III}, etc.

Where w_I, w_{II}, w_{III}, etc., = the loads, in pounds.

Where Σ = the sign of summation.

The same formulæ, of course, would hold good for any point of beam.

In a cantilever (supported and built in at one end only), the greatest bending moment is *always* at the point of support.

For a uniform load, it is equal to the product of the whole load into one-half of the length of the free end of cantilever, or

$$m = \frac{u.l}{2} \quad (25)$$

Where $m =$ the amount, in lbs. inch, of the greatest bending moment (at point of support).

Where $u =$ the amount of the whole uniform load, in pounds.

Where $l =$ the length, in inches, of the free end of cantilever.

For a load concentrated at the free end of a cantilever, the greatest bending moment is at the point of support, and is equal to the product of the load into the length of the free end of cantilever, or

$$m = w.\, l \quad (26)$$

Where $m =$ the amount, in lbs. inch, of the greatest bending moment (at point of support).

Where $w =$ the load, in pounds, concentrated at free end.

Where $l =$ the length, in inches, of free end of cantilever.

For a load concentrated at any point of a cantilever, the greatest bending moment is at the point of support, and is equal to the product of the load into its distance from the point of support, or

$$m = w \cdot x \qquad (27)$$

Where $m =$ the amount, in lbs. inch, of the greatest bending moment (at point of support).

Where $w =$ the load, in pounds, at any point.

Where $x =$ the distance, in inches, from load to point of support of cantilever.

Note, that in all cases, when measuring the distance of a load, we must take the shortest distance (at right angles) of the vertical neutral axis of the load, (that is, of a vertical line through the centre of gravity of the load.)

4. *Find the Required Cross-section.*

To do this it is necessary first to find what will be the required moment of resistance.

If the cross-section of the beam is *uniform* above and below the neutral axis, we use Formula (18), viz.:—

$$r = \frac{m}{\left(\frac{k}{f}\right)}$$

If the cross-section is *unsymmetrical*, that is, not uniform above and below the neutral axis, we use for the *fibres above the neutral axis*, formula (19), viz.:—

$$r = \frac{m}{\left(\frac{c}{f}\right)}$$

and for the *fibres below the neutral axis*, Formula (20), viz.:—

$$r = \frac{m}{\left(\frac{t}{f}\right)}$$

In the latter two cases, for economy, the cross-section should be so designed that the respective distances of the upper and lower edges (extreme fibres), from the neutral axis, should be proportioned to their respective stresses or capacities to resist compression and tension. This will be more fully explained under cast-iron lintels.

A simple example will more fully explain all of the above rules.

Example.

Three weights of respectively 500 *lbs.*, 1000 *lbs., and* 1500 *lbs., are placed on a beam of* 17′ 6″ (*or* 210″) *clear span,* 2′ 6″ (*or* 30″), 7′ 6″ (*or* 90″), *and* 10′ 0″ (*or* 120″) *from the left-hand support. The modulus of rupture of the material is* 2800 *lbs. per square inch. The factor-of-safety to be used is* 4. *The beam to be of uniform cross-section. What size of beam should be used?*

1. *Find Reactions* (*see formulæ* 16 *and* 17).

Reaction p will be in pounds, $= \frac{500.180}{210} + \frac{1000.120}{210} + \frac{1500.90}{210} =$ $1642\frac{6}{7}$ pounds.

Reaction q will be in pounds, $= \frac{500.30}{210} + \frac{1000.90}{210} + \frac{1500.120}{210} =$ $1357\frac{1}{7}$ pounds.

Check, $p + q$ must equal whole load, and we have in effect: —

$p + q = 1642\frac{6}{7} + 1357\frac{1}{7} = 3000$, which

being equal to the sum of the loads is correct, for: —

$500 + 1000 + 1500 = 3000.$

2. *Find Point of Greatest Bending Moment.*

Begin at p, pass over load 500, plus load 1000, and we still need to pass $142\frac{6}{7}$ pounds of load to make up amount of reaction p ($1642\frac{6}{7}$ lbs.); therefore, the greatest bending moment must be at load 1500; *check*, begin at q and we arrive only at the first load (1500) before passing amount of reaction q ($1357\frac{1}{7}$ lbs.), therefore, at load 1500 is the point sought.

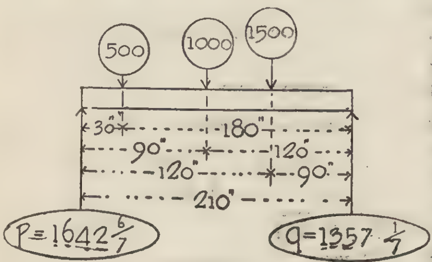


Fig. 10.

3. *Find Amount of Greatest Bending Moment.*

Suppose the beam cut at load 1500, then take the left-hand side of beam, and we have for the bending moment at the point where the beam is cut.

$$\begin{aligned} m &= 1642\tfrac{6}{7}.120 - (500.90 + 1000.30 + 1500.0) \\ &= 197143 - (45000 + 30000 + 0) \\ &= 197143 - 75000 \\ &= 122143 \text{ lbs. inch.} \end{aligned}$$

As a check on the calculation, take the right-hand side of beam and we should have: —

$$m = 1357\tfrac{1}{7}.90 - 1500.0$$
$$= 122143 - 0$$
$$= 122143 \text{ lbs. inch,}$$

which, of course, proves the correctness of former calculation.

4. *Find the Required Cross-section of Beam.*

We must first find the required moment of resistance, and as the cross-section is to be uniform, we use formula (18), viz.: —

$$r = \frac{m}{\left(\frac{k}{f}\right)}$$

Now, $m = 122143$, and $\frac{k}{f} = \frac{2800}{4} = 700$, therefore,

$$r = \frac{122143}{700} = 174{,}49 \text{ or say} = 174{,}5$$

Consulting Table I, fourth column, for section No. 2, we find

$r = \frac{bd^2}{6}$, we have, therefore,

$$\frac{bd^2}{6} = 174{,}5 \text{ or } bd^2 = 1047.$$

If the size of either b or d is fixed by local conditions, we can, of course, find the other size (d or b) very simply; for instance, if for certain reasons of design we did not want the beam to be more than 4″ wide, we should have

$$b = 4, \text{ therefore, } 4.d^2 = 1047, \text{ and}$$
$$d^2 = \frac{1047}{4} = 262, \text{ therefore, } d = \text{(about) } 16'',$$

or, if we did not want the beam to be over 12″ deep, we should have

$$d = 12, \text{ and } d^2 = 12.12 = 144, \text{ therefore,}$$
$$b.144 = 1047, \text{ and } b = \frac{1047}{144} = 7{,}2'' \text{ or say } 7\tfrac{1}{4}''.$$

The deepest beam the most economical. One thing is very important and, *must be remembered*, that the *deeper* the beam is, the more economical, and the stiffer will it be. If the beam is too shallow, it might deflect so as to be utterly unserviceable, besides using very much more material. As a rule, it will therefore be necessary to calculate the beam for deflection as well as for its transverse strength.

Safe deflection. The deflection should not exceed 0,″03 that is, three one-hundredths of an inch for each foot of span, or else the plastering would be apt to crack, we have then the formula: —

$$\delta = L.\ 0{,}03 \qquad (28)$$

Where $\delta =$ the greatest allowable total deflection, in inches, at centre of beam, to prevent plaster cracking.

Where $L =$ the length of span, in feet.

In case the beam is so unevenly loaded that the greatest deflection will not be at the centre, but at some other point, use:—

$$\delta = X.\ 0{,}06 \tag{29}$$

Where $\delta =$ the greatest allowable total deflection, in inches, at point of greatest deflection.

Where $X =$ the distance, in feet, to *nearer* support from point of greatest deflection.

If the beam is not stiffened sideways, it should also be calculated for lateral flexure. These matters will be more fully explained when treating of beams and girders.

COMPARATIVE STRENGTH AND STIFFNESS OF BEAMS AND CANTILEVERS.

(1) If a beam supported at both ends and loaded uniformly will safely carry an amount of load $= u$; then will the same beam:

(2) if both ends are built in solidly and load uniformly distributed, carry $1\frac{1}{2}.\ u$,

(3) if one end is supported and other built in solidly and load uniformly distributed, carry $1.\ u$,

(4) if both ends are built in solidly and load applied in centre, carry $1.\ u$,

(5) if one end is supported and other built in solidly and load applied in centre, carry $\frac{2}{3}.\ u$,

(6) if both ends are supported and load applied in centre, carry $\frac{1}{2}.\ u$,

(7) if one end is built in solidly and other end free (cantilever) and load uniformly distributed, carry $\frac{1}{4}.\ u$,

(8) if one end is built in solidly and other end free (cantilever) and load applied at free end, carry $\frac{1}{8}.\ u$.

That is, in cases (1), (3) and (4) the effect would be the same with the same amount of load; in case (2) the beam could safely carry $1\frac{1}{2}$ times as much load as in case (1); in case (5) the beam could safely carry only $\frac{2}{3}$ as much as in case (1), etc., *provided that the length of span is the same in each case.*[2]

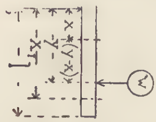
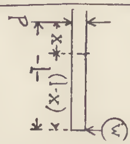
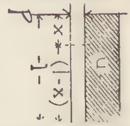
If the amount of deflection in case (1) were δ_{I} then would the amount of deflection in the other cases be as follows:

Case (2) $\delta_{II} = \frac{1}{5}.\ \delta_{I}$	Case (4) $\delta_{IV} = \frac{2}{5}.\ \delta_{I}$	Case (7) $\delta_{VII} = 9\frac{3}{5}.\ \delta_{I}$
Case (3) $\delta_{III} = \frac{2}{5}.\ \delta_{I}$	Case (5) $\delta_{V} = \frac{3}{4}.\ \delta_{I}$	Case (8) $\delta_{VIII} = 25\frac{3}{5}.\ \delta_{I}$
	Case (6) $\delta_{VI} = 1\frac{3}{5}.\ \delta_{I}$	

[2] To count on the end of a beam being built in solidly would be very bad practice in most cases of building construction; as, for instance, a wooden beam with end built in solidly could not fall out in case of fire, and would be apt to throw the wall. Even where practicable, it would require very careful supervision to get the beam built in properly; then, too, it causes upward strains which must be overcome, complicating the calculations unnecessarily. In most cases where it is necessary to "build in" beam ends, the additional strength and diminished deflection thereby secured had better be credited as an additional margin of safety. The above rules for deflection do not hold good if the beam is not of uniform cross-section throughout; the deflection being greater as the variation in cross-section is greater.

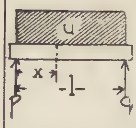
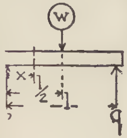
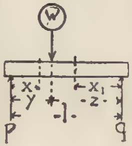
TABLE

BENDING-MOMENT (m) AND AMOUNT OF SHEARING-STRAIN (s)

		
Load at any point of Cantilever.	Load at free end of cantilever.	Uniform load on cantilever.
If y smaller than $\frac{l}{2}$: $s = o$, $m = o$ If y greater than $\frac{l}{2}$: $s = w$, $m = w.\left(y - \frac{l}{2}\right)$	$s = w$ $m = \frac{w.l}{2}$	$s = \frac{u}{2}$ $m = \frac{u.l}{8}$
If x smaller than y use: $s = w$, $m = w.(y-x)$ If x greater than y use: $s = o$, $m = o$ If $x = y$ use: $s = o$, $w = o$	For x use: $s = w$, $m = w.(l-x)$ At free end use: $s = o$, $m = o$	For x use: $m = \frac{u.(l-x)^2}{2.l}$, $s = \frac{u.(l-x)}{l}$ At free end use: $s = o$, $m = o$
$m = w.y$ at support p	$m = w.l$ at support p	$m = \frac{ul}{2}$ at support p
$s = w$ at support p	$s = w$ at support p	$s = u$ at support p
$\delta = \frac{1}{5} \cdot \frac{w.y.l^2}{e.i}$ at free end $\delta = \frac{1}{3} \cdot \frac{w.y^3}{e.i}$ at load	$\delta = \frac{1}{3} \cdot \frac{w.l^3}{e.i}$ at free end	$\delta = \frac{1}{8} \cdot \frac{u.l^3}{e.i}$ at free end

VII. (See foot-note p. 60.)

OF BEAMS AND CANTILEVERS FOR VARIOUS LOADS.

Manner of Loading.	Description.	m and s at centre.	m and s at any point distant x from support p.	Location and amount of greatest bending-moment m.	Location and amount of greatest shearing-strain s.	Location and amount of greatest deflection δ.
	Uniform load on beam supported at both ends.	$m = \frac{u.l}{8}$ $s = o$	When x greater than $\frac{l}{2}$ use $(l-x)$ in place of x. $m = \frac{u}{2}.x.\left(1-\frac{x}{l}\right)$ $s = u.\left(\frac{1}{2} - \frac{x}{l}\right)$	at centre $m = \frac{u.l}{8}$	at support p or q. $s = \frac{u}{2}$	at centre $\delta = \frac{5}{384}.\frac{u.l^3}{e.i}$
	Load at centre of beam supported at both ends.	$m = \frac{w.l}{4}$ $s = o$	$m = \frac{w}{2}.x$ $s = \frac{w}{2}$	at centre $m = \frac{w.l}{4}$	at support p or q $s = \frac{w}{2}$	at centre $\delta = \frac{1}{48}.\frac{w.l^3}{e.i}$
	Load at any point on beam supported at both ends.	If y greater than $\frac{l}{2}$ $\left\{ m = \frac{w.z}{2},\ s = \frac{w.z}{l} \right.$ If y smaller than $\frac{l}{2}$ $\left\{ m = \frac{w.y}{2},\ s = \frac{w.y}{l} \right.$	At load use: $\left\{ m = \frac{w.y.z}{l},\ s = o \right.$ For x use: $\left\{ m = \frac{w.z.x}{l},\ s = \frac{w.z}{l} \right.$ For x_1 use: $\left\{ m = \frac{w.y.x_1}{l},\ s = \frac{w.y}{l} \right.$	at load $m = \frac{w.y.z}{l}$	at nearer support p If y smaller than $\frac{l}{2}$ $s = \frac{w.z}{l}$ at support q. If y greater than $\frac{l}{2}$ $s = \frac{w.y}{l}$	near centre $\delta = \frac{w.y.z}{9.l.e.i}.\sqrt{\frac{y.(l+z)^3}{3}}$

Strength of beams of different cross-sections. The comparative transverse strength of two or more rectangular beams or cantilevers is directly as the product of their breadth into the square of their depth, provided the span, material and manner of supporting and loading are the same, or

$$x = b\, d^2 \qquad (30)$$

Where x = a figure for comparing strength of beams of equal spans.

Where b = the breadth of beam, in inches.

Where d = the depth of beam, in inches.

Example.

What is the comparative strength between a 3″ x 12″ beam, and a 6″ x 12″ beam? Also, between a 4″ x 12″ beam, and a 3″ x 16″ beam? All beams of same material and span, and similarly supported and loaded.

The strength of the 3″ x 12″ beam would be

$$x_{I} = 3.\ 12.\ 12 = 432.$$

The strength of the 6″ x 12″ beam would be

$$x_{II} = 6.\ 12.\ 12 = 864, \text{ therefore,}$$

the latter beam would be just twice as strong as the former.

Again, the strength of the 4″ x 12″ beam would be

$$x_{III} = 4.\ 12.\ 12 = 576$$

and the strength of the 3″ x 16″ beam would be

$$x_{IIII} = 3.\ 16.\ 16 = 768.$$

The latter beam would therefore be $\frac{768}{576}$ or just $1\frac{1}{3}$ times as strong as the former, while the amount of material in each beam is the same, as

$$4.\ 12 = 3.\ 16 = 48 \text{ square inches in each.}$$

The reason the last beam is so much stronger is on account of its greater depth.

Strength of beams of different lengths. The comparative transverse strength of two or more beams or cantilevers of same cross-section and material, but of unequal spans, is inversely as their lengths, provided manner of supporting and loading are the same. That is, a beam of twenty-foot span is only half as strong as a beam of ten-foot span, a quarter as strong as one of five-foot span, etc.

All measurements in Table VII are in inches; all weights in pounds; e = modulus of elasticity in pounds inch; i = moment of inertia of cross-section of beam or cantilever around its neutral axis in inches; m = bending-moment in pounds inch; s = amount of shearing strain in pounds; δ = total amount of deflection in inches.

Stiffness of beams of different lengths. The stiffness of beams or cantilevers of same cross-section and material (and similarly loaded and supported), however, diminishes very rapidly, as the length of span increases, or what is the same thing, the deflection increases much more rapidly in proportion than the length; the comparative stiffness or deflection being directly as the cube of their respective lengths or L^3.

That is if a beam 10 feet long deflects under a certain load one-third of an inch, the same beam with same load, but 20 feet long will deflect an amount x as follows:

$$x : \tfrac{1}{3} = 20^3 : 10^3, \text{ or } x = \frac{20^3 \cdot \frac{1}{3}}{10^3} = \frac{8000}{3000} = 2\tfrac{2}{3}''$$

Stiffness of beams of different cross-sections. The comparative stiffness, that is amount of deflection of two or more beams or cantilevers, similarly supported and loaded, and of same material and span, but of different cross-sections, is inversely as the product of their respective breadths into the cubes of their respective depths or

$$x = \frac{1}{b.d^3} \qquad (31)$$

Where x = a figure for comparing the deflection of beams of same material, span and load.

Where b = the breadth of beam, in inches.

Where d = the depth of beam, in inches.

Example.

If a beam 3″ x 8″ *deflects* ½″ *under a certain load, what will a beam* 4″ x 12″ *deflect, if of same material and span, similarly supported and with same load?*

For the first beam we should have

$$x_{\text{I}} = \frac{1}{3.8^3} = \frac{1}{1536} = 0{,}00065$$

For the second beam we should have

$$x_{\text{II}} = \frac{1}{4.12^3} = \frac{1}{6912} = 0{,}00014$$

The deflection of the latter beam will be as

$$\delta : 0'',5 = 0{,}00014 : 0{,}00065, \text{ or } \delta = 0'',108$$

Strength of beams of different lengths & cross-sections. The comparative strength of rectangular beams or cantilevers of different cross-sections and spans, but of same materials and similarly loaded and supported, is, of course, directly as the product of their breadth into the

squares of their depths, divided by their length of span, or

$$x = \frac{b.d^2}{L} \tag{32}$$

Where x = a figure for comparing the strength of different beams of same material, but of different cross-sections and spans.

Where b = the breadth, in inches.

Where d = the depth, in inches.

Where L = the length of span, in feet.

Stiffness of beams of different lengths & cross-sections. The comparative stiffness or amount of deflection of different rectangular beams or cantilevers of same material, and similarly loaded and supported, but of different cross-sections and spans, would be directly as the cubes of their respective lengths, divided by the product of their respective breadths into the cubes of their depths or

$$x = \frac{L^3}{b.d^3} \tag{33}$$

Where x = a figure for comparing the amount of deflections of beams of same material and load, but of different spans and cross-sections.

Where L = the length of span, in feet.

Where b = the breadth, in inches.

Where d = the depth in inches.

Strength and deflection of wooden beams, one inch thick. If it is desired to calculate a wooden girder supported at both ends and to carry its full safe uniform load, and yet not to deflect enough to crack plaster, the following will simplify the calculation :

TABLE VIII.

	Spruce.	Georgia pine.	White pine.	White oak.	Hemlock.
Calculate (x) for transverse strength only if d is greater than	$1\frac{1}{6}L$	L	$1\frac{1}{19}L$	$1\frac{1}{5}L$	$\frac{14}{15}L$
Calculate (x_1) for deflection only if d is less than	$1\frac{1}{6}L$	L	$1\frac{1}{19}L$	$1\frac{1}{5}L$	$\frac{14}{15}L$

Where L = the length of span, in feet.

Where d = the depth of beam, in inches.

Where x or x_1 = is found according to Table IX.

To find the safe load (x) or (x_1) *per running foot of span*, which a beam supported at both ends, and 1″ *thick* will carry, use the following table. (Beams two inches thick will safely carry twice as much

per running foot, as found per table, beams three inches thick three times as much, four inches thick four times as much, etc.)

TABLE IX.

	Spruce.	Georgia pine.	White pine.	White oak.	Hemlock.
If calculating for transverse strength only use	$x = 111 \left(\frac{d}{L}\right)^2$	$x = 133 \left(\frac{d}{L}\right)^2$	$x = 100 \left(\frac{d}{L}\right)^2$	$x = 122 \left(\frac{d}{L}\right)^2$	$x = 83 \left(\frac{d}{L}\right)^2$
If calculating for a deflection (not to crack plaster) use	$x_1 = 95 \left(\frac{d}{L}\right)^3$	$x_1 = 133 \left(\frac{d}{L}\right)^3$	$x_1 = 95 \left(\frac{d}{L}\right)^3$	$x_1 = 100 \left(\frac{d}{L}\right)^3$	$x_1 = 89 \left(\frac{d}{L}\right)^3$

Where $x =$ the safe load in lbs., per running foot of span, which a beam one-inch thick will carry regardless of deflection, if supported at both ends, and

$x_1 =$ the same, but without deflecting the beam enough to crack plaster; for thicker beams multiply x or x_1 by breadth, in inches.

Calculate for either x or x_1 as indicated in Table VIII.

Where $d =$ the depth of beam, in inches.

Where $L =$ the length of span in feet.

If a beam is differently supported, or not uniformly loaded, also for cantilevers, add or deduct from above result, as directed in cases 1 to 8, page 57.

Example.

A floor of 19′ *clear span is to be built with spruce beams, to carry* 100 *lbs. per square foot; what size beams would be the most economical?*

According to Table VIII, if $d = 1\frac{1}{6}.\ L = 1\frac{1}{6}.\ 19 = 22\frac{1}{6}''$ we can calculate for either deflection or rupture and the result would be the same. If we make the beam deeper it will be so stiff that it will break before deflecting enough to crack plastering underneath; while if we make the beam more shallow it will deflect enough to crack plaster before it carries its total safe load. The former would be more economical of material, but, of course, in practice we should certainly not make a wooden beam as deep as 22″. Whatever depth we select, therefore, less than 22″, we need calculate for deflection only. We have, then, according to Table IX, second column,

$$x_1 = 95.\left(\frac{d}{L}\right)^3$$

If we use a beam 12″ deep, we should have

$$x_1 = 95.\frac{12^3}{19^3} = 24$$

or a beam 1″ x 12″ would carry 24 lbs. per foot; as the load is 100 lbs. per foot we should need a beam $\frac{100}{24} = 4\frac{1}{6}$ wide, or say a beam 4″ x 12″, and of course 12″ from centres.

If we use a beam 14″ deep we should have

$$x_1 = 95.\frac{14.^3}{19^3} = 38$$

or a beam 1″ x 14″ would carry 38 lbs. per foot, we need, therefore, a beam of width

$$b = \frac{100}{38} = 2\frac{12}{19}''$$

or we must use a beam say 3″ x 14″ and 12″ from centre, or a beam 4″ x 14″ and 16″ from centre. For if the beams are 16″ from centres each beam will carry per running foot $1\frac{1}{3}.100$ lbs. $= 133$ lbs. and a 4″ x 14″ will carry per foot

$$4.x_1 = 4.38 = 152.$$

We could even spread the beams farther apart, except for the difficulty of keeping the cross-furring strips sufficiently stiff for lathing.

Of course the 14″ beam is the most economical, for in the 12″ beam we use 4″ x 12″ = 48 square inches (cross-section) of material, and our beam is a trifle weak. While with the 14″ beam we use only 3″ x 14″ = 42 square inches of material, and our beam has strength to spare. The 4″ x 14″ beam 16″ from centres would be just as strong and use just as much material as the 3″ x 14″ beam 12″ from centres. If we wished to be still more economical of material, we might use a still deeper beam, but in that case it would be less than 3″ thick and might twist and warp. If the beam is not cross-bridged or supported sideways it might be necessary to calculate its strength for lateral flexure. That it will not shear off transversely we can see readily, as the load is so light, nor is there much danger of longitudinal shearing, still for absolute safety it would be better to calculate each strain.

Strength of columns different lengths. The comparative strength of columns of same cross-section is approximately inversely as the square of their lengths. Thus, if x be the strength of a column, whose length

is L, and x_1 be the strength of a column whose length is L_1, then we have *approximately*

$$x : x_1 = L_1^2 : L^2, \text{ or } x_1 = \frac{x.\ L^2}{L_1^2} \qquad (34)$$

Where $x_1 =$ approximately the strength of a column, L_1 feet long.

Where $x =$ the strength (previously ascertained or known), of a column of same cross-section, and L feet long.

Where L and $L_1 =$ the respective lengths of columns in feet.

The nearer L and L_1 are to each other the closer will be the result.

Strength of columns different cross-sections. The comparative strength *per square inch* of cross-section of columns of same length, but of different cross-sections, is, approximately, as their least outside diameter, or side, or

$$x : x_1 = b : b_1, \text{ or } x_1 = \frac{x.\ b_1}{b} \qquad (35)$$

Where $x_1 =$ approximately the strength of a column, per square inch, whose least side or diameter (outside) is $= b_1$.

Where $x =$ the strength per square inch (previously ascertained or known) of a column of same length, but whose least side or diameter (outside) is $= b$.

The more similar and the nearer in size the respective cross-sections are, the closer will be the result. That is, the comparison between two circular columns, each 1″ thick, will be very much nearer correct than between two circular columns, one ¾″ thick and the other 2″ thick, or between a square and a circular column. The thicker the shell of a column the less it will carry per square inch. The formulæ (34 and 35) are hardly exact enough for safe practice, but will do for ascertaining approximately the necessary size of column, before making the detailed calculation required by formula (3).

The approximate thickness required for the flanges of plate girders is as follows:

Approximate thickness of flange of plate girders.

$$x = \frac{\frac{r}{d} - a_1}{b} \qquad (36)$$

Where $x =$ the approximate thickness, in inches, of either flange of a riveted girder.

Where $b =$ the breadth of flange, *less rivet holes*, all in inches.

Where $d =$ the total depth of girder in inches.

Where $r =$ the moment of resistance in inches.

Where $a_1 =$ the area (less rivet holes) of cross-section of both angles at flange, in square inches.

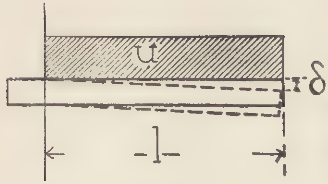


Fig. 11.

DEFLECTION.

The derivation of the following formulæ would be too lengthy to go into here, it will suffice for all practical purposes to give them. They are all based on the moduli of elasticity of the different materials, and are as follows:

FOR A CANTILEVER, UNIFORMLY LOADED.

$$\delta = \frac{1}{8} \cdot \frac{u.l^3}{e.i} \tag{37}$$

FOR A CANTILEVER, LOADED AT FREE END.

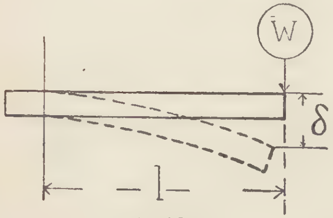


Fig. 12.

$$\delta = \frac{1}{3} \cdot \frac{w.l^3}{e.i} \tag{38}$$

FOR A BEAM, UNIFORMLY LOADED.

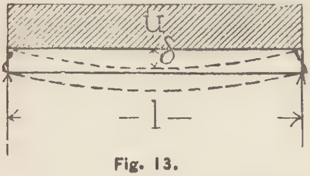


Fig. 13.

$$\delta = \frac{5}{384} \cdot \frac{u.l^3}{e.i} \tag{39}$$

FOR A BEAM, LOADED AT CENTRE.

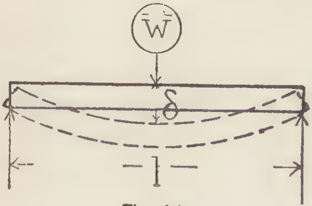


Fig. 14.

$$\delta = \frac{1}{48} \cdot \frac{w.l^3}{e.i} \tag{40}$$

FOR A BEAM, LOADED AT ANY POINT.

Greatest deflection is near the centre, not at the point where load is applied.[1]

$$\delta = \frac{w.m.n.(l+n)}{9.l.e.i.} \cdot \sqrt{\frac{m.(l+n)}{3}} \qquad (41)$$

Where u = uniform load, in pounds.

Where w = concentrated load, in pounds.

Where l = length of span, in inches.

Where e = the modulus of elasticity, in pounds-inch, of the material, see Tables IV and V.

Where i = the moment of inertia, of cross-section, in inches.

Where m and n = the respective distances to supports, in inches.

Where δ = the greatest deflection, in inches (see Formulæ 28 and 29).

FOR A CANTILEVER, LOADED AT ANY POINT.

Greatest deflection is at free end; if y = distance from support to load, in inches, then:[2]

$$\delta = \frac{1}{3} \cdot \frac{w.y.l^2}{e.i} \qquad (42)$$

EXPANSION AND CONTRACTION OF MATERIALS.

Expansion of iron trusses. All long iron trusses, *say about eighty feet long, or over*, should not be built-in solidly at both ends; otherwise the expansion and contraction due to variations of the temperature will either burst one of the supports, or else cause the truss to deflect so much, as to crack, and possibly endanger the work overhead. One end should be left free to move (lengthwise of truss) on rollers, but otherwise braced and anchored, the anchor sliding through slits in truss, as necessary. The expansion of iron for each additional single degree of temperature, Fahrenheit, is about equal to $\frac{1}{145000}$ of its length, that is, a truss 145

[1] The point of greatest deflection can never be further from the centre of beam than 2-25 of the entire length of span. It can as a rule, therefore, be safely assumed to be at the centre. If it is desired to find its exact location, use

$$x = \sqrt{\frac{l^2 - n^2}{3}} \qquad (43)$$

where n = the distance from weight to *nearer* support; x = the distance of point of greatest deflection from *farther* support; and l = the length of span; x, l and m should all be expressed either in feet or inches.

[2] Formula (42) is approximate only, but sufficiently exact for practical use.

feet long at 10° Fahrenheit, would gain in length (if the temperature advanced to 100° Fahrenheit), — $\frac{90.145}{145000} = \frac{9}{100}$ of a foot, or, say, $1\frac{1}{12}$ inches, so that at 100° Fahrenheit the truss would be 145 feet and $1\frac{1}{12}$ inches long; this amount of expansion would necessitate rollers under one end. Of course the contraction would be in the same proportion. The approximate expansion of other materials for each additional degree Fahrenheit would be (in parts of their lengths), as follows:

Expansion and contraction of materials.

Wrought-iron	$\frac{1}{145000}$	Pewter	$\frac{1}{78000}$
Cast-iron	$\frac{1}{162000}$	Platina	$\frac{1}{208000}$
Steel	$\frac{1}{151000}$	Zinc	$\frac{1}{62000}$
Antimony	$\frac{1}{166000}$	Glass	$\frac{1}{210000}$
Gold, annealed	$\frac{1}{123000}$	Granite	$\frac{1}{208000}$
Bismuth	$\frac{1}{130000}$	Fire Brick	$\frac{1}{365000}$
Copper	$\frac{1}{104000}$	Hard Brick	$\frac{1}{600000}$
Brass	$\frac{1}{95000}$	White Marble	$\frac{1}{173000}$
Silver	$\frac{1}{95000}$	Slate	$\frac{1}{173000}$
Gun metal	$\frac{1}{90000}$	Sandstone	$\frac{1}{103000}$
Tin	$\frac{1}{87000}$	White pine	$\frac{1}{440000}$
Lead	$\frac{1}{63000}$	Cement	$\frac{1}{120000}$
Solder	$\frac{1}{70000}$		

The tension due to each additional degree of Fahrenheit would be equal to the modulus of elasticity of any material multiplied by the above fraction; or about 186 pounds per square inch of cross-section, for wrought-iron. Above figures are for linear dimensions, the area of the superficial extension would be equal to the square of the linear, while the contents of the cubical extension would be equal to the cube of the linear.

Water is at its maximum density at about 39° Fahrenheit; above that it expands by additional heat, and below that point it expands by less heat. At 32° Fahrenheit water freezes, and in so doing expands nearly $\frac{1}{12}$ part of its bulk, this strain equal to about 30000 lbs. per square inch will burst iron or other pipes not sufficiently strong to resist such a pressure. The above table of expansions might be useful in many calculations of expansions in buildings; for instance, were

we to make the sandstone copings of a building in 10-foot lengths, and assume the variation of temperature from summer sun to winter cold would be about 150° Fahrenheit, each stone would expand $\frac{150.10}{103000}=\frac{1}{68}$ of a foot, or, say, about $\frac{1}{6}$ inches, quite sufficient to open the mortar joint and let the water in. The stones should, therefore, be much shorter.

GRAPHICAL METHOD OF CALCULATING STRAINS. — NOTATION.

Notation. The calculation of strains in trusses and arches is based on the law known as the "Parallelogram of Forces." Before going into same it will be necessary to explain the notation used. If Fig. 15 represents a truss, and the arrows the loads, and the two reactions (or supporting forces), we should call the left reaction O A and the right reaction F O. The loads would be, taking them in their order, A B, B C, C D, D E and E F. The foot, or lower half, of left rafter would be called B K, the upper half C I, while the respective parts of right rafter would be G E and H D. The King-post (tie) is I H, and the struts K I and H G, while the lower ties are K O and O G.

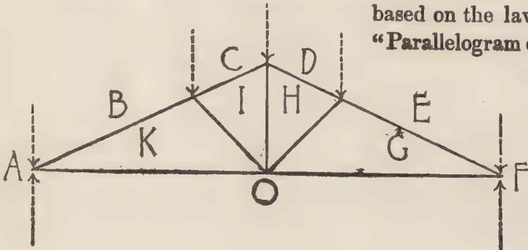


Fig. 15.

In the strain diagram, Fig. 16 (which will be explained presently), the notation is as usual; that is, loads A B, B C, C D, etc., are represented in the strain diagram by the lines *ab*, *bc*, *cd*, etc. Rafter pieces B K, C I, D H and E G are in the strain diagram *b k*, *c i*, *d h* and *e g* (*g* and *k* falling on the same point). I H in Fig 15 becomes *i h* in strain diagram.

K I becomes *k i*, H G becomes *h g*, O K becomes *o k*, G O becomes *g o*, O A becomes *o a* and F O becomes *f o*.

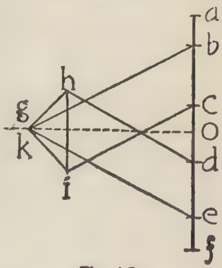


Fig. 16.

Or, in the drawing of the truss itself the lines are called, not by letters placed at the ends of the lines, but by letters placed *each side* of the lines, the lines being between; it is also usual to put these

letters in capitals to distinguish them from the letters representing the strain diagram, which are, as usual, at *each end* of the line they represent.

One thing is *very important*, however, and that is, always to read the pieces off in the *correct direction* and in their *proper order*. For instance, if we were examining the joint at middle of left rafter we must read off the pieces in their proper order, as B C, C I, I K, K B, and not jump, as B C, I K, C I, etc., as this would lead to error. Still more important is it to read around the joint in one direction, as from left to right (Fig. 17), that is, in the direction of the arrow. If we were to reverse the reading of the pieces, we should find the direction of the strain or stress reversed in the strain diagram. For instance, if we read K I and then find its corresponding line *k i* in the strain diagram, we find its direction downward, that is, pulling away from the joint, which would make K I a tie-rod, which, of course, is wrong, as we know it is a strut. If, however, we had read correctly *i k* it would be pushing upwards, which, of course, is correct and is the action of a strut.

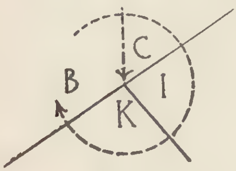


Fig. 17.

When we come to examine the joint at O, however, we reverse the above and here have to read *k i*, which is in the same relative direction for the point O, as was *i k* for the point at centre of left rafter.

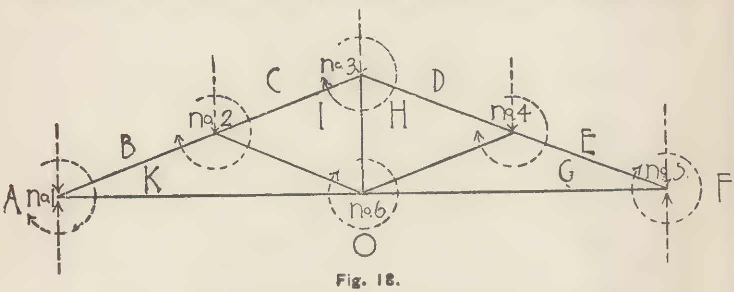


Fig. 18.

The arrows in the accompanying figure (18) show how each joint must be read, and *remember* always to read the pieces in their proper succession.

It makes no difference with which joint or with which piece of the joint we begin, so long as we read in correct succession and direction, thus: for joint No 1 we can read

A B, B K, K O and O A
or K O, O A, A B and B K,

or B K, K O, O A and A B, etc.

In the strain sheet of course we read in the same succession, and it will be found that the lines, as read, point *always* in the correct direction of the strain or stress.

PARALLELOGRAM OF FORCES.

Parallelogram of Forces. If a ball lying at the point A, Fig. 19, is propelled by a power sufficient to drive it in the direction of B, and as far as B in one minute, and at B is again propelled by a power sufficient to drive it in the direction of and as far as the point C in another minute, it will, of course, arrive at C at the end of two minutes, and by the route A B C.

If, on the other hand, both powers had been applied to the ball simultaneously, while lying at A, Fig. 20, it stands to reason that the ball would have reached C, but in one minute and by the route A C. A C (or E D), is, therefore, called the resultant of the forces A E and D A. If, now, we were to apply to the ball, while at A, simultaneously with the forces D A and A E, a third force (E D) sufficient to force the ball in the opposite direction to A C (that is, in the direction of C A), a distance equal to C A in one minute it stands to reason that the ball would remain perfectly motionless at A, as C A being the resultant (that is, the result) of the other two forces, if we oppose them with a power just equal to their own result, it stands to reason that they are completely neutralized. Now, applying this to a more practical case, if we had two sticks lying on A E and D A, Fig. 21, and holding the ball in place, and we apply to the ball a force E D = C A and in the direction C A, we can easily find how *much* each stick must resist or push against the ball. Draw a line *e d*, Fig. 22, parallel to E D, and of a length at any convenient scale equal in amount to force E D; through *e*, Fig. 22, draw *a e* parallel to A E, and through *d* draw *d a* parallel to D A, then the triangle *eda* (not *ead*) is the strain diagram for the Fig. 21, and *d a*,

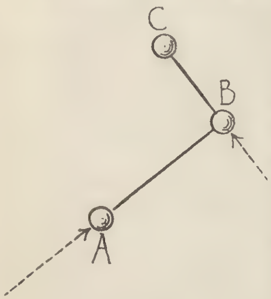


Fig. 19.

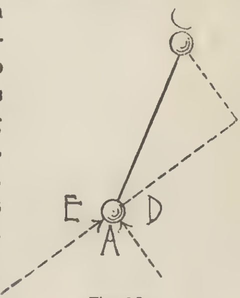
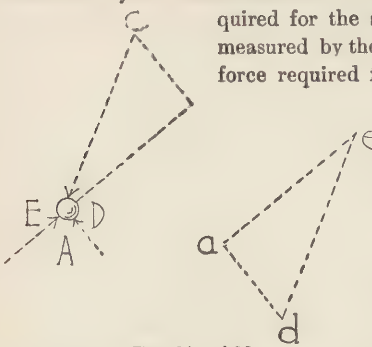


Fig. 20.

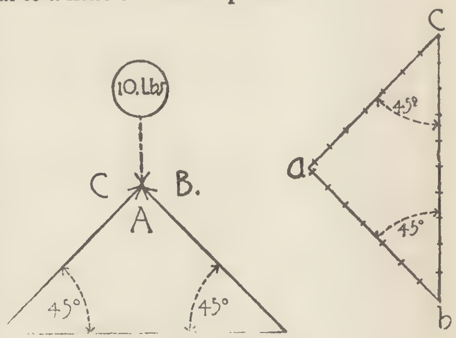
measured by the same scale as *e d*, is the amount of force required for the stick D A to exert, while *a e*, measured by the same scale, is the amount of force required for the stick A E to exert. If in place of the force E D we had had a load, the same truths would hold good, but we should represent the load by a force acting downward in a vertical and plumb line.



Figs. 21 and 22.

Thus, if two sticks, B A and A C, Fig. 23, are supporting a load of ten pounds at their summit, and the inclination of each stick from a horizontal line is 45°, we proceed in the same manner. Draw *c b*, Fig. 24, at any scale equal to ten units, through *b* and *c* draw *b a* and *a c* at angles of 45° each, with *c b*, then measure the number of (scale measure) units in *b a* and *a c*, which, of course, we find to be a little over seven. Therefore, each stick must resist with a force equal to a little over seven pounds.

Now, to find the direction of the forces. In Fig. 23 we read C B, B A and A C, the corresponding parts in the strain diagram, Fig. 24, are *c b*, *b a* and *a c*. Now the direction of *c b* is downwards, therefore C B acts downwards, which is, of course, the effect of a weight.



Figs. 23 and 24.

The direction, however, of *b a* and *a c* is upwards, therefore B A and A C must be pushing upwards, or *towards* the weight, and therefore they are in compression. The same truths hold good no matter how many forces we have acting at any point; that is, if the point remains in equilibrium (all the forces neutralizing each other), we can construct a strain diagram which will always be a *closed* polygon with as many sides as there are forces, and each side equal and parallel to one of the forces, and the sides being in the same succession

to each other as the forces are. We can now proceed to dissect a simple truss.

Roof Trusses. Take a roof truss with two rafters and a single tie-beam. The rafters are supposed to be loaded uniformly, and to be strong enough not to give way transversely, but to transfer safely one-half of the load on each rafter to be supported on each joint at the ends of the rafter. We consider each joint separately.

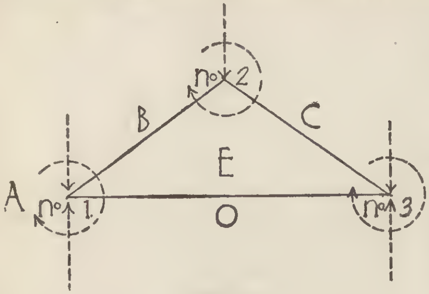


Fig. 25.

Take joint No. 1, Fig. 25.

We have four forces, one O A (the left-hand reaction), being equal to half the load on the whole truss; next, A B, equal to half the load on the rafter B E. Then we have the force acting along B E, of which we do not as yet know amount or direction (up or down), but only know that it is parallel to B E; the same is all we know, as yet, of the force E O. Now draw, at any scale, Fig. 26, No. 1, *o a* = and parallel to O A, then from *a* draw *a b* = and parallel to A B (*a b* will, of course, lap over part of *o a*, but this does not affect anything). Then from *b* draw *b e* parallel to B E, and through *o* draw *e o* parallel to E O. Now, in reading off strains, begin at O A, then pass in succession to A B, B E and E O. Follow on the strain diagram Fig. 26, No. 1, the direction as read off, with the finger (that is, *o a*, *a b*, *b e* and *e o*), and we have the actual directions of the strains.

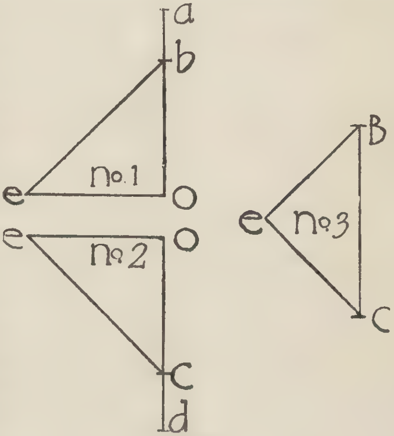


Fig. 26.

Thus *o a* is up, therefore pushing up; *a b* is down, therefore pushing down; *b e* is downwards, therefore pushing against joint No. 1 (and we know it is compression); lastly, *e o* is

pushing to the right, therefore pulling away from the joint No. 1, and we know it is a tie-rod. In a similar manner we examine the joints 2 and 3, getting the strain diagrams No. 2 and No. 3 of Fig. 26. In Fig. 27, we get the same results exactly as in the above three diagrams of Fig. 26, only for simplicity they are combined into one diagram. If the single (combination) diagram, Fig. 27, should prove confusing to the student, let him make a separate diagram for each joint, if he will, as in Fig. 26. The above gives the principle of calculating the strength of trusses, graphically, and will be more fully used later on in practical examples.

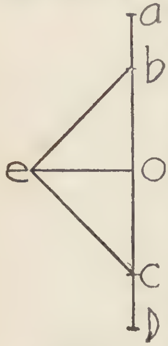


Fig. 27.

Should the student desire a fuller knowledge of the subject, let him refer to "*Greene's Analysis of Roof Trusses*," which is simple, short, and one of the best manual on the subject.

Line of Pressure Central.

In roof and other trusses the line of pressure or tension will always be co-incident with the central line or longitudinal axis of each piece. Each joint should, therefore, be so designed that the central lines or axes of all the pieces will go through one point. Thus, for instance, the foot of a king post should be designed as per Fig. 28.

In roof-trusses where the rafters support purlins, the rafters must not only be made strong enough to resist the compressive strain on them, but *in addition to this enough material must be added* to stand the transverse strain. Each part of the rafter is treated as a separate beam, supported at each joint, and the amount of reaction at each joint must be taken as the load at the joint. The same holds good of the tie-beam, when it has a ceiling or other weights suspended from it; of course these weights must all be shown by arrows on the drawing of the truss, so as to get their full allowance in the strain diagram. Strains in opposite directions, of course, counteract each other; the stress, therefore, to be exerted by the material need only be equal to the difference between the amounts of the opposing strains, and, of course, this stress will be directed against the larger strain.

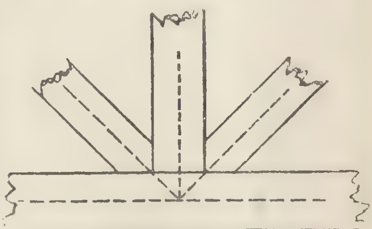


Fig. 28.

THE ARCH.

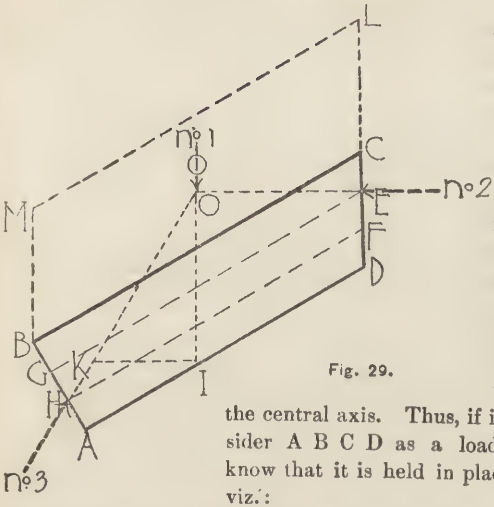


Fig. 29.

We consider an arch as a truss with a succession of straight pieces; we can calculate it graphically the same as any other truss, only we will find that the absence of central or inner members (struts and ties) will force the line of pressure, as a rule, far away from the central axis. Thus, if in Fig. 29, we consider A B C D as a loaded half-arch, we know that it is held in place by three forces, viz.:

1. The load B C L M which acts through its centre of gravity as indicated by arrow No. 1.

2. A horizontal force No 2 at the crown C D, which keeps the arch from spreading to the right.

3. A force at the base B A (indicated by the arrow No. 3), which keeps the arch from spreading at the base. Now we know the direction and amount of No. 1, and can easily find Nos. 2 and 3. In an arch lightly loaded, No. 2 is always assumed to act at two-thirds way down C D, that is at F (where C E = E F = F D = $\frac{1}{3}$ C D). In an arch heavily loaded, No. 2 is always assumed to act one-third way down C D, that is at E; further the force No. 3 is always assumed to act through a point two-thirds way down B A, that is at H (where B G = G H = H A = $\frac{1}{3}$ B A). The reason for these assumptions need not be gone into here. Therefore to find forces Nos. 2 and 3 proceed as follows: If the arch is heavily loaded, draw No. 2 horizontally through E (C E being equal to $\frac{1}{3}$ C D), prolong No 2 till it intersects No. 1 at O, then draw O H (H A being equal to $\frac{1}{3}$ B A), which gives the direction of the resistance No. 3. We now have the three forces acting on the arch concentrated at the point O, and can easily find the amounts of each by using the parallelogram of forces. Make O I vertical and (at any scale) equal to whole load (or No. 1), draw I K horizontally, till it intersects O H at K; then scale I K,

and K O (at same scale as O I), which will give the amount of the forces Nos. 2 and 3. The line of pressure of this arch A B C D, is therefore not through the central axis, but along E O H (a curve drawn through E and H with the lines Nos. 2 and 3 as tangents is the real line of pressure).

Line of pressure not central.

Now let in Fig. 30, A B C E F D A represent a half-arch. We can examine A B C D same as before, and obtain I K = to force

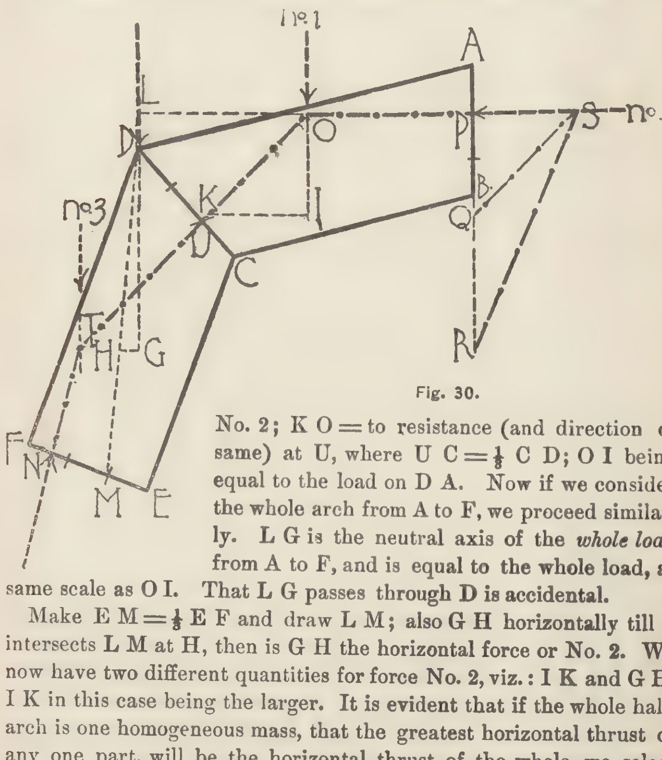


Fig. 30.

No. 2; K O = to resistance (and direction of same) at U, where U C = $\frac{1}{3}$ C D; O I being equal to the load on D A. Now if we consider the whole arch from A to F, we proceed similarly. L G is the neutral axis of the *whole load* from A to F, and is equal to the whole load, at same scale as O I. That L G passes through D is accidental.

Make E M = $\frac{1}{3}$ E F and draw L M; also G H horizontally till it intersects L M at H, then is G H the horizontal force or No. 2. We now have two different quantities for force No. 2, viz.: I K and G H, I K in this case being the larger. It is evident that if the whole half-arch is one homogeneous mass, that the greatest horizontal thrust of any one part, will be the horizontal thrust of the whole, we select therefore the larger force or I K as the amount of the horizontal thrust.

Now make S P = to I K and P Q = to No. 1, or load on A D and P R = to L G or whole load on F A, at any scale, then draw Q S and R S. Now at O we have the three forces concentrated, which act on the part of arch A B C D, viz.: Load No. 1 (= P Q), horizontal force No 2 (= S P) and resistance K O (= Q S). Now let No. 3 represent the vertical neutral axis of the part of whole load on F D, then

prolong K O until it intersects No. 3 at T; then at T we have the three forces acting on the part of arch E C D F, viz.: The load No. 3 (= Q R), the thrust from A B C D, viz.: O T (= S Q), and the resistance N T (= R S). To obtain N T draw through T a line parallel to R S, of course R S giving not only the direction, but also the amount of the resistance N T. The line of pressure of this arch therefore passes along P O, O T, T N. A curve drawn through points P, U and N — (that is, where the former lines intersect the joints A B, D C, F E) — and with lines P O, O T and T N as tangents is the real line of pressure. Of course the more parts we divide the arch into, the more points and tangents will we have, and the nearer will our line of pressure approach the real curve.

Now if this line of pressure would always pass through the exact centre or axis of the arch, the compression on each joint would of course be uniformly spread over the whole joint, and the amount of this compression on each square-inch of the joint would be equal to the amount of (line of) pressure at said joint, divided by the area of the cross-section of the arch in square inches, at the joint, but this rarely occurs, and as the position of the line of pressure varies from the central axis so will the strains on the cross section vary also.

Stress at intrados and extrados. Let the line A B in all the following figures represent the section of any joint of an arch (the thickness of arch being overlooked) C D the amount and actual position of line of pressure at said joint and the small arrows the stress or resistance of arch at the joint.

We see then that when C D is in the centre of A B, Fig. 31, the stress is uniform, that is the joint is, uniformly compressed, the amount of compression being equal to the average as above. As the line of pressure C D approaches one side, Fig. 32, the amount of compression on that side increases, while on the further side it decreases, until the line of pressure C D, reaches one-third way, Fig. 33 (that is, D B=⅓ A B). Then we see there is no compres-

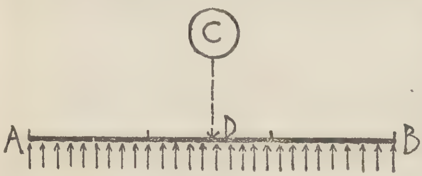


Fig. 31.

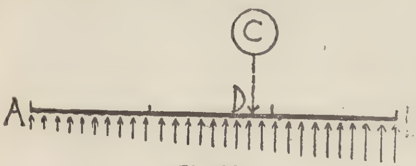


Fig. 32.

sion at A, but at B the compression is equal to just double the average as it was in Fig. 31. Now, as C D passes beyond the central third of A B, Fig. 34, the compression at the nearer side increases still further, while the further side begins to be subjected to stress in the opposite direction or tension, this action increasing of course the further C D is moved from the central third. This means that the edge of arch section at B would be subject to very severe crushing, while the other edge (at A) would tend to separate or open. When the line C D passes on to the edge B, the nearer two-thirds of arch joint will be in compression, and the further third in tension. As the line passes out of joint, and further and further away from B, less and less of the joint is in compression, while more and more is in tension, until the line of pressure C D gets so far away from the joint finally, that one-half of the joint would be in tension, and the other half in compression.

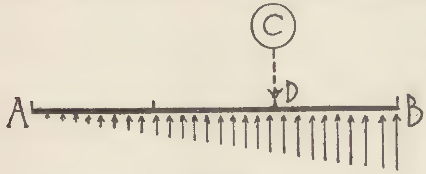


Fig. 33.

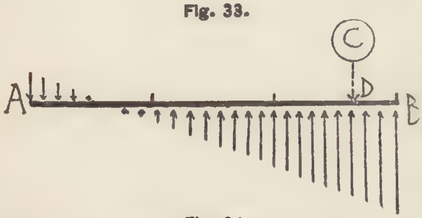


Fig. 34.

Tension means that the joint is tending to open upwards, and as arches are manifestly more fit to resist crushing of the joints than opening, it becomes apparent why it is dangerous to have the line of pressure far from the central axis. Still, too severe crushing strains must be avoided also, and hence the desirability of trying to get the line of pressure into the inner third of arch ring, if possible.

But the fact of the line of pressure coming outside of the inner third of arch ring, or even entirely outside of the arch, does not necessarily mean that the arch is unstable; in these cases, however, we must calculate the exact strains on the extreme fibres of the joint at both the inner and outer edges of the arch (intrados and extrados), and see to it that these strains do not exceed the *safe* stress for the material.

The formulæ to be used, are:

For the fibres at the edge nearest to the line of pressure

$$v = \frac{p}{a} + 6 . \frac{x . p}{a . d} \qquad (44)$$

And for the fibres at the edge furthest from the line of pressure

$$v = \frac{p}{a} - 6.\ \frac{x.p}{a.d} \qquad (45)$$

Where v = the stress in lbs., required to be exerted by the extreme edge fibres (at intrados or extrados).

Where x = the distance of line of pressure from centre of joint in inches.

Where a = the area of cross section of arch at the joint, in square inches.

Where p = the total amount of pressure at the joint in lbs.

Where d = the depth of arch ring at the joint in inches, measured from intrados to extrados.

When the result of the formulæ (44) and (45) is a positive quantity the stress v should not exceed $\left(\frac{c}{f}\right)$, that is the safe compressive stress of the material. When, however, the result of the formula (45) yields a negative quantity, the stress v should not exceed $\left(\frac{t}{f}\right)$, that is the safe tensile stress of the material, or mortar.

The whole subject of arches will be treated much more fully later on in the chapter on arches.

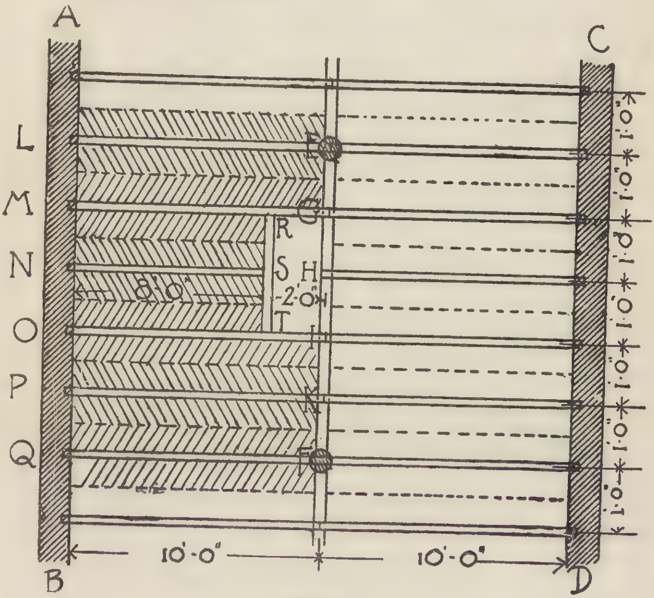


Fig. 35.

TO ASCERTAIN AMOUNT OF LOADS.

Let A B C D be a floor plan of a building, A B and C D are the walls, E and F the columns, with a girder between, the other lines being floor beams, all 12″ between centres; on the left side a well-hole is framed 2′ x 2′. Let the load assumed be 100 pounds per square foot of floor, which includes the weight of construction. Each of the right-hand beams, also the three left-hand beams E L, K P and F Q will each carry, of course, ten square feet of floor, or

Load on Beams.

$10.100 = 1000$ pounds each uniform load. Each will transfer one-half of this load to the girder and the other half to the wall. The tail beam S N will carry 8 square feet of floor, or

$8.100 = 800$ pounds uniform load. One-half of this load will be transferred to the wall, the other half to the header R T, which will therefore carry a load of 400 pounds at its centre, one-half of which will be transferred to each trimmer.

The trimmer beam G M carries a uniform load, one-half foot wide, its entire length, or fifty pounds a foot (on the off-side from well-hole), or

$50.10 = 500$ pounds uniform load, one-half of which is transferred to the girder and the other half to the wall. The trimmer also carries a similar load of fifty pounds a foot on the well-hole side, but only between M and R, which is eight feet long, or

$50.8 = 400$ pounds, the centre of this load is located, of course, half way between M and R, or four feet from support M, and six feet from support G, therefore M will carry (react)

$$\frac{6.400}{10} = 240 \text{ pounds and G will carry}$$

$$\frac{4.400}{10} = 160 \text{ pounds.}$$

See Formulæ (14) and (15).

We also have a load of 200 pounds at R, transferred from the header on to the trimmer; as R is two feet from G, and eight feet from M, we will find by the same formulæ, that G carries

$$\frac{8.200}{10} = 160 \text{ pounds and M carries}$$

$$\frac{2.200}{10} = 40 \text{ pounds.}$$

So that we find the loads which the trimmer transfers to G and M as follows:

At $M = 250 + 240 + 40 = 530$ pounds.

" $G = 250 + 160 + 160 = 570$ pounds.

The loads which trimmer O I transfers to wall and girder will, of course, be similar. We therefore find the total loading, as follows:

Load on Walls.

On the wall A B:

At L = 500 pounds.
" M = 530 pounds.
" N = 400 pounds.
" O = 530 pounds.
" P = 500 pounds.
" Q = 500 pounds.
Total on wall A B = 2960 pounds.

On the wall C D we have six equal loads of 500 pounds each, a total of 3000 pounds.

Load on Girder.

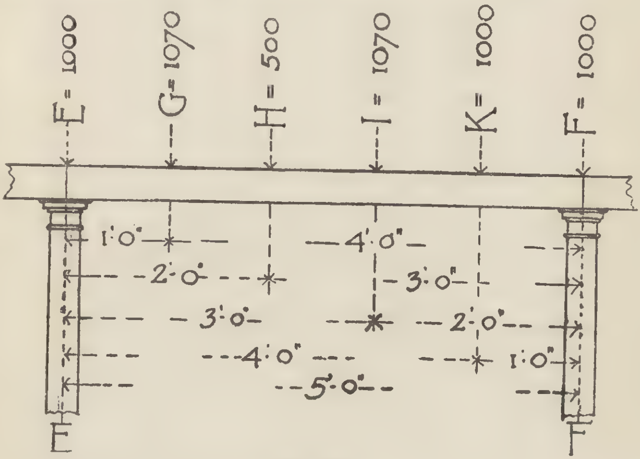


Fig. 36.

On the girder E F, we have:

At E from the left side 500 pounds, from the right 500 pounds.	Total 1000 pounds.
At G from the left side 570 pounds, from the right 500 pounds.	Total 1070 pounds
At H from the left side nothing, from the right 500 pounds.	Total 500 pounds
At I from the left side 570 pounds, from the right 500 pounds.	Total 1070 pounds.
At K from the left side 500 pounds, from the right 500 pounds.	Total 1000 pounds.

At F from the left side 500 pounds, from the right 500 pounds. Total 1000 pounds.

Total on girder 5640 pounds.

As the girder is neither uniformly nor symmetrically loaded, we must calculate by Formulæ (16) and (17), the amount of each reaction, which will, of course, give the load coming on the columns E and F. (These columns will, of course carry additional loads, from the girders on opposite side, further, the weight of the column should be added, also whatever load comes on the column at floor above.)

Girder E F then transfers to columns,

At $E = 1000 + (\frac{4}{5}.\ 1070) + (\frac{3}{5}.\ 500) + (\frac{2}{5}.1070) + (\frac{1}{5}.\ 1000) + (0.\ 1000) = 2784$ pounds.

At $F = 1000 + (\frac{4}{5}.\ 1000) + (\frac{3}{5}.\ 1070) + (\frac{2}{5}.\ 500) + (\frac{1}{5}.\ 1070) + (0.\ 1000) = 2856$ pounds.

As a check the loads at E and F must equal the whole load on the girder, and we have, in effect,

$$2784 + 2856 = 5640.$$

Now as a check on the whole calculation the load on the two columns and two walls should equal the whole load. The whole load being $20' \times 6' \times 100$ pounds minus the well-hole $2' \times 2' \times 100$ pounds, or $12000 - 400 = 11600$ pounds.

And we have in effect,

Load on A B	=	2960 pounds.
" C D	=	3000 pounds.
" two columns	=	5640 pounds.
Total loads	=	11600 pounds.

We therefore can calculate the strength of all the beams, headers and trimmers and girders, with loads on, as above given.[1]

For the columns and walls, we must however add, the weight of walls and columns above, including all the loads coming on walls and columns above the point we are calculating for, also whatever load comes on the columns from the other sides. If there are openings in a wall, one-half the load over each opening goes to the pier each side of the opening, including, of course, all loads on the wall above the opening.

Load over Wall openings.

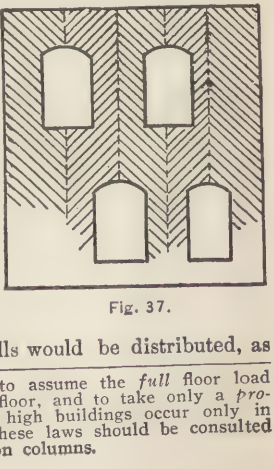


Fig. 37.

Thus in Figure 37, the weight of walls would be distributed, as

[1] In very high buildings it is customary to assume the *full* floor load only on the column immediately under the floor, and to take only a *proportion* of the floor loads from above. As high buildings occur only in large cities, and their building laws vary, these laws should be consulted in making any modifications of floor loads on columns.

indicated by etched lines; where, however, the opening in the wall is very small compared to the mass of wall-space over, it would, of course, be absurd to consider all this load as on the arch, and practically, after the mortar has set, it would not be, but only an amount about equal to the part enclosed by dotted lines in Figure 38, the inclined lines being at an angle of 60° with the horizon. Where only part of the wall is calculated to be carried on the opening, the wooden centre should be left in until the mortar of the entire wall has set. In case of beams or lintels the wall should be built up until the intended amount of load is on them, leaving them free underneath; after the intended load is on them, they should be shored up, until the rest of wall is built and thoroughly set.

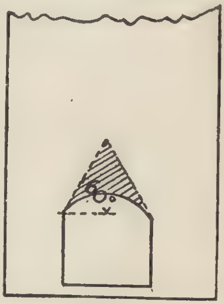


Fig. 38.

Wind Pressure and Snow. Wind-pressure on a roof is generally assumed at a certain load per square foot superficial measurement of roof, and added to the actual (dead) weight of roof; except in large roofs, or where one foot of truss rests on rollers, when it is important to assume the wind as a separate force, acting at right angles to incline of rafter.

The load of snow on roofs is generally omitted, when wind is allowed for, as, if the roof is very steep snow will not remain on it, while the wind pressure will be very severe; while, if the roof is flat there will be no wind pressure, the allowance for which will, of course, offset the load of snow.

If the roof should not be steep enough for snow to slide off, a heavy wind would probably blow the snow off.

In case of "continuous girders," that is, beams or girders supported at three or more points and passing over the intermediate supports without being broken, it is usual to allow more load on the central supports, than the formulæ (14) to (17) would give. This subject will be more fully dealt with in the chapter on beams and girders.

FATIGUE.

If a load or strain is applied to a material and then removed, the material is supposed to recover its first condition (provided it has not been strained beyond the limit of elasticity). This practically, however, is not the case, and it is found that a small load or strain often applied and removed will do more damage (fatigue the material more) than a larger one left on steadily. Most loads in buildings

are stationary or "dead" loads. But where there are "moving" loads, such as people moving, dancing, marching, etc., or machinery vibrating, goods being carted and dumped, etc., it is usual to assume larger loads than will ever be imposed; sometimes going so far as to double the actual intended load, or what amounts to the same thing, doubling (or increasing) the factor-of-safety, in that case retaining, of course, the actual intended load in the calculations. This is a matter in which the architect must exercise his judgment in each individual case.

Moving Loads.

Dead Weight of Wind Pressure on Walls.

The vertical effect of wind at right angles to a vertical wall, is generally assumed as equal to 15 pounds vertical load for each square foot of outside vertical wall surface exposed to wind. Thus on a wall 100 feet high, there would be (for wind pressure) an additional vertical load to be added to it, per *running foot* horizontal, of 150 lbs. ten feet below roof, 300 lbs. twenty feet below roof, etc., and of 1500 lbs. per running foot at ground level. This load is supposed to act at the centre of thickness of wall.

CHAPTER II.

FOUNDATIONS.

Nature of Soils. The nature of the soils usually met with on building sites are: rock, gravel, sand, clay, loamy earth, "made" ground and marsh (soft wet soil).

If the soil is hard and *practically* non-compressible, it is a good foundation and needs no treatment; otherwise it must be carefully prepared to resist the weight to be superimposed.

Stepping Courses. The base-courses of all foundation walls must be spread (or stepped out) sufficiently to so distribute the weight that there may be no appreciable settlement (compression) in the soil.

Two important laws must be observed:—

1. All base-courses must be so proportioned as to produce exactly the same pressure per square inch on the soil under all parts of building where the soil is the same. Where in the same building we meet with different kinds of soils, the base-courses must be so proportioned as to produce the same relative pressure per square inch on the different soils, as will produce an equal settlement (compression) in each.

2. Whenever possible, the base-course should be so spread that its neutral axis will correspond with the neutral axis of the superimposed weight; otherwise there will be danger of the foundation walls settling unevenly and tipping the walls above, producing unsightly or even dangerous cracks.

Example.

In a church the gable wall is 1′ 6″ thick, and is loaded (including weight of all walls, floors and roofs coming on same) at the rate of 52 lbs. per square inch. The small piers are 12″ x 12″ and 5′ high, and carry a floor space equal to 14′ x 10′. What should be the size of base-courses, it being assumed that the soil will safely stand a pressure of 30 lbs. per square inch?

If we were to consider the wall only, we should have the total pressure on the soil per running inch of wall, $18.52 = 936$ lbs.

Dividing this by 30 lbs., the safe pressure, we should need $\frac{936}{30} =$ 31,2″ or say 32″ width of foundation, or we should step out *each side* of foundation wall an amount $\frac{32-18}{2} = 7''$ each side.

Now the load on pier, assuming the floor at 100 lbs. per square

foot, would be $14 \times 10 \times 100 = 14000$ lbs. To this must be added the weight of the pier itself. There are 5 cubic feet of brickwork (weighing 112 pounds per foot)$=5.112=560$ lbs., or, including base-course, a total load of say 15000 lbs. This is distributed over an average of 144 square inches; therefore pressure per square inch under pier.

$$\frac{15000}{144} = 104 \text{ or, say, } 100 \text{ lbs.}$$

We must therefore make the foundation under pier very much wider, in order to avoid unequal settlements. The safe pressure per square inch we assumed to be 30 lbs.; therefore the area required would be $= \frac{15000}{30} = 500$ square inches, or a square about 22″x22″.

We therefore shall have to step out *each side* of the pier an amount $\frac{22-12}{2} = 5''$.

The safe compressions for different soils are given in Table V, but in most cases it is a matter for experienced judgment or else experiment.[1]

Testing Soils. It is usual to bore holes at intervals, considerably deeper than the walls are intended to go, at some spot where no pressure is to take place, thus enabling the architect to judge somewhat of the nature of the soil. If this is not sufficient, he takes a crowbar, and, running it down, his experienced touch should be able to tell whether the soil is solid or not. If this is not sufficient, a small pumping or tube testing-machine should be obtained, and samples of the soil, at different points of the lot, bottled for every one or two feet in depth. These can be taken to the office and examined at leisure. The boring should be continued if possible, until hard bottom is struck.

If the ground is soft, new made, or easily compressible, experiment as follows: Level the ground off, and lay down four blocks each, say 3″x3″; on these lay a stout platform. Alongside of platform plant a stick, with top level of platform marked on same. Now pile weight onto platform gradually, and let same stand. As soon as platform begins to sink appreciably below the mark on stick, you have the practical ultimate resistance of the foundation; this divided by 36 gives the ultimate resistance of the foundation per square inch. One-tenth of this only should be considered as a safe load for a permanent building.

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[1] In most large cities the building laws cover the amount of safe compressions to be allowed on various soils.

Drainage of Soil. Drainage is essential to make a building healthy, but can hardly be gone into in these articles. Sometimes it is also necessary to keep the foundations from being undermined.

It is usual to lead off all surface or spring water by means of blind drains, built underground with stone, gravel, loose tile, agricultural tile, half-tile, etc. To keep dampness out, walls are cemented and then asphalted, both on the outsides. If the wall is of brick, the cementing can be omitted. **Damp-proofing.** Damp-courses of slate or asphalt are built into walls horizontally, to keep dampness from rising by capillary attraction.[1] Cellar bottoms are concreted and then asphalted; where there is pressure

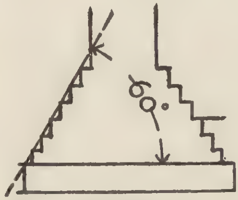


Fig. 39.


Fig. 40.

of water from underneath, such as springs, tide-water, etc., the asphalt has to be sufficiently weighted down to resist same, either with brick paving or concrete. Frequently courses of tarred paper between concrete layers are used for damp-proofing; these are carried up walls, around and up piers, pipes, etc., being careful to make all tight, to well above water level.

Where there are water-courses they should be diverted from the foundations, but never dammed up. They can often be led into iron or other wells sunk for this purpose, and from there pumped into the building to be used to flush water-closets, or for manufacturing or other purposes. Clay, particularly in vertical or inclined layers, and sand are the foundations most dangerously affected by water as they are apt to be washed out.

Where a very wide base-course is required by the nature of the soil, it is usual to step out the wall above gradually; the angle of stepping should never be more acute than 60°, or, as shown in Figure 39. Care must also be taken that the stepped-out courses are sufficiently wide to project well in under each other and wall, to prevent same breaking through foundation, as indicated in Figure 40.

Where, on account of party lines or other buildings, the stepping

[1] When it is preferred to use liquid asphalt in place of a prepared asphalt damp-course, a key should be built along the entire length of the wall to keep the wall above from slipping. This key consists of a course of brick, 8 in. wide, laid about centre of wall, and top and *sides* of key must be asphalted, too.

out of a foundation wall has to be done entirely at one side, the stepping should be even steeper than 60°, if possible; and particular attention must be paid to anchoring the walls together as soon and as thoroughly as possible, in order to avoid all danger of the foundation wall tipping outwardly.

Where a front or other wall is composed of isolated piers, it is well to combine all their foundations into one, and to step the piers down for this purpose, as shown in Figure 41. Where there is not sufficient depth for this purpose, inverted arches must be resorted to.

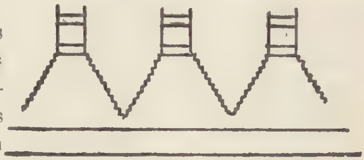


Fig. 41.

The manner of calculating the strength of inverted arches will be given under the article on arches. Inverted arches are not recommended, however (except where the foundation wall is by necessity very shallow), as it requires great care and good mechanics to build them well.

Inverted arches.

Two things must particularly be looked out for: 1, That the end arch has sufficient pier or other abutment; otherwise it will throw the pier out, as indicated in Figure 42. (This will form part of calculation of strength of arch.) Where there is danger of this, ironwork should be resorted to, to tie back the last pier.

Fig. 42.

Size of Skew-back.

2. The skew-back of the arch should be sufficiently wide to take its proportionate share of load from the pier (that is, amount of the two skew-backs should be proportioned to balance of pier or centre part of pier, as the width of opening is to width of pier); otherwise the pier would be apt to crack and settle past arch, as shown in Figure 43.

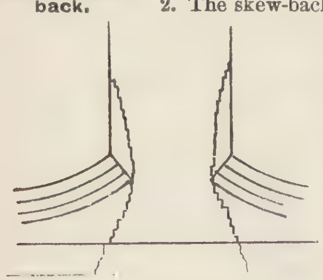


Fig. 43.

An easy way of getting the width of skew-back graphically is given below. In Figure 44,

draw A B horizontally at springing-line of inverted arch; bisect A C at F, and C B at E. Draw E O at random to vertical through F; then draw O C, and parallel to O C draw G D; then is C D the required skew-back.[1]

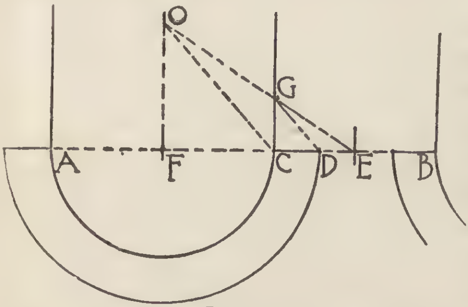


Fig. 44.

A good way to do is to give the arches wide skew-backs, and then to introduce a thick granite or blue-stone pier stone over them, as shown in Figure 45. This will force all down evenly and avoid cracks. The stone must be thick enough not to break at dotted lines, and should be carefully bedded.

Example.

A foundation pier carrying 150000 *lbs. is* 5′ *wide and* 3′ *broad The inverted arches are each* 24″ *deep. What thickness should the granite block have?*

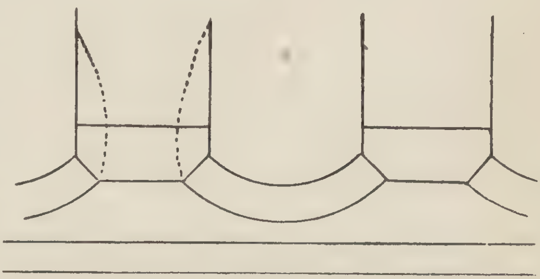


Fig. 45.

We have here virtually a granite beam, 60″ long and 36″ broad, supported at two points (the centre lines of skew-backs) 36″ apart. The load is a uniform load of 150000 lbs. The safe modulus of rupture, according to Table V, for average granite is $\left(\frac{k}{f}\right) = 180$ lbs.

[1] In reality C D should be somewhat larger than the amount thus obtained; but this can be overlooked, except in cases where the pier approaches in width the width of opening. In such cases, however, stepping can generally be resorted to in place of inverted arches. Then, too, if the opening were very wide and the line of pressure came very much outside of central third of C D, it might be necessary to still further increase the width of skew-back, C D.

The bending moment on this beam, according to Formula (21) is

$$m = \frac{u.l}{8} = \frac{150\,000.36}{8} = 675\,000$$

The moment of resistance, r, is, from Formula (18)

$$r = \frac{m}{\left(\frac{k}{f}\right)} = \frac{675\,000}{180} = 3\,750$$

From Table I, No. 3, we find

$$r = \frac{bd^2}{6}, \text{ therefore}$$

$$\frac{bd^2}{6} = 3750 \text{ ; now, as } b = 36, \text{ transpose and we have}$$

$$d^2 = \frac{3750.6}{36} = 625.$$

Therefore $d = 25''$ or say $24''$. The size of granite block would have to be $5' \times 3' \times 2'$.

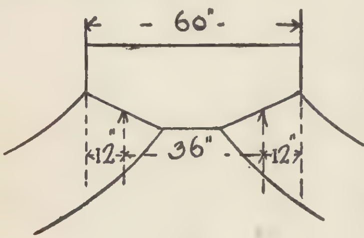


Fig. 46.

As this would be a very unwieldy block, it might be split in two lengthwise of pier; that is, two stones, each $5' \times 18'' \times 2'$ should be used, and clamped together. Before building piers, the arch should be allowed to get thoroughly set and hardened, to avoid any after shrinkage of the joints.

A parabolic arch is best. Next in order is a pointed arch, then a semi-circular, next elliptic, and poorest of all, a segmental arch, if it is very flat. But, as before mentioned, avoid inverted arches, if possible, on account of the difficulty of their proper execution.

Rock foundations. A rock foundation makes an excellent one, and needs little treatment, but is apt to be troublesome because of water. Remove all rotten rock and step off all slanting surfaces, to make

Fig. 47.

level beds, filling all crevices with concrete, as shown in Figure 47 :

In no case build a wall on a slanting foundation.

Look out for springs and water in rock foundations. Where soft

soils are met in connection with rock, try and dig down to solid rock, or, if this is impossible, on account of the nature of the case or expense, dig as deep as possible and put in as wide a concrete base-course as possible. If the bad spot is but a small one, arch over from rock to rock, as shown in Figure 48:

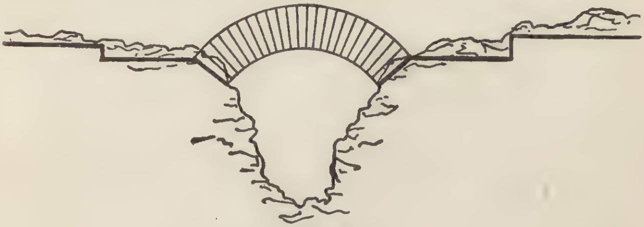


Fig. 48.

Sand, gravel and clay. Good hard sand or even quicksand makes an excellent foundation, if it can be kept from shifting and clear of water. To accomplish this purpose it is frequently "sheath-piled" each side of the base course.

Gravel and sand mixed make an excellent, if not the best foundation; it is practically incompressible, and the driest, most easily drained and healthiest soil to build on.

Clay is a good foundation, if in horizontal layers and of sufficient thickness to bear the superimposed weight. It is, however, a very treacherous material, and apt to swell and break up with water and frost. Clay in inclined or vertical layers cannot be trusted for important buildings, neither can loamy earth, made ground or marsh. If the base-course cannot be sufficiently spread to reduce the load to a minimum, pile-driving has to be resorted to. This is done in many different ways. **Short piles.** If there is a layer of hard soil not far down, short piles are driven to reach down to same. These should be of sufficient diameter not to bend under their load; they should be calculated the same as columns. The tops should be well tied together and braced, to keep them from wobbling or spreading.

Example.

Georgia-pine piles of 16″ *diameter are driven through a layer of soft soil* 15′ *deep, until they rest on hard bottom. What will each pile safely carry?*

The pile evidently is a circular column 15′ long, of 16″ diameter, solid, and we should say with rounded ends, as, of course, its bear-

ings cannot be perfect. From Formula (3) we find, then, that the pile will safely carry a load.

$$w = \frac{a.\left(\frac{c}{f}\right)}{1 + \frac{l^2.n}{\rho^2}},$$ now from Table I, Section No. 7, and fifth column, we have

$$a = \frac{22}{7}.\,\mathfrak{r}^2 = \frac{22.8^2}{7} = 201.$$

From the same table we find, for Section No. 7, last column,

$$\rho^2 = \frac{\mathfrak{r}^2}{4} = \frac{8^2}{4} = 16.$$

From Table IV we find for Georgia pine, along fibres, $\left(\frac{c}{f}\right) = 750.$

And from Table II, for wood with rounded ends,

$n = 0{,}00067$, therefore:

$$w = \frac{201.750}{1 + \frac{180^2.\,0{,}00067}{16}} = \frac{150750}{2{,}357} = 63958,$$

or say 30 tons to each pile.

Sand piles. Sometimes large holes are bored to the hard soil and filled with sand, making "sand piles." This, of course, can only be done where the intermediate ground is sufficiently firm to keep the sand from escaping laterally. Sometimes holes are dug down and filled in with concrete, or brick piers are built down; or large iron cylinders are sunk down and the space inside of them driven full of piles, or else excavated and filled in with concrete or other masonry, or even sand, well soaked and packed. If filled with sand, there should first be a layer of concrete, to keep the sand from possibly escaping at the bottom.

Where no hard soil can be struck, piles are driven over a large area, and numerous enough to consolidate the ground; they should not be closer than two feet in the clear each way, or they will cut up the ground too much. The danger here is that they may press the ground out laterally, or cause it to rise where not weighted. Sometimes, by sheath-piling each side, the ground can be sufficiently compressed between the piles, thereby being kept from escaping laterally.

Long piles. But by far the most usual way of driving piles is where they resist the load by means of the friction of their sides against the ground. In such cases it is usual to drive experimental piles, to ascertain just how much the pile descends at the last blow of the hammer or ram; also the amount of fall and weight of ram, and

then to compute the load the pile is capable of resisting: one-tenth of this might be considered safe.

The formula then is: —

$$w = \frac{r.\,f}{10.\,s} \qquad (46)$$

Where $w =$ the safe load on each pile, in lbs.

" $r =$ the weight of ram used, in lbs.

" $f =$ the distance the ram falls, in inches.

" $s =$ the set, in inches, or distance the pile is driven at the last blow.

Where there is the least doubt about the stability of the pile, use three-fourths w, and if the piles drive very unevenly, use only one-half w.

Some engineers prefer to assume a fixed rule for all piles. Professor Rankine allows 200 lbs. per square inch of area of head of pile. French engineers allow a pile to carry 50000 lbs., provided it does not sink perceptibly under a ram falling 4′ and weighing 1350 lbs., or does not sink half an inch under thirty blows. There are many other such rules, but the writer would recommend the use of the above formula, as it is based on each individual experiment, and is therefore manifestly safer.

Example.

An experimental pile is found to sink one-half inch under the last blow of a ram weighing 1500 *lbs., and falling* 12′. *What will each pile safely carry?*

According to formula (46), the safe load w would be

$w = \frac{r.\,f}{10.\,s}$ we have,

$r = 1500$ lbs.

$f = 12.12 = 144''$, and

$s = \frac{1}{2}''$, therefore

$w = \frac{1500.144}{10.\frac{1}{2}} = 43200$ lbs.

If several other piles should give about the same result, we would take the average of all, or else allow say 20 tons on each pile. If, however, some piles were found to sink considerably more than others, it would be better to allow but 10 tons or 15 tons, according to the amount of irregularity of the soil.

All cases of pile-driving require experience, judgment, and more or less experiment; in fact all foundations do.

All piles should be straight, solid timbers, free from projecting

branches or large knots. They can be of hemlock, spruce or white pine, but preferably, of course, of yellow pine or oak.

There is danger, where they are near the seashore, of their being destroyed by worms. To guard against this, the bark is sometimes shrunken on; that is, the tree is girdled (the bark cut all around near the root) before the tree is felled, and the sap ceasing to flow, the bark shrinks on very tightly.

Others prefer piles without bark, and char the piles, coat them with asphalt, or fill the pores with creosote. Copper sheets are the best (and the most expensive) covering.

Piles should be of sufficient size not to break in driving, and should, as a rule, be about 30′ long, and say 15″ to 18″ diameter at the top. They should not be driven closer than about 2′ 6″ in the clear, or they will be apt to break the ground all up. The feet should be shod with wrought-iron shoes, pointed, and the heads protected with wrought-iron bands, to keep them from splitting under the blows of the ram.

Sheath-piling. In sheath-piling it is usual to take boards (hemlock, spruce, white pine, yellow pine or oak) from 2″ to 6″ thick. Guide-piles are driven and cross pieces bolted to the insides of them. The intermediate piles are then driven between the guide piles, making a solid wooden wall each side, from 2″ to 6″ thick. Sometimes the sheath-piles are tongued and grooved. The feet of the piles are cut to a point, so as to drive more easily. The tops are covered with wrought-iron caps, which slip over them and are removed after the piles are driven.

Iron piles. Piles are sometimes made of iron; cast-iron being preferable, as it will stand longer under water. Screw-piles are made of iron, with large, screw-shaped flanges attached to the foot, and they are screwed down into the ground like a gimlet.

Sheath-piling is sometimes made of cast-iron plates with vertical strengthening ribs, and sometimes of steel plates, channels or other metal sections.

Where piles are driven under water, great care must be taken that they are *entirely immersed*, and at all times so. They should be cut off to a uniform level, below the lowest low-water mark. If they are alternately wet and dry, they will soon be destroyed by decay.

Base-courses over piles. After the piles are cut to a level, tenons are often cut on their tops, and these are made to fit mortises in heavy wooden girders which go over them, and on which the superstructure

The sizes given on this page are for heavy buildings; for very light work use piles of 8″ to 9″ diameter at tops, about 20 feet long and 16″ apart.

rests. This is usual for docks, ferry-houses, etc. For other buildings we frequently see concrete packed between and over their tops; this, however, is a very bad practice, as the concrete surrounding the tops is apt to decay them. It is better to cover the piles with 3″ x 12″ or similar planks (well lag-screwed to piles, where it is necessary to steady the latter) and then to build the concrete base course on these planks.[1]

Better yet, and the best method, is to get large-sized building-stones, with levelled beds, and to rest these directly on the piles. In this case care must be taken that piles come at least under each corner of the stone, or oftener, to keep it from tipping, and that the stone has a full bearing on each pile-head. On top of stone build the usual base-courses. Now-a-days the concrete is frequently reinforced and strengthened by steel grillage beams, where stone courses would be too expensive.

Piles should be as nearly uniform as possible (particularly in the case of short piles resting on hard ground), for otherwise their respective powers of resistance will vary very much.

Slip-Joints. It is well to connect all very heavy parts of buildings (such as towers, chimneys, etc.) by vertical slip-joints with rest of building. The slip-joint should be carried through the foundation-walls and base-courses, as well as above.

Where there are very high chimneys or towers, or unbraced walls, the foundation must be spread sufficiently to overcome the leverage produced by wind. These points will be more fully explained in the chapter on "Walls and Piers."

Action of Frost. All base-courses should be carried low enough to be below frost, which will penetrate from three to five feet deep in our latitudes. The reason of this is that the frost tends to swell or expand the ground (on account of its dampness) in all directions, and does it with so much force that it would be apt to lift the base-course bodily, causing cracks and possible failure above.

Concrete Piles. In the last few years concrete piles have begun to supersede wooden piles. The latter owing to the cutting of our forests, are becoming scarce, difficult to obtain of good quality and are very expensive.

The concrete pile is made in a mold of concrete reinforced with longitudinal iron wires or small rods, generally wound with wiring. They are of all shapes, many being polygonal (many sided) and some have flutings in their sides running the entire length, as a

[1] The strength of planks is calculated the same as for beams laid on their flat sides.

rule they are iron shod at the foot, and some iron capping device —which is later withdrawn—protects their heads from damage by the blow of the hammer, the latter being manipulated by hand, steam, or other power.

These piles are vastly superior to wooden piles in every way, and in most cases to iron piles.

They are not liable to decay, etc., and they do not rust. Such iron as they contain is perfectly protected by the surrounding concrete, which is now-a-days acknowledged to be the best protection to a once well cleansed and properly coated piece of iron.

Even should the pile crack, which is very infrequent, the parts would still hold and act together, owing to the wiring.

Where the foundations have to go very deep to hard pan, or into deep water, concrete piles are the most economical solution.

They avoid the great expense of pumping, nearly all excavating, blasting or levelling of bottom, and the weight and expense of solid deep masonry or concrete piers.

One great advantage of reinforçed concrete piles is: that they need not be cut off below water level, but can stick up through the ground to the underside of base courses, just below the cellar bottom.

The most curious adaptation of reinforced concrete piles is their use as telegraph poles.

The writer believes that before long they will be used as sleepers for ties for R. R. tracks to avoid noise and shock of the present system of iron ties or plates against rails.

Sometimes a solid concrete wall is built all around the building between continuous inner and outer caissons, which are built inside and outside (all around) thus forming a continuous open well to receive the solid continuous concrete foundation wall.

This has the advantage that the caisson can be built large enough to allow of waterproofing the outside of the concrete, and then excavating the entire interior surface of the lot down to rock, for cellars and sub-cellars, which will be waterproof against side pressure of water.

If there should be springs or fissures in rock from which water rises, the entire bottom should be stoned up and drained to certain basins (sump tanks) to be built below bottom, which must of course, be pumped out continuously.

The stone is then covered with a five or six-inch layer of concrete, then waterproofed, as previously described, and then sufficient weight of concrete put over this to hold down the water.

Such a continuous foundation would be very expensive.

It is customary therefore to put down piers only, under the outer

(or long inner) walls, and near the cellar bottom to span from pier to pier with steel beams, or riveted girders.

The foundations proper begin near the cellar level with grillage beams and concrete.

Grillage Foundations. Where buildings are very heavy and not carried to rock, it becomes necessary to spread the foundation to such an extent that the cost would become excessive, were stepping up in mason work attempted.

In such cases grillage is resorted to. A thin layer of concrete, say one foot thick, is spread over the necessary surface. On this steel beams are laid parallel to each other, at right angles to wall above, and close enough together for each to bear on as much soil as its share of the superimposed load will be: for instance, if the load per running foot is 100000 pounds, (including an estimated allowance for grillage and other foundation work) and the ground is capable of carrying safely 35 pounds per square inch or say 5000 pounds per square foot, the foundation would have to be 20 feet wide, now if we place our grillage beams 1 foot between centres, each beam will be 20 feet long, will bear on 20 square feet of concrete, and the latter on 20 square feet of ground.

The steel beams should be calculated upside down, that is, as supported in the center as a double lever, each arm projecting 10 feet and bearing a uniform load of 100000 pounds, that is, 50000 pounds uniform load on each lever arm.

After the beams are in place, they are filled in between solidly with concrete and another layer of concrete placed over them to protect them.

Beams should of course, be treated same as for other parts of building, thoroughly cleansed of rust, dirt, sand, scales, etc., then oiled, and then painted two or more coats. On top of the upper layer of concrete the wall can then be begun without any stepping courses.

If the wall is on a party line, and therefore at the end of the grillage beams, struts from above or truss work above the grillage beams has to be resorted to, to force down the inner ends of the grillage beams uniformly with their outer ends at party lines; in this case each beam would be supported at both ends and load a uniform one.

The example given above would be bad practice, except in cases where foundations must necessarily be shallow; for it makes an unnecessarily long span, and besides makes a centre support, and beams a cantilever each side of wall.

To remedy this, other layers of grillage above the lower one are

resorted to, at right angles to each other; or in other words the stepping up is done in layers of concrete surrounding steel beams.

The top layer of beams should always be at right angles to wall.

These of course make the load a uniform pressure on the lower grillage and leave only their projecting length as levers the same as on the other layers, excepting possibly the top one, which, if its beams project much beyond the wall either side, should be calculated for a central support with two lever arms.

In every case the beam should be considered reversed with the resistance of the ground as an upward pressure: and this pressure a uniform load.

In each layer the space *between* the grillage beams is filled with concrete, but no concrete is placed *over* the grillage beams, the bottom flange of one layer being laid on top flange of layer below, but all exposed ends and the top layer of beams should be thoroughly protected with a layer of concrete.

Of course all the concrete should be of a good quality—Portland cement concrete, as hereinafter explained.

Great care should be taken to get even bearing *under* all parts of grillage.

Where there are columns, etc., sort of floating islands of grillage should be resorted to no matter of what shape, square, rectangular, polygonal, or even figures with acute angles, so long as its part of the load is properly carried to each square foot of the ground underneath.

When the entire foundation is made of one solid mass of grillage and concrete, steel struts, riveted trusses, girders and other devices must be resorted to, to see that every square foot of the concrete mass receives its share of load to be transmitted to the ground. Otherwise the upward pressure of the latter, on the non-loaded parts, will tend to crack the concrete mass and cause the building ultimately to settle.

Open Caissons and Pumping. When rock can be found at a moderate distance below the water level, open caissons should be used—that is water tight boxes built of tongued and grooved sheath piling with joints glued together with a resinous mixture. Sometimes steel sections, with plates between, are used for sheath piling.

After the sheath piling is driven, excavation begins, and, as this proceeds, timbers and braces are used to keep the piling from being forced inwardly. Sheath piling is about 12 feet long, so that at about every 10 feet of depth a new box or caisson has to be driven

down. This is usually one foot smaller all around, (that is inside of) than the caisson above.

Borings should therefore be made to ascertain depth of rock, the lower caisson made size required for lowest part of pier, and for every ten feet of height two feet added each way for width of upper caissons. This gives the required size for starting the top caisson. Pumps are used to keep the water down.

As is well known, the atmospheric pressure will allow a suction theoretically of a column of water not over 32 feet high, but practically no pumps can do this, so that 28 feet is about the limit that a pump can raise water by suction, as pumps in open caissons have to do.

As the depth of the caisson increases it is necessary therefore to lower the pumps, which is done by allowing sufficient offset on one side to receive them at the lower levels. Where there are many caissons being sunk at once, a central pumping plant is established and connected with each.

Closed Caissons. When the column of water is so deep, or its pressure so great as to make pumping impractical or too expensive, closed caissons are resorted to. These consist of air and water tight boxes with tops, but no bottoms, two (or more), superimposed above each other, and called the lower and upper chambers. Each chamber has an outlet opening at the top sufficiently large to pass a bucket. The air is put under sufficient pressure in the lower chamber to keep out the water; and air is also supplied, under pressure, to the upper chambers. The outlet openings have trap doors which can be opened from below and shut air tight.

The men, tools, or buckets are first entered into the upper chamber, the upper outlet closed and then air pressure put into the upper chamber.

When this has been brought up to the same pressure as in the chamber below, the outlet to lower opening is opened.

The same process of going from chamber to chamber is resorted to when leaving, except that air in upper chamber is reduced before leaving.

The pressure has to be sufficient in the lower chamber to keep out water or the men would drown.

The men in lower chamber dig out the ground under them and around under the edges of lower caisson, and weights (usually pig iron) piled on top force the whole caisson down. When rock is reached it is cleaned off, levelled (sometimes stepped) and then concrete lowered into lower chamber to build pier.

CHAPTER III.

CELLAR AND RETAINING WALLS.

ANALYTICAL SOLUTION.

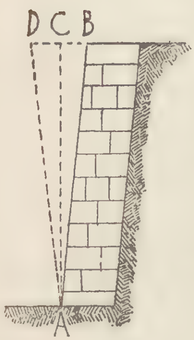


Fig. 49.

THE architect is sometimes called upon to build retaining-walls in connection with terraces, ornamental bridges, city reservoirs, or similar problems. Then, too, all cellar walls, where not adjoining other buildings, become retaining walls; hence the necessity to know how to ascertain their strength. Some writers distinguish between "face-walls" and "retaining-walls"; a face-wall being built in front of and against ground which has not been disturbed and is not likely to slide; a retaining-wall being a wall that has a filled-in backing. On this theory a face-wall would have a purely ornamental duty, and would receive no thrust, care being taken during excavation and building-operations not to allow damp or frost to get into the ground so as to prevent its rotting or losing its natural tenacity, and to drain off all surface or underground water. It seems to the writer, however, that the only walls that can *safely* be considered as "face-walls" are those built against rock, and that all walls built against other banks should be calculated as retaining-walls.

Most Economical Section. The cross-section of retaining-walls vary, according to circumstances, but the outside surface of wall is generally built with a "batter" (slope) *towards* the earth. The most economical wall is one where both the outside and back surfaces batter *towards* the earth. As one or both surfaces become nearly vertical the wall requires more material to do the same work, and

the most extravagant design of all is where the back face batters away from the earth; of course, the outside exposed surface of wall must either batter towards the earth (A B in Figure 49) or be vertical, (A C); it cannot batter away from the ground, otherwise the wall would overhang (as shown at A D). Where the courses of masonry are built at right angles to the outside surface the wall will be stronger than where they are all horizontal.

Thus, for the same amount of material in a wall, and same height, Figure 50 will do the most work, or be the strongest retaining-wall, Figure 51 the next strongest, Figure 52 the next, Figure 53 next,

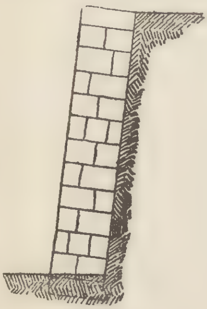


Fig. 50.

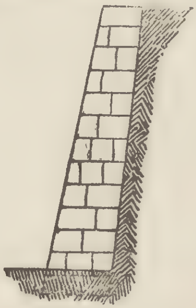


Fig. 51.

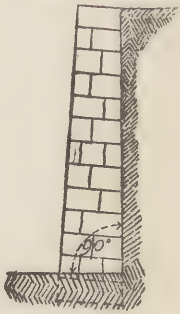


Fig. 52.

Figure 54 next, and Figure 55 the weakest. In Figure 50 and Figure 52 the joints are at right angles to the outside surface; in the

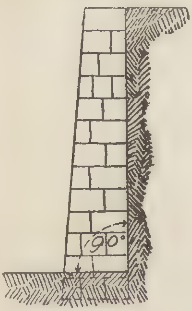


Fig. 53.

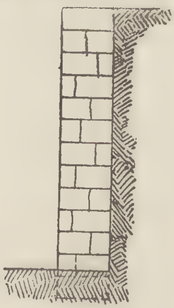


Fig. 54.

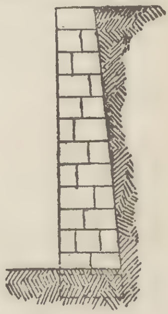


Fig. 55.

other figures they are all horizontal. For reservoirs, however, the shapes of Figures 53 or 55 are often employed.

To calculate the resistance of a retaining-wall proceed as follows:

Height of Line of Pressure. The central line or axis of the pressure O P or p of backing will be at one-third of the height of back surface, measured from the ground lines,[1] that is at O in Figure 56, where A O $= \frac{1}{3}$, A B.

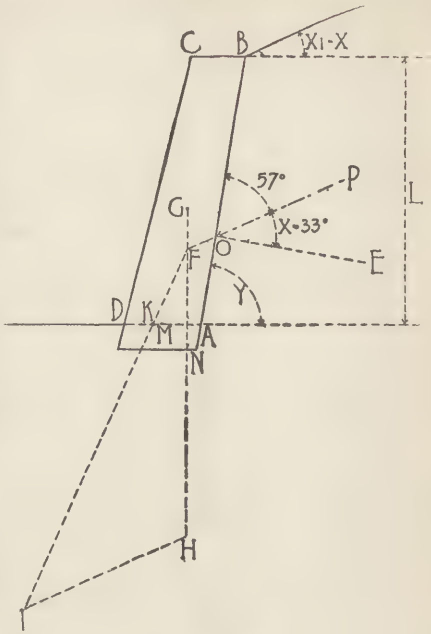


Fig. 56.

The direction of the pressure-line (except for reservoirs) is usually assumed to form an angle of 57° with the back surface of wall, or

$\angle$ P O B = 57°.

For water it is assumed normal, that is, at right angles to the back surface of wall.

If it is desired, however, to be very exact, erect O E perpendicular to back surface, and make angle E O P, or ($\angle$ x) = the angle of friction of the filling-in or backing. This angle can be found from Table X.

Amount of pressure — General case. The amount (p) of the pressure P O is found from the following formulæ:

If the backing is filled in higher that the wall,[2]

Backing higher than Wall.

$$p = \frac{w.L^2}{2} \cdot \frac{\sin^2.\ (y-x)}{\sin^2.\ y.\ \sin.\ (y+x)} \tag{47}$$

If the backing is filled in only to the top level of wall,

Backing level with Wall.

$$p = \frac{w.L^2}{2} \cdot \frac{\sin.\ x}{\sin\ (y+2\,x)} \cdot \left(\sqrt{\cot.\ x - \cot\ (y+2x)} - \sqrt{\cot.\ y - \cot\ (y+2x)}\right) \tag{48}$$

[1] Where the earth in front of the outside surface of wall C D is not packed very solidly below the grade line and against the wall, the total height of wall N B (including part underground) should be taken, in place of A B (the height above grade line).

[2] The top slope of backing in this case should never form an angle with the horizon, greater than the friction angle.

Where p = the total amount of pressure, in pounds, per each running foot in length of wall.

Where w = the weight, in pounds, per cubic foot of backing.

Where L = the height of retaining wall above ground, in feet. See foot note 1, p. 102.

Where y = the angle formed by the back surface of wall with the horizon.

Where x = the angle of friction of the backing as per Table X.

TABLE X.[1]

Material.	Weight per cubic foot. w.	Angle of friction. x.
AVERAGE (except water)	120	33°
Very compact earth	115	55°
Dry clay	100	45°
Sharp pebbles	110	45°
Dry loam	100	40°
Sharp broken stones	100	38°
Dry rammed earth	110	37°
Dry sand	112	32°
Dry gravel	110	32°
Wet rammed earth	125	27°
Wet sand	125	24°
Wet gravel	125	24°
Round pebbles	110	23°
Wet loam	130	17°
Wet clay	125	17°
Salt water	64	0°
Rain water	62½	0°

Even those who do not understand trigonometry can use the above formulæ.

It will simply be necessary to add or subtract, etc., the numbers of degrees of the angles y and x, and then find from any table of *natural* sines, cosines, etc., the corresponding value for the amount of the new angle. The value, so found, can then be squared, multiplied, square root extracted, etc., same as any other arithmetical problem. Should the number of degrees of the new angle be more than 90°, subtract 90° from the angle and use the positive cosine of the *difference* in place of the sine of whole, or the tangent of the *difference* in place of the co-tangent of the whole; in the latter case the value of the tangent will be a negative one, and should have the negative sign prefixed.

Thus, if $x = 33°$ and $y = 50°$, formula (47) would become:

$$p = \frac{w.\,L^2}{2} \cdot \frac{\sin.^2\,(17)°}{\sin.^2\,50.°\ \sin.\,83°}$$

[1] Above table of friction angles is taken from Klasen's "*Hochbau und Brückenbau-Constructionen.*" As a rule it will do to assume the angle of friction at 33° and the weight of backing at 120 lbs. per cubic foot, except in the case of water.

The values of which, found in a table of natural sines, etc., is:

$$p = \frac{w.\,L^2}{2} \cdot \frac{0{,}2924^2}{0{,}766^2.\ 0{,}9925}$$
$$= \frac{w.\,L^2}{2} \cdot 0{,}1468 \quad = 0{,}734.\,w.L^2$$

Similarly, in formula (48), we should have for the quantity:

$$\sqrt{\cot.\,x - \cot.\,(y + 2x)} = \sqrt{\cot.\,33^\circ - \cot.\,116^\circ}$$
$$= \sqrt{\cot.\,33^\circ - [-\mathrm{tg}.\,(116^\circ - 90^\circ)]}$$
$$= \sqrt{\cot.\,33^\circ + \mathrm{tg}.\,26^\circ}$$
$$= \sqrt{1{,}5399 + 0{,}4877}$$
$$= \sqrt{2{,}0276} \quad = 1{,}424$$

Average Case. As already mentioned, however, the angle of friction — (except for water when it is $= 0^\circ$, that is, normal to the back surface of wall) — is usually assumed at 33°; this would reduce above formulæ to a very much more convenient form, viz.:

For the average angle of friction (33°)

If the backing is higher than the wall:

Backing higher than Wall.

$$p = \frac{w.\,L^2}{2} \cdot \frac{(10 - n.\,0{,}55)^2.\ \sqrt{144 + n^2}}{(10 + n.\,0{,}55).\ \ 144} \qquad (49)$$

Or, if the backing is level with top of wall:

Backing level with Wall.

$$p = \frac{w.\,L^2}{2} \cdot \frac{\sqrt{144 + n^2}}{9 + n.1{,}7} \cdot \left(\sqrt{1{,}54 - \frac{n.\,0{,}4 - 11}{5 + n.\,0{,}9}} - \sqrt{\frac{n}{12} - \frac{n.\,0{,}4 - 11}{5 + n.\,0{,}9}}\right)^2 \qquad (50)$$

Where p, w and L same as for formulæ (47) and (48).

Where $n =$ amount of slope or batter in inches (per foot height of wall) of rear surface of wall.

Thus, if the rear surface sloped *towards* the backing three inches (for each foot in height) we should have a positive quantity, or

$$n = +3.$$

If the rear surface sloped *away* from the backing three inches (per foot of height), n would become negative, or

$$n = -3.$$

When the rear surface of wall is vertical, there would be no slope, and we would have

Cellar Walls. $n = 0$.

The latter is the case generally for all cellar walls, which would still further simplify the formula, or, for *cellar walls* where weight of soil or backing varies materially from 120 pounds per cubic foot.

Cellar Walls—general case. $p = w.\ L^2.\ 0{,}138.$ (51)

For *cellar walls*, where the weight of soil or backing can be safely assumed to weigh 120 pounds per cubic foot

Cellar Walls—usual case. $p = 16\frac{2}{3}.\ L^2.$ (52)

Where $p =$ the total amount of pressure, in pounds, per each running foot in length of wall.

Where $w =$ the weight, in pounds, per cubic foot of backing.

Where $L =$ the height, in feet, of ground line above cellar bottom.

For different slopes of the back surface of retaining walls (assuming friction angle at 33°) we should have the following table; + denoting slope *towards* backing, – denoting slope *away* from backing.

TABLE XI.

Slope of back surface of wall in inches per foot of height.	Value of p for backings of different weights per cubic foot.	Value of p for the average backing, assumed to weigh 120 lbs. per cubic foot.
+4″	$p=0{,}072.\ w.\ L^2$	$p=8\frac{2}{3}.\ L^2$
+3″	$p=0{,}088.\ w.\ L^2$	$p=11\ .\ L^2$
+2″	$p=0{,}098.\ w.\ L^2$	$p=12\ .\ L^2$
+1″	$p=0{,}112.\ w.\ L^2$	$p=13\frac{1}{2}.\ L^2$
0″	$p=0{,}138.\ w.\ L^2$	$p=16\frac{2}{3}.\ L^2$
–1″	$p=0{,}157.\ w.\ L^2$	$p=19\ .\ L^2$
–2″	$p=0{,}185.\ w.\ L^2$	$p=22\frac{1}{4}.\ L^2$
–3″	$p=0{,}205.\ w.\ L^2$	$p=24\frac{2}{3}.\ L^2$
–4″	$p=0{,}258.\ w.\ L^2$	$p=31\ .\ L^2$

Now having found the amount of pressure p from the most convenient formula, or from Table XI, and referring back to Figure 56, proceed as follows:

To find Curve of Pressure. Find the centre of gravity G of the mass A B C D,[1] from G draw the vertical axis G H, continue P O till it intersects G H at F. Make F H equal to the weight in pounds of the mass A B C D (one foot thick), at any convenient scale, and at same scale make $H\ I = p$ and parallel to P O, then draw I F and it is the resultant of the pressure of the earth, and the resistance of the retaining wall. Its point of intersection K with the base D A is a point of the curve of pressure.

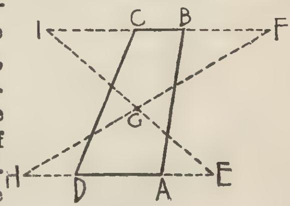


Fig. 57.

Amount of Stress at Joint. To find the exact amount of pressure on the joint D A use formula (44) for the edge of joint nearest to the point K or edge D, and formula (45) for the edge of joint farthest from the point K, or edge A.

[1] To find the centre of gravity of a trapezoid A B C D, Fig. 57, prolong C B until B F = C I = D A and prolong D A until A E = D H = C B, draw E I and H F and their point of intersection G is the centre of gravity of the whole.

Formula (44) was

$$v = \frac{p}{a} + 6 . \frac{x.p}{a.d}$$

If M be the centre of D A, that is D M = M A = $\frac{1}{2}$. D A, and remembering that the piece of wall we are calculating, is only one running foot (or one foot thick), we should have

For $x =$ K M; expressed in inches.

For $a =$ A D. 12; (A D expressed in inches).

For $d =$ A D; in inches, and

For $p =$ F I, in lbs., measured at same scale as F H and H I; or, the stress at D, (the nearer edge of joint) would be v, in pounds, per square inch,

$$v = \frac{\text{F I}}{\text{A D. 12}} + 6 . \frac{\text{KM.FI}}{\text{12. AD}^2}$$

Remembering to measure all parts in inches *except* F I, which must be measured at same scale as was used to lay out F H and H I. Similarly we should obtain the stress at A in pounds per square inch.

$$v = \frac{\text{F I}}{\text{AD. 12}} - 6 . \frac{\text{KM.FI}}{\text{12.AD}^2}$$

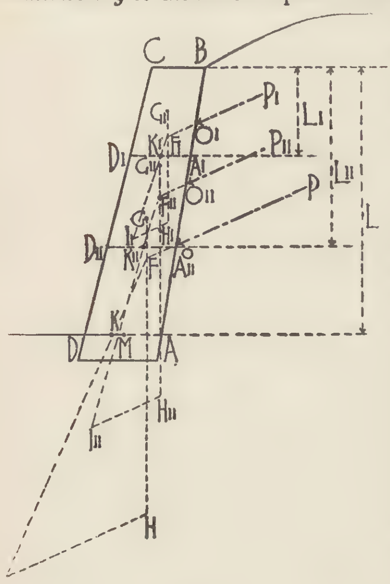


Fig. 58.

v should not exceed the safe crushing strength of the material if positive; or if v is negative, the safe tensile strength of the mortar. If we find the wall too weak, we must enlarge A D, or if too strong, we can diminish it; in either case, finding the new centre of gravity G of the new mass A B C D and repeating the operation from that point; the pressure, of course, remaining the same so long as the slope of back surface remains unaltered. If the wall is a very high one, it should be divided into several sections in height, and each section examined separately, the base of each section being treated the same as if it were the joint at the ground line, and the *whole* mass of wall *in* the section and *above* the section being taken in each time.

Thus in Figure 58, when examining the part A_{I} B C D_{I} we should find O_{I} P_{I} for the part only, using L_{I} as its height, the point O_{I} being at one third the height of A_{I} B or A_{I} $O_{I} = \frac{1}{3}$. A_{I} B; G_{I} would be the centre of gravity of A_{I} B C D_{II}, while F_{I} H_{I} would be equal to the weight of its mass, one foot thick; this gives one point of the curve of pressure at K_{II}, with the amount of pressure $= F_{I}$ I_{II}, so that we can examine the pressures on the fibres at D_{I} and A_{I}. Similarly when comparing the section A_{II} B C D_{II} we have the height L_{III}, and so find the amount of pressure O_{II} P_{III}, applied at O_{III}, where A_{II} $O_{II} = \frac{1}{3}$ A_{II} B; G_{II} is centre of gravity of A_{II} B C D_{II} while F_{II} H_{II} is equal to the weight of A_{II} B C D_{III}, one foot thick, and F_{II} I_{II} gives us the amount of pressure on the joint, and another point K_{II} of curve of pressure, so that we can examine the stress on the fibres at D_{II} and A_{II}. For the whole mass A B C D we, of course, proceed as before.

Reservoir Walls. For reservoirs the line of pressure O P is always at right angles to the back surface of the wall, so that we can simplify formula (50) and use for rain water:

$$p = 31\tfrac{1}{4}.\ L^2 \tag{53}$$

For salt water:

$$p = 32.\ L^2 \tag{54}$$

Where $p =$ the amount of pressure, in pounds, on one running foot in length of wall, and at one-third the height of water, measured from the bottom, and p taken normal to back surface of wall,

Where $L =$ the depth of water in feet.

If Backing is Loaded. Where there is a superimposed weight on the backing of a retaining-wall proceed as follows:

In Figure 59 draw the angle C A D $= x$, the angle of friction of the material. Then take the amount of load, in pounds, coming on B C and one running foot of it in thickness (at right angles to B C), divide

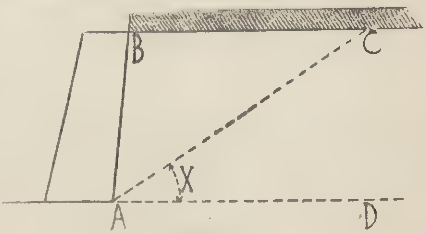


Fig. 59.

this by the area, in feet, of the triangle A B C and add the quotient to w, the weight of the backing per cubic foot, then proceed as before, inserting the sum w_{I} in place of w in formulæ (47) to (51) and in Table XI, when calculating p; or $w_{I} = w + \frac{2.z}{B.L}$ (55)

Where w_1 = the amount, in pounds, to be used in all the formulæ (47) to (51) and in Table XI, in place of w.

Where w = weight of soil or backing, in pounds, per cubic foot.

Where z = the total superimposed load on backing, in pounds, per running foot in length of wall.

Where B = length, in feet, of B C, as found in Figure 59.

Where L = the height of wall, in feet.

Where there is a superimposed load on the backing, the central line of pressure p should be assumed as striking the back surface of wall higher than one-third its height, the point selected, being at a height X from base; where X is found as per formula (56)

$$X = \frac{L.\,(w_1 - \frac{2}{3}\,w)}{2w_1 - w} \tag{56}$$

Where X = the height, in feet, from base, at which pressure is applied, when there is a superimposed load on the backing.

Where L = the height, in feet, of wall.

Where w = the weight, in pounds, per cubic foot of backing.

Where w_1 = is found from formula (55)

When calculating the pressure against cellar walls, only the actual weight of the material of walls, floors and roof should be assumed as coming on the wall, and no addition should be made for wind nor for load on floors, as these cannot always be relied on to be on hand. The additional compression due to them should, however, be added afterwards.[1]

GRAPHICAL METHOD.

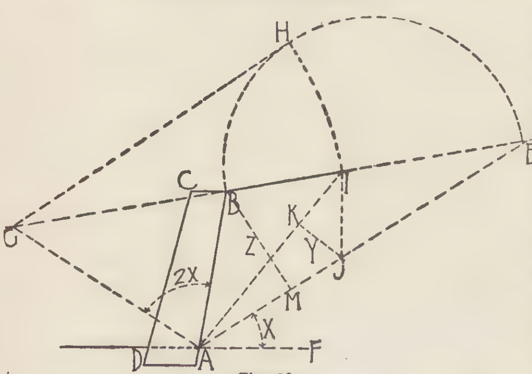


Fig. 60.

The graphical method of calculating retaining-walls is much easier than the analytical, being less liable to cause errors, and is recommended for office use, though the analytical method

[1] Where a wall is not to be kept braced until the superimposed wall, etc., is on it, these should of course be entirely omitted from the calculation, and the wall must be made heavy enough to stand alone.

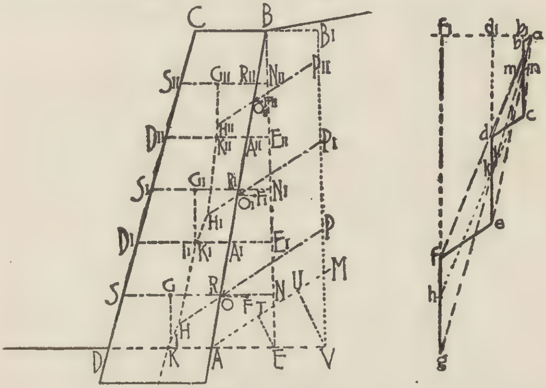
might often serve as a check for detecting errors, when undertaking important work.

If A B C D is the section of a retaining wall and B I the top line of backing, draw angle F A M = x = the angle of friction, usually assumed at 33° (except for water); continue B I to its intersection at E with A M; over B E draw a semi-circle, with B E as diameter; make angle B A G = 2 x (usually 66°), continuing line A G till it intersects the continuation of B I at G; draw G H tangent to semi-circle over B E; make G I = G H; draw I A, also I J parallel to B A; draw J K at right angles to I A; also B M at right angles to A E. Now for the sake of clearness we will make a new drawing of the wall A B C D in Figure 61.

Calling B M = Z and K J = Y (both in Figure 60) make A E = Q (Figure 61) where Q is found from formula (57) following: —

$$Q = \frac{Y.\,Z.\,s}{L.\,m} \qquad (57)$$

Where Q = the length of A E in Figure 61, in feet,
Where Y = the length of K J in Figure 60, in feet,[1]
Where Z = the length of B M in Figure 60, in feet,



Figs. 61 and 62.

Where s = the weight of one cubic foot of backing, in lbs.
Where m = the weight of one cubic foot of wall, in lbs.
Where L = the height of backing, in feet, at wall.

[1] If the incline of line B I to the horizon is equal to the angle of friction, as is often the case, find A G as before and use this length in place of K J or Y, which, of course, it will be impossible to find, as A M and B I would be parallel and would have no point of intersection; of course, B I should never be steeper than A M, or else all of the soil steeper than the line of angle of friction would be apt to slide.

Sections must have equal heights. Draw E B, divide the wall into any number of sections of *equal height*, in this case we will say three sections, $A\ A_{I}\ D_{I}\ D$; $A_{I}\ A_{II}\ D_{II}\ D_{I}$ and $A_{II}\ B\ C\ D_{II}$. Find the centres of gravity of the different parts, viz.: G, G_{I} and G_{II}, also F, F_{I} and F_{II}. Bisect $D\ D_{I}$ at S, also $D_{I}\ D_{II}$ at S_{I} and $D_{II}\ C$ at S_{II}. Draw $S\ N$, $S_{I}\ N_{I}$ and $S_{II}\ N_{II}$ horizontally. Through G, G_{I} and G_{II} draw vertical axes, and through F, F_{I} and F_{II} horizontal axes, till they intersect A B at O, O_{I} and O_{II}. Draw $O\ P$, $O_{I}\ P_{I}$ and $O_{II}\ P_{II}$ parallel to M A, where angle $M\ A\ E = x =$ angle of friction of soil, or backing. In strain diagram Figure 62 make $a\ b_{I} = R_{II}\ N_{II}$; also $b_{I}\ d_{I} = R_{I}\ N_{I}$ and $d_{I}\ f_{I} = R\ N$. From b_{I}, d_{I} and f_{I} draw the vertical lines. Now begin at a; draw $a\ b$ parallel to M A; make $b\ c = S_{II}\ R_{II}$; draw $c\ d$ parallel to M A; make $d\ e = S_{I}\ R_{I}$; draw $e\ f$ parallel to M A and make $f\ g = S\ R$. Draw $a\ c$, $a\ d$, $a\ e$, $a\ f$ and $a\ g$. Now returning to Figure 61, prolong $P_{II}\ O_{II}$ till it intersects the vertical axis through G_{II} at H_{II}; draw $H_{II}\ H_{I}$ parallel to $a\ c$ till it intersects $P_{I}\ O_{I}$ at H_{I}; draw $H_{I}\ I_{I}$ parallel to $a\ d$ till it intersects the vertical axis through G_{I} at I_{I}; draw $I_{I}\ H$ parallel to $a\ e$ till it intersects P O at H; draw H I parallel to $a\ f$ till it intersects the vertical through G at I; draw I K parallel to $a\ g$. Then will points K, K_{I} and K_{II} be points of the curve of pressure. The amount of pressure at K_{II} will be $a\ c$, at K_{I} it will be $a\ e$, and at K it will be $a\ g$, from which, of course, the strains on the edges D, D_{I} and D_{II}, also A, A_{I} and A_{II} can be calculated by formulæ (44) and (45). To obtain scale, by which

Scale of strain diagram. to measure $a\ c$, $a\ e$ and $a\ g$, make $g\ h$, Figure 62 at any scale equal to the weight, in pounds, of the part of wall $A\ A_{I}\ D_{I}\ D$ one foot thick, draw $h\ i$ parallel $f\ a$, then $g\ i$ measured at same scale as $g\ h$, is the amount of pressure, in pounds, at K. Similarly make $e\ k =$ weight of centre part, and $c\ m =$ weight of upper part, draw $k\ l$ parallel $d\ a$, and $m\ n$ parallel $b\ a$, then is $e\ l$ the pressure at K_{I} and $c\ n$ the pressure at K_{II}, both measured at same scale; or, a still more simple method would be to take the weight of $A\ A_{I}\ D_{I}\ D$, in pounds, and one foot thick, and divide this weight by the length of $g\ f$ in inches; the result being the number of pounds per inch to be used, when measuring lengths, etc., in Figure 62. The above graphical method is very convenient for high walls, where it is desirable to examine many joints, but *care must be taken to be sure to get the parts all of equal height*, otherwise, the result would be incorrect.

If backing loaded. In case of a superimposed weight find w_{I}, as directed in formula (55), make A T at any scale equal to w and $A\ U = w_{I}$, draw T E and parallel thereto U V, draw V B,

parallel to E B and use V B_1 in place of E B, proceeding otherwise as before. The points O of application of pressure P O, will be slightly changed, particularly in the upper part, as they will be horizontally opposite the centres of gravity of the enlarged trapezoids, and in the upper case this point would be much higher, the figure now being a trapezoid, instead of a triangle as before.

Buttressed walls. Where a wall is made very thin and then buttressed at intervals, all calculations can be made the same as for walls of same thickness throughout, but the vertical axis through centre of gravity of wall should be shifted so as to pass through the centre of gravity of the whole mass, including buttresses; and the weight of thin part of wall should be increased proportionately to the amount of buttress, thus: If a 12″ wall is buttressed every 5 feet (apart) with 2′ x 2′ buttresses, proceed as follows:

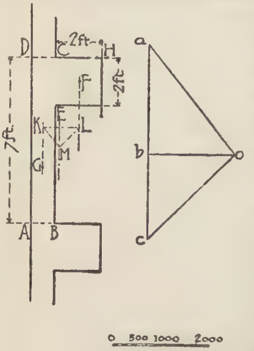


Fig. 63.

Find the centre of gravity G of the part of wall A B C D (in plan) Figure 63, also centre of gravity F of part E I H C, draw lines through F and G parallel to wall. Now make $a\ b$ parallel to wall and at any scale equal to weight or area of A B C D and $b\ c$ equal to that of E I H C. From any point o draw the lines $o\ a$, $o\ b$ and $o\ c$; now draw K L (anywhere between parallel lines F and G), but parallel to $b\ o$, and from L draw L M parallel to $o\ c$, and from K draw K M parallel to $a\ o$, a line through their point of intersection M drawn parallel to wall is the neutral axis of the whole mass. When drawing the vertical section of wall-part A B C D, Figure 64, therefore, instead of locating the neutral axis through the centre of wall it will be as far outside as M is from B C, in Figure 63; that is, at G H, Figure 64.

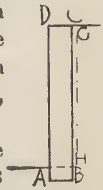


Fig. 64.

When considering the weight per cubic foot of wall, we add the proportionate share of buttress; now in Figure 63 there are 4 cubic feet of buttress to every 7 feet of wall, so that we must add to the usual weight w per cubic foot of wall $\frac{4}{7}\,w$

or $\qquad w.\ (1 + \frac{4}{7})$

To put this in a formula.

$$w_{\text{u}} = w\left(1 + \frac{A}{A_1}\right) \qquad (59)$$

Where w_{11} = the weight per cubic foot, in pounds, to be used for buttressed walls, after finding the neutral axis of the whole mass.

Where w = the actual weight, in pounds, per cubic foot of the material.

Where A = the area in square feet of one buttress.

Where A_1 = the area in square feet of wall from side of one buttress to corresponding side of next buttress.

Walls with counterforts. Buttresses, however, will not be of very much value, unless they are placed quite close together. Buttresses on the back surface of a wall are of very little value, unless thoroughly bonded and anchored to walls; these latter are called counterforts. It is wiser and cheaper in most cases to use the additional masonry in thickening out the lower part of wall its entire length.

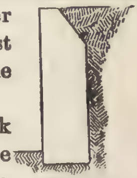


Fig. 65.

Resistance to frost. Where frost is to be resisted the back part of wall should be sloped, for the depth frost is likely to penetrate (from 3 to 4 feet in our climate), and finished smoothly with cement, and then asphalted, to allow the frozen earth to slide upwards, see Figure 65.

Example I.

Cellar wall to frame dwelling. *A two story and attic frame house has a 12″ brick foundation wall, the distance from cellar bottom to ground level being 6 feet. The angle of friction of ground to be assumed at 33° and the weight per cubic foot at 120 pounds. Is the wall safe?*

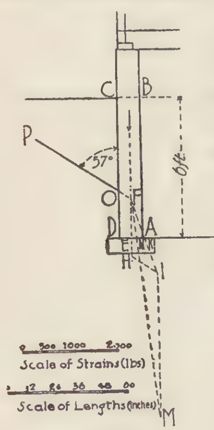


Fig. 66.

The weight of wall and superstructure must, of course, be taken at its minimum, when calculating its resistance to the ground, we shall, therefore, examine one of the sides on which no beams or rafters rest. The weight will consist, therefore, only of brick wall and frame wall over. We examine only one running foot of wall, and have

8 cubic feet of brick at 112 lbs.	=	896 lbs
24 feet (high) of frame wall at 15 lbs.	=	360 "
Total weight resisting pressure	=	1256 "

The pressure itself will be according to formula (52)

$$p = 16\tfrac{2}{3}.\ L^2 = 16\tfrac{2}{3}.\ 36$$
$$= 600 \text{ lbs.}$$

Now make D O = $\frac{1}{3}$. D C. Make angle P O C = 57°; prolong P O till it intersects the

vertical neutral axis of wall[1] at F; make F H = 1256 pounds, at any scale, draw H I parallel to P O, and make H I = p = 600 pounds at same scale. Draw I F, then is its point of intersection K (with D A) a point of the curve of pressure, and F I (measured at same scale) is the amount of pressure p to be used in formulæ (44) and (45). By careful drawing we will find that K comes ½″ beyond A (outside of A D), or 6½″ from centre E of A D. F I we find measures 1660 units, therefore $p = 1660$ pounds.

To find the actual stress or resistance v of edge of fibres of brickwork at A use (44), viz.:

$$v = \frac{p}{a} + 6 . \frac{x . p}{a . d}$$

and as $p = 1660$ and $x = \text{E K} = 6\tfrac{1}{2}''$ and $a = 12 . 12 = 144$ inches and $d = \text{A D} = 12''$ we have:

$$v = \frac{1660}{144} + 6 . \frac{6\tfrac{1}{2} . 1660}{144 . 12} = 11\tfrac{1}{2} + 37\tfrac{1}{2} = + 49 \text{ pounds}$$

as this is a positive quantity it will be compression.

The resistance of edge-fibres at D will be according to formula (45)

$$v = \frac{1660}{144} - 6 . \frac{6\tfrac{1}{2} . 1660}{144 . 12} = 11\tfrac{1}{2} - 37\tfrac{1}{2} = - 26 \text{ pounds}$$

as this is a negative quantity D will be subjected to tension; that is, there is a tendency for A B C D to tip over around the point A, the point D tending to rise. The amount of tension at D is more than ordinary brickwork will safely stand, according to Table V, still, as it would only amount to 26 pounds on the extreme edge-fibres and would diminish rapidly on the fibres nearer the centre, we can consider the wall safe, even if of but fairly good brickwork, particularly as the first-story beams and girders and the end and possible cross-walls, will all help to stiffen the wall. Had we taken a foot-slice of the wall under the side carrying the beams, we should have had an additional amount of weight resisting the pressure. If the beams were 18 ft. span, we should have three floors each 9 ft. long and with load weighing, say, 90 pounds per foot; to this must be added the roof, or about 13 ft. à 50 pounds, the additional load being:

Floors,	3. 9. 90 =	2430
Roof,	13. 50 =	650
Total		3080

Now make I M = 3080 pounds at same scale as F H, etc., draw M F and its point of intersection N with D A would be a point of

[1] It should really be the vertical neutral axis of the whole weight, which would be a trifle nearer to D C than centre of wall.

the curve of pressure, and F M, at same scale the amount of pressure on D A for the bearing walls of house; E N we find measures $2\frac{1}{2}''$, and F M measures 4600 units or pounds. The stress at A, then, would be:

$$v = \frac{4600}{144} + 6 . \frac{2\frac{1}{2} . 4600}{144 . 12} = + 72 \text{ pounds.}$$

and the stress at D would be:

$$v = \frac{4600}{144} - 6 . \frac{2\frac{1}{2} . 4600}{144 . 12} = - 8 \text{ pounds.}$$

There will, therefore, be absolutely no doubt about the safety of bearing walls.

Example II.

Cellar wall deeper than adjoining building. *A cellar wall A B C D is to be carried* 15 *feet below the level of adjoining cellar; for particular reasons the neighboring wall cannot be underpinned. It is desirable not to make the wall **A** B C D over* 2′ 4″ *thick. Would this be safe? The soil is wet loam.*

In the first place, before excavating we must sheath-pile along line C D, then as we excavate we must secure horizontal timbers along the sheath-piling and brace these from opposite side of excavation. The sheath-piling and horizontal timbers must be built in and left in wall. The braces will have to be built around and must not be removed until the whole weight is on the wall.

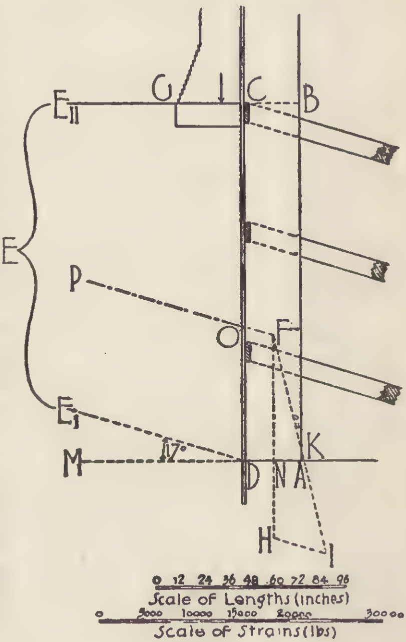


Fig. 67.

The weight of the wall C G, per running foot of length, including floors and roofs, we find to be 13000 pounds, but to this we must add the possible loads coming on floors, which we find to be 6000 pounds additional, or 19000 pounds total, possible maximum load. This load will

be distributed over the area of C D E. In calculating the **weight of** A B C D resisting the pressure, we must take, of course, only the minimum weight; that is, the actual weight of construction and omit all loads on floors, as these may not always be present. The weight of walls and unloaded floors coming on A B C D, and including the weight of A B C D itself, we find to be 21500 pounds per running foot. Now to find the pressure p, proceed as follows: Make angle E_{I} D M $= 17°$, the angle of friction of wet loam (See Table X), and prolong D E_{I} till it intersects C E_{II} at E. Now C E, we find, measures 49 feet; C D or L is 15 feet; then, instead of using w in formula (51), we must use w_{II} as found from formula (55), viz.:

$$w_{I} = w + \frac{2.\ 19000}{CE.\ L}$$

w for wet loam (Table X) is 130 pounds; therefore,

$$w_{I} = 130 + \frac{2.\ 19000}{49.\ 15} = 130 + 48,\ 7 = 179.$$

Inserting this value for w in formula (51) we have:

$$p = 179.\ 15^2.\ 0,138 = 5558.$$

The height X from D at which P O is applied is found from formula 56, and is:

$$X = \frac{15.\ (179 - \frac{2}{3}.130)}{2.179 - 130} = \frac{15.\ (179 - 87)}{358 - 130} =$$

$6',053 = 6'0\frac{5}{8}''$

Make D O $=$ X $= 6'0\frac{5}{8}''$; draw P O parallel to E D till it intersects the vertical neutral axis of wall (centre line) at F; make F H (vertically) at any scale equal to 21500, draw H I parallel to P O and make H I $= p = 5558$ pounds at same scale, draw I F, then is its point of intersection with the prolongation of A D at K a point of the curve of pressure, and F I measured at same scale as F H is the amount of pressure on joint. The distance K from centre of joint N we find is $14\frac{1}{2}''$, F I measures 23800 units or pounds; the stress (v) at A, therefore, will be, formula (44):

$$v = \frac{23800}{28.12} + 6.\frac{14\frac{1}{2}.\ 23800}{28.12.28} = + 292$$

While the stress at D would be formula (45)

$$v = \frac{23800}{28.12} - 6.\frac{14\frac{1}{2}.23800}{28.12.28} = - 150$$

Or the edge at A would be subjected to a compression of 292 pounds, while the edge at D would be submitted to a tension of 150 pounds per square inch, both strains much beyond the safe limit of even the best masonry. The wall will, therefore, have to be thickened and a new calculation made.

Example III.

Wall to stage pit. *A stage-pit 30 feet deep is to be enclosed by a stone-wall, 3 feet thick at the top and increasing 4 inches in thickness for every 5 feet of depth. The wall, etc., coming over this wall weighs 25000 pounds per running foot, but cannot be included in the calculation, as peculiar circumstances will not allow braces to be kept against the cellar wall, until the superimposed weight is on it. The surrounding ground to be taken as the average, that is, 120 pounds weight per cubic foot, and with an angle of friction of 33°.*

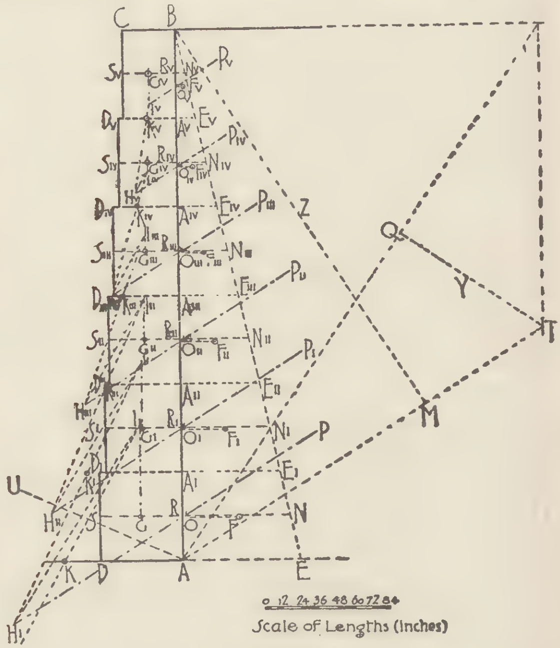


Fig. 68.

Find B M (= Z) and Q T (= Y) by making angles T A E = 33° and B A U = 66° and then proceeding as explained for Figure 60. We scale B M and Q T at same scale as height of wall A B is drawn, and find:

B M = Z = 25 ft. 6″ = $25\frac{1}{2}$

Q T = Y = 9 ft. 8″ = $9\frac{2}{3}$; assuming each cubic foot of wall to weigh 150 pounds we find Q from Formula (57)

$$Q = \frac{9\frac{2}{3} . 25\frac{1}{2} . 120}{30 . 150} = 6{,}573 = 6' \, 7''.$$

Make A E $=$ Q $= 6' \, 7''$ and draw B E.

At *equal heights*, that is, every 5 feet, in this case, draw the joint lines D E, D_{I} E_{II}, D_{II} E_{III}, etc. Find the centres of gravity F, F_{II}, F_{III}, etc., of the six parts of A E B, (see foot-note, p. 101) and also the centres of gravity G, G_{II}, G_{III}, etc., of the six parts of the wall itself, which, in the latter case, will be at the centre of each part.

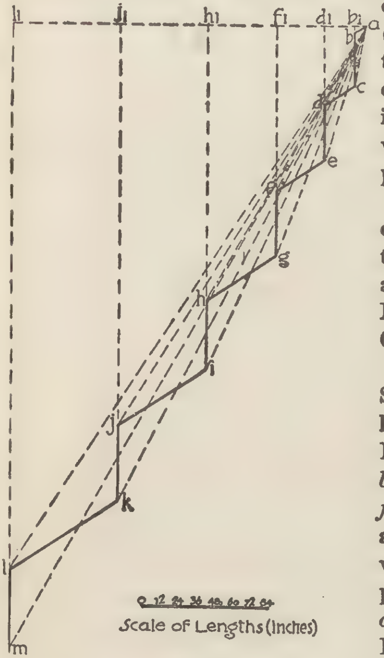


Fig. 69.

Horizontally, opposite the centres F, F_{II}, F_{III}, etc., apply the pressures P O, P_{I} O_{II}, etc., against wall, and parallel to M A. Through centres G, G_{II}, G_{III}, etc., draw vertical axes.

Draw the lines S N, S_{I} N_{II}, S_{II} N_{III}, etc., at half the vertical height of each section. Now in Figure 69 make $a\, b_{I} = R_{V}\, N_{V}$; $b_{I}\, d_{I} = R_{IV}\, N_{IV}$; $d_{I}\, f_{I} = R_{III}\, N_{III}$; $f_{I}\, h_{I} = R_{II}\, N_{II}$; $h_{I}\, j_{I} = R_{I}\, N_{I}$; and $j_{I}\, l_{I} = R\ N$. Draw the vertical lines through these points. Now begin at a, make $a\,b$ parallel to P O, make $b\,c = R_{V}\, S_{V}$; draw $c\,d$ parallel P O; make $d\,e = R_{IV}\, S_{IV}$; draw $e\,f$ parallel P O, make $f\,g = R_{III}\, S_{III}$, and similarly $g\,h$, $i\,j$ and $k\,l$ parallel P O, and $h\,i = R_{II}\, S_{II}$; $j\,k = R_{I}\, S_{I}$ and $l\,m = R\,S$. Draw from a lines to all the points c, d, e, f, g, etc. Now in Figure 68 begin at P_{V} O_{V}, prolong it till it intersects vertical axis G_{V} at I_{V}, draw I_{V} H_{V} parallel $a\,c$ till it intersects P_{IV} O_{IV} at H_{V}; draw H_{V} I_{IV} parallel to $a\,d$ till it intersects vertical axis G_{IV} at I_{IV}; draw I_{IV} H_{IV} parallel to $a\,e$ till it intersects P_{III} O_{III} at H_{IV} and similarly H_{IV} I_{III} parallel $a\,f$; I_{III} H_{III} parallel $a\,g$; H_{III} I_{II} parallel $a\,h$; I_{II} H_{II} parallel to $a\,i$; H_{II} I_{I} parallel to $a\,j$; I_{I} H_{I} parallel to $a\,k$; H_{I} I parallel to $a\,l$, and I K parallel to $a\,m$.

The points of intersection K, K_{II}, K_{III}, etc., are points of the curve of pressure. To find the amount of the pressure at each point, find

weight per running foot of length of any part of wall, say, the bottom part ($A A_{,} D_{,} D$) the contents are 5′ high, 4′ 8″ wide, 1′ thick $= 5.4\frac{2}{3}.1$ $= 23\frac{1}{3}$ cubic feet à 150 pounds

$= 3500$ pounds.

Divide the weight by the length of $m\,l$ in inches, and we have the number of pounds per inch, by which to measure the pressures. As $m\,l$ measures 56 inches, each inch will represent $\frac{3500}{56} = 62\frac{1}{2}$ lbs.

Now let us examine any joint, say, $A_{,,,}\,D_{,,,}$; $I_{,,,}\,H_{,,,}$ which intersects $A_{,,,}\,D_{,,,}$ at $K_{,,,}$ is parallel to $a\,g$. Now $a\,g$ scales 166 inches, therefore, pressure at $K_{,,,} = 166.\ 62\frac{1}{2} = 10375$ pounds. In measuring the distance of $K_{,,,}$ from centre of joint in the following, remember that the width of $A_{,,,}\,D_{,,,}$ is 44 inches, the width of masonry above joint, and *not* 48″ (the width of masonry below). $A_{,,,}\,K_{,,,}$ scales 38″, therefore, distance x of $K_{,,,}$ from centre of joint is $x = 38 - 22 = 16''$

We have, then, from Formula (44)

stress at $D_{,,,}$; $v = \dfrac{10375}{44.12} + 6.\dfrac{16.10375}{44.12.44} = +\ 63$ pounds.

and from formula (45)

stress at $A_{,,,}$; $v = \dfrac{10375}{44.12} - 6.\dfrac{16.10375}{44.12.44} = -\ 23$ pounds.

The joint $A_{,,,}\,D_{,,,}$, therefore, would be more than safe.

Let us try the bottom joint A D similarly. I K is parallel to $a\,m$ now $a\,m$ scales 480″, therefore, the pressure at K is $p = 480.\ 62\frac{1}{2} =$ 30000 pounds.

Now K is distant 53 inches from centre of joint, therefore, stress at D is $v = \dfrac{30000}{56.12} + 6.\dfrac{53.30000}{56.12.56} = +\ 298$ pounds.

and stress at A is $v = \dfrac{30000}{56.12} - 6.\dfrac{53.30000}{56.12.56} = -\ 209$ pounds.

The wall would evidently have to be thickened at the base. If we could only brace the wall until the superimposed weight were on it, this might not be necessary. If we could do this we should lengthen $b\,c$ an amount of inches equal to the amount of this load divided by $62\frac{1}{2}$ (the number of pounds per inch), or $b\,c$ instead of being 36 inches long would be:

$$36 + \frac{25000}{62\frac{1}{2}} = 436 \text{ inches long.}$$

While this lengthening of $b\,c$ would make the lines of pressure $a\,c$, $a\,e$, $a\,f$, etc., very much longer, and consequently the actual pressure very much greater, it will also make them very much steeper and consequently bring this pressure so much nearer the centre of each joint,

that the pressure will distribute itself over the joint much more evenly, and the worst danger (from tension) will probably be entirely removed.

Example IV.

Reservoir Wall. *A stone reservoir wall is plumb on the outside,* 2 *feet wide at the top and* 5 *feet wide at the bottom; the wall is* 21 *feet high, and the possible depth of water* 20 *feet. Is the wall safe?*

Divide the wall into three parts in height; that is, $D\,D_{\prime} = D_{\prime}\,D_{\prime\prime} = D_{\prime\prime}\,C$. Find the weight of the parts from each joint to top, per running foot of length of wall, figuring the stonework at 150 pounds per cubic foot, and we have:

Weight of $A_{\prime\prime}\,B\,C\,D_{\prime\prime}$ $= 2975$ pounds.

Weight of $A_{\prime}\,B\,C\,D_{\prime}$ $= 7140$ pounds.

Weight of $A\,B\,C\,D$ $= 12495$ pounds.

Find centres of gravity of the parts $A\,B\,C\,D$ (at G), of $A_{\prime}\,B\,C\,D_{\prime}$ (at $G_{\prime}$) and of $A_{\prime\prime}\,B\,C\,D_{\prime\prime}$ (at $G_{\prime\prime}$). Apply the pressures $P\,O$ at $\frac{1}{3}$ height of $A\,E$; $P_{\prime}\,O_{\prime}$ at $\frac{1}{3}$ height of $A_{\prime}\,E$ and $P_{\prime\prime}\,O_{\prime\prime}$ at $\frac{1}{3}$ height of $A_{\prime\prime}\,E$, where E top level of water.

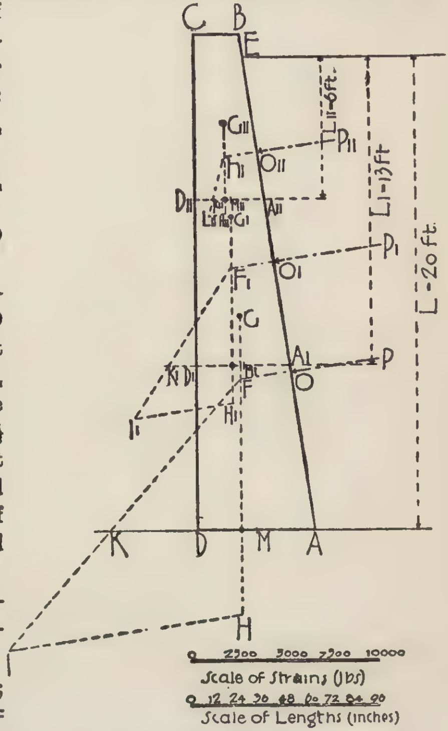


Fig. 70.

The amount of pressures will be from formula (53).

For part $A_{\prime\prime}\,E$; $P_{\prime\prime}\,O_{\prime\prime} = 31\frac{1}{4}.\ L^2_{\prime\prime} = 31\frac{1}{4}.\ 6^2 = 1125$ pounds.

For part $A_{\prime}\,E$; $P_{\prime}\,O_{\prime} = 31\frac{1}{4}.\ L^2_{\prime} = 31\frac{1}{4}.\ 13^2 = 5281$ pounds.

For part $A\,E$; $P\,O = 31\frac{1}{4}.\ L^2 = 31\frac{1}{4}.\ 20^2 = 12500$ pounds.

The pressures $P\,O$, $P_{\prime}\,O_{\prime\prime}$, etc., will be applied at right angles to $A\,E$, prolong these lines, till they intersect the vertical axes through

(the centres of gravity) G, $G_{\prime}$ and $G_{\prime\prime}$ at F, $F_{\prime}$ and $F_{\prime\prime}$. Then make

$F_{\prime\prime}\,H_{\prime\prime} = 2975$, weight of upper part.

$F_{\prime}\,H_{\prime} = 7140$, weight of $A_{\prime}$ B C $D_{\prime\prime}$, and

$F\ H = 12495$, weight of A B C D.

Draw through H, $H_{\prime}$ and $H_{\prime\prime}$ the lines parallel to pressure lines making

$$H_{\prime\prime}\,I_{\prime\prime} = P_{\prime\prime}\,O_{\prime\prime} = 1125$$
$$H_{\prime}\,I_{\prime} = P_{\prime}\,O_{\prime} = 5281$$
$$H\ I = P\ O = 12500$$

Draw $I_{\prime\prime}$ $F_{\prime\prime}$, $I_{\prime}$ $F_{\prime}$ and I F, then will their lengths represent the amounts of pressure at points $K_{\prime\prime}$, $K_{\prime}$ and K on the joints $A_{\prime\prime}$ $D_{\prime\prime}$, $A_{\prime}$ $D_{\prime}$ and A D.

$F_{\prime\prime}\,I_{\prime\prime}$	measures	3300	units or	pounds.
$F_{\prime}\,I_{\prime}$	"	9500	"	"
F I	"	18800	"	"

By scaling we find that

$K_{\prime\prime}$	is $10\frac{1}{2}$	inches from centre of	$D_{\prime\prime}$ $A_{\prime\prime}$
$K_{\prime}$	is 37	" "	$D_{\prime}$ $A_{\prime}$
K	is 74	" "	D A

The stresses to be exerted by the wall will, therefore, be

$$\text{at } D_{\prime\prime};\ v = \frac{3300}{36.12} + 6.\frac{10\frac{1}{2}.3300}{36.12.36} = +\ 21 \text{ pounds.}$$

$$\text{at } A_{\prime\prime};\ v = \frac{3300}{36.12} - 6.\frac{10\frac{1}{2}.3300}{36.12.36} = -\ 6 \text{ pounds.}$$

$$\text{at } D_{\prime};\ v = \frac{9500}{48.12} + 6.\frac{37.9500}{48.12.48} = +\ 93 \text{ pounds.}$$

$$\text{at } A_{\prime};\ v = \frac{9500}{48.12} - 6.\frac{37.9500}{48.12.48} = -\ 60 \text{ pounds.}$$

$$\text{at } D;\ v = \frac{18800}{60.12} + 6.\frac{74.18800}{60.12.60} = +\ 219 \text{ pounds.}$$

$$\text{at } A;\ v = \frac{18800}{60.12} - 6.\frac{74.18800}{60.12.60} = -\ 167 \text{ pounds.}$$

From the above it would appear that none of the joints are subject to excessive compression: further that joint $D_{\prime\prime}A_{\prime\prime}$ is more than safe, but that the joints $D_{\prime}$ $A_{\prime}$ and D A are subject to such severe tension that they cannot be passed as safe. The wall should, therefore, be redesigned, making the upper joint lighter and the lower two joints much wider.

CHAPTER IV.

WALLS AND PIERS.[1]

WALLS are usually built of brick or stone, which are sometimes, though rarely, laid up dry, but usually with mortar filling all the joints. The object of mortar is threefold:

Object of mortar. 1. To keep out wet and changes of temperature by filling all the crevices and joints.

2. To cement the whole into one mass, keeping the several parts from separating, and,

3. To form a sort of cushion, to distribute the crushing evenly, taking up any inequalities of the brick or stone, in their beds, which might fracture each other by bearing on one or two spots only.

To attain the first object, "grouting" is often resorted to. That is, the material is laid up with the joints only partly filled, and liquid cement-mortar is poured on till it runs into and fills all the joints. Theoretically this is often condemned, as it is apt to lead to careless and dirty work and the overlooking of the filling of some parts; but *practically* it makes the best work and is to be recommended, except, of course, in freezing weather, when as little water as possible should be used.

To attain the second object, of cementing the whole into one mass, it is necessary that the mortar should adhere firmly to all parts, and this necessitates soaking thoroughly the bricks or stones, as otherwise they will absorb the dampness from the mortar, which will crumble to dust and fail to set for want of water. Then, too, the brick and stone need washing, as any dust on them is apt to keep the mortar from clinching to them. In winter, of course, all soaking must be avoided, and as the mortar will not set so quickly, a little lime is added, to keep it warm and prevent freezing.

Thickness of Joints. To attain the third object, the mortar joint must be made thick enough to take up any inequalities of the brick or stone. It is, therefore, impossible to set any standard for

[1] See Chapter VIII on "Reinforced Concrete Construction."

joints, as the more irregular the beds of the brick or stone, the larger should be the joint. For general brickwork it will do to assume that the joints shall not average over one-quarter of an inch above irregularities. Specify, therefore, that, say, eight courses of brick laid up "in the wall" shall not exceed by more than two inches in height eight courses of brick laid up "dry." For front work it is usual to gauge the brick, to get them of exactly even width, and to lay them up with one-eighth inch joints, using, as a rule, "putty" mortar. While this makes the prettiest wall, it is the weakest, as the mortar has little strength, and the joint being so small it is impossible to bond the facing back, except every five or six courses in height. "Putty" mortar is made of lime, water and white lead, care being taken to avoid all sand or grit in the mortar or on the beds.

Quality of mortars. The best mortar consists of English Portland cement and sharp, clean, coarse sand. The less sand the stronger the mortar.

Sand for all mortars should be free from earth, salt, or other impurities. It should be carefully screened, and for very important work should be washed. The coarser and sharper the sand the better the cement will stick to it. English Portland cement will stand as much as three or four parts of sand. Next to English come the German Portland cements, which are nearly as good. Then the American Portland, and lastly the Rosendale and Virginia cements. Good qualities of Rosendale cements will stand as much as two-and-a-half of sand. Of limes, the French lime of Teil is the strongest and most expensive. Good, hard-burned lime makes a fairly good mortar. It should be thoroughly slacked, as otherwise, if it should absorb any dampness afterwards, it will begin to burn and swell again. At least forty-eight hours should be allowed the lime for slacking, and it is very desirable to strain it to avoid unslacked lumps. Lime will take more sand than cement, and can be mixed with from two to four of sand, much depending on the quality of the sand, and particularly on the "fatness" of the lime. It is better to use plenty of sand (with lime) rather than too little; it is a matter, however, for practical judgment and experiment, and while the specification should call for but two parts of sand to one of lime, the architect should feel at liberty to allow more sand if thought desirable. Lime and Rosendale cement are often mixed in equal proportions, and from three to five parts of sand added; that is, one of lime, one of cement, and three to five of sand. It is advisable to specify that all parts shall be actually measured in barrels, to avoid such tricks, for instance, as hiring a

decrepit laborer to shovel cement or lime, while two or three of the strongest laborers are shovelling sand, it being called one of cement to two or three of sand. A little lime should be added, even to the very best mortars, in winter, to prevent their freezing.

Frozen walls. When a wall has been frozen, it should be taken down and re-built. Never build on ice, but use salt, if necessary, to thaw it; sweep off the salt-water, which is apt to rot the mortar, and then take off a few courses of brick before continuing the work. Protect walls from rain and frost in winter by using boards and tarpaulins. Some writers claim that it does no harm for a wall to freeze; this may be so, provided all parts freeze together and are kept frozen until set, and that they do not alternately freeze and thaw, which latter will undoubtedly rot the mortar.

Plaster-of-Paris makes a good mortar, but is expensive and cannot stand dampness. Cements or limes that will set under water are called hydraulic.

Quickness of setting is a very desirable point in cements. All cement-mortars, therefore, must be used perfectly fresh; any that has begun to set, or has frozen, should be condemned, though many contractors have a trick of cutting it up and using it over with fresh mortar. To keep dampness out of cellar-walls the outside should be plastered with a mortar of some good hydraulic cement, with not more than one part of sand to one part of cement; this cement should be scratched, roughened, and then the cement covered outside with a heavy coat of asphalt, put on hot and with the trowel. In brick walls, the coat of cement can be omitted and the joints raked out, the asphalt being applied directly against the brick. This asphalt should be made to form a tight joint, with the slate or asphalt damp-course, which is built through bottom of wall, to stop the rise of dampness from capillary attraction.

In ordinary rubble stonework the mortar should be as strong as possible, as this class of work depends entirely on the mortar for its strength.

For the strengths of different mortars, see Table V.

Some cements are apt to swell in setting, and should be avoided.

Smoke flues. Where flues or unplastered walls are built, the joints should be "*struck*," that is, scraped smooth with the trowel. No flues should be "pargetted"; that is, plastered over, as the smoke rots the mortar, particles fall, and the soot accumulating in the crevices is apt to set fire to the chimney. Joints of chimneys are liable to be eaten out from the same reason, and the loose por-

tions fall or are scraped out when the flues are cleaned, leaving dangerous cracks for fire to escape through. It is best, therefore, to line up chimneys inside with burned earthenware or fire-clay pipes. If iron pipes are used, cast-iron is preferable; wrought-iron, unless very thick, will soon be eaten away. Where walls are to be plastered, the joints are left as rough as possible, to form a good clinch.[1]

Wall furrings. Outside walls are not plastered directly on the inside, unless hollow; otherwise, the dampness would strike through and the plaster not only be constantly damp, but it would ultimately fall off. Outside walls, unless hollow, are always "furred." In fireproof work, from one to four-inch thick blocks are used for this purpose. These blocks are sometimes cast of ashes, lime, etc., but are a very poor lot and not very lasting. Generally they are made of burnt clay, fire-clay or porous terra-cotta. The latter is the best, as, besides the advantages of being lighter, warmer and more damp-proof, it can be cut, sawed, nailed into, etc., and holds a nail or screw as firmly as wood. These blocks are laid up independently of the wall, but occasionally anchored to the same by iron anchors. The plastering is applied directly to the blocks. Where space is very desirable, or the economy of saving expense of fire proof furring is necessary, or it is desired to avoid the dangers of wooden furrings, viz: fire-spaces, vermin, rats, mice, etc., Antihydrine should be applied immediately to the inside surface of the brick or concrete wall, care being taken to have this surface smooth and free from cracks or holes of any kind.

Antihydrine—the original wall damp-proofing material—has to-day many imitators, some good, many worse than useless. The architect, if he cannot obtain the genuine, should make thorough practical tests, as otherwise he may later have to remove all plastering.

Even where fire proof furrings are used it is desirable to coat them with Antihydrine, or similar good material, to avoid subsequent staining of plasters. Similar damp-proofing material applied to fire proof ceilings and partitions, will stop the staining and frequent ruin of subsequent decorations.

In all cases the first coat of plaster should be put on while the Antihydrine is still tacky. The damp-proofing material is best ap-

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[1] Except in cases where Antihydrine or similar damp-proof materials, such as tar, etc., take the place of furrings. In such cases the walls and joints should be as smooth as possible, as even a small pin-hole may cause dampness and staining of plaster.

plied with a whitewash brush, after thorough mixing, and should be gone over to find breaks or holes in it, before plastering.

The architect should allow no thinning or adulteration, on any excuse whatever, of this or any other damp-proofing material.

In cheaper and non-fireproof work, furrings are made of vertical strips of wood about two inches wide, and from one to two inches thick, according to the regularity of the backing. For very fine work, sometimes, an independent four-inch frame is built inside of the stone-wall, and only anchored to same occasionally by iron anchors. Where there are inside blinds, a three or four inch furring is used (or a fireproof furring), and this is built on the floor beams, as far inside of the wall as the shutter-boxes demand. To the wooden furrings the lath are nailed. Furrings are set, as a rule, sixteen inches apart, the lath being four feet long; this affords four nailings to each lath. Sometimes the furrings are set twelve inches apart, affording five nailings. All ceilings are cross-furred every twelve inches, on account of stiffness, and the strips should not be less than one-and-three-eighths inches thick, to afford strength for nailing. Furring-strips take up considerable of the strain of settlements and shrinkage, and prevent cracks in plastering by distributing the strain to several strips. To still further help this object, the "heading-joints" of lath should not all be on the same strip, but should be frequently broken (say, every foot or two), and should then be on some other strip. Laths should be separated sufficiently (about three-eighths inch) to allow the plaster to be well worked through the joint and get a strong grip or "clinch" on the back of the laths. If a building is properly built, theoretically correct in every respect, it should not show a single crack in plastering. Practically, however, this is impossible.

Shrinkage of Joints. But there never need be any fear of shrinkage or settlement, in a well-constructed building, where the foundations, joints and timbers are properly proportioned. The danger is never from the *amount* of settlements or shrinkage, but from the *inequality* of same in different parts of the building. Inequalities in settlements are avoided by properly proportioning the foundations. Inequalities in shrinkage of the joints, though quite as important, are frequently overlooked by the careless architect. He will build in the same building one wall of brick with many joints, another of stones of all heights and with few joints, and then put iron columns in the centre, making no allowance whatever for the difference in shrinkage. If he makes any, it is probably to call for the most exact setting of the columns, for the hardest and quickest-setting Portland cement for the stonework,

and probably be content with lime for the brickwork. To avoid uneven shrinkages, allowances should be made for same. Brickwork will shrink, according to its quality, from one-sixteenth to one-eighth inch per story, ten to twelve feet high, and according to the total height of wall. The higher the wall, the greater the weight on the joints and the greater the shrinkage. Iron columns should, therefore, be made a trifle shorter than the story requires, the beams being set out of level, lower at the column. The plan should provide for the top of lowest column to be one-sixteenth or one-eighth inch low, while the top of highest column would be as many times one-sixteenth or one-eighth inch low as there were stories; or if there were eight stories, the top of bottom column for the very best brickwork would be, say, one-sixteenth inch low, and the top of highest column would be one-half inch low. Stone walls should have stone backings in courses as high as front stones, if possible; if not, the backing should be set in the hardest and quickest-setting cement.

Slip-joints.

Stone walls should be connected to brick walls by means of slip-joints. By this method the writer has built a city stone-front, some 150 feet high and over 50 feet wide, connected to brick walls at each side, without a single stone sill, or transom, or lintel cracking in the front. The slip-joint should carry through foundations and base courses where the pressure is not equal on all parts of the foundation. If for the sake of design, it is necessary to use long columns or pilasters, in connection with coursed stone backings, the columns or pilasters must either be strong enough to do the whole work of the wall, or else must be bedded in putty-mortar with generous top and bottom joints, to allow for shrinkage of the more frequent joints behind them; otherwise, they are apt to be shattered. Such unconstructional designs had, however, better be avoided. In no case should a wall be built of part iron uprights and part masonry; one or the other must be strong enough to do the work alone; no reliance could be placed on their acting together. In

Shrinkage of timber.

frame walls, care should be taken to get the amount of "cross" timbering in inner and outer walls about equal, and to have as little of it as possible. Timber will shrink "*across*" the grain from one-fourth to one-half inch per foot. Where the outer walls are of masonry, and inner partitions or girders are of wood, great care must be taken that the shrinkage of each floor is taken up by itself. If the shrinkage of all beams and girders is transferred to the bottom, it makes a tremendous strain on the building and will ruin the plastering. To

effect this, posts and columns should bear *directly* on each other, and the girders be attached to their sides or to brackets, but by no means should the girder run between the upper and lower posts or columns. If there are stud-partitions, the head pieces should be as thin as possible, and the studs to upper partitions should rest *directly* on the head of lower partitions.

All beds level. In masonry, all beds should be as nearly level as possible, to avoid unequal crushing. Particularly is this the case with cut stonework. If the front of the stone comes closer than the backing (which is foolishly done sometimes to make a small-looking joint), the face of the stone will surely split off. If the back of a joint is broken off carelessly, and small stones inserted in the back of a joint to form a support to larger stones, they will act as wedges, and the stone will crack up the centre of joint and wall. Stones should be bedded, therefore, perfectly level and solid, except the front of joint for about three-fourths inch back from the face, which should not be bedded solid, but with "putty"-mortar.

Cement Stains. Light-colored stones, particularly lime-stones, are apt to stain if brought in connection with cement-mortar. A good treatment for such stones is to coat the back, sides and beds with Antihydrine or similar damp-proofing material. This should be put on both sides, top, bottom and back of stone before setting. No space should be left for marking, but contractor be compelled to cut marks with *chisel, before* coating. After stones are set it is well to give an additional coat on back, covering joints as well. La Farge or similar white, non-staining cements, though expensive, will still further insure immunity against unsightly stains, after cleaning down.

Natural bed. All stones should be laid on their "natural beds"; that is, in the same position as taken from the quarry. This will bring the layers of each stone into horizontal positions, on top of each other, and avoid the "peeling" so frequently seen. Ashlar should be well anchored to the backing. The joints should be filled with putty-mortar, and should be sufficiently large to take up the shrinkage of the backing.

Size of Stones. Stones should not be so large as to risk the danger of their being improperly bedded and so breaking. Professor Rankine recommends for soft stones, such as sand and lime stones, which will crush with less than 5000 pounds pressure per square inch, that the length shall not exceed three times the depth, nor the breadth one-and-a-half times the depth. For hard stones, which will resist 5000 pounds compression per square inch, he

allows the length to be from four to five times the depth, and the breadth three times the depth. Stones are sometimes joined with "rebated" joints, or "dove-tail" joints, the latter particularly in circular work, such as domes or light-houses. The practice (Early Italian Renaissance) of making stone-work in courses of long stones, alternately thin and thick, and breaking joints centrally, is very pretty in effect, but very apt to break every thin stone immediately over joints in big stones, as can be seen, for instance, in the Memorial Arch, Washington Square, New York City, and in innumerable other examples.

Sills bedded hollow. All sills in either stone or brick walls should be bedded at the ends only, and the centre part left hollow until the walls are thoroughly set and settled; otherwise, as the piers go down, the part or panels between them, not being so much weighted, will refuse to set or settle equally with them, and will force up the centre of sill and break it. Where there are lintels across openings in one piece, with central mullion, the lintel should either be jointed on the mullion, or else the mullion bedded in putty at the top. Otherwise, the lintel will break; or, if it be very strong, the mullion will split; for, as the piers set or settle, the lintel tends to go down with them, and, meeting the mullion, must either force it down, or else break it, or break itself.[1]

Slip-joints. Walls of uneven height, even where of the same material, should be connected to each other by means of a slip-joint, so as to provide for the uneven shrinkage. Slip-joints must be so designed that while they allow independent vertical movement to each part, neither can separate from the other in any other direction. Figures 71 to 73 give a few examples.

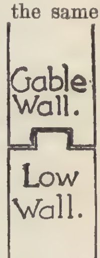


Fig. 71.

Figure 71 shows the plan of a gable-wall connected with a lower wall, by means of a slip-joint. Figure 72 shows the corner of a front stone-wall connected similarly with side-walk of brick. Figure 73 the corner of a tower or chimney connected with a lower wall. The joint must be built plumb from top to bottom. Where the higher wall sets over the tongue above lower wall, one or two inches must be left hollow over the tongue, to allow for settlement or shrinkage of the higher wall and to prevent its resting on the tongue and possibly cracking it off. Where iron anchors are used in connection with slip-joints, they should be so arranged as to allow free vertical movement.

[1] Top joints of mullions in such cases had better not be filled at all until "cleaning-down," and then only with putty mortar. In meanwhile keep joint clear by sawing it out occasionally.

Such joints and anchors must be designed with reference to each special case. In stepped-foundations (on shelving-rock, etc.), or in walls of uneven heights where slip-joints are impracticable, the foundations or walls should be built up to each successive level and be allowed to set thoroughly before building further. A hard and quick-setting cement should be used, and the joints made as small as possible. In no case should one wall of a building be carried up much higher than the others, where slip-joints are not to be used. When building on top of old work, clean same off thoroughly or the mortar will not take hold (clinch). In summer, soak the old work thoroughly.

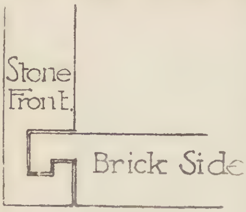


Fig. 72.

Old and new walls. Where new work has to be built against old work, a slip-joint should be used, if possible, or else a straight joint should be used with slip-anchors, and after the new work has thoroughly set, bond-stones can be cut in. In such cases, the foundations should be spread as much as possible, to avoid serious settlements. In all work involving old and new walls combined, the quickest and hardest-setting cements should be used. Sometimes it is advisable not to load walls until they have set, unless all walls are loaded alike, as the uneven weights on green walls are apt to crack them. All walls should be well braced, and wooden centres left in till they have set thoroughly.

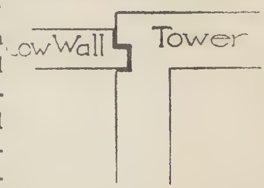


Fig. 73.

Timber in walls. Timber of any kind, in walls, should be avoided, if possible.

It should only be used for temporary support, as it is liable to rot, shrink, burn out, or to absorb dampness and swell, in either case causing settlements or cracks, even if not endangering the wall. In no case bond a wall with timbers. Where it is necessary to nail into a wall, wooden plugs are sometimes driven into the joints; they are very bad, however, and liable to shake the wall in driving. Wooden strips, let in, weaken the wall just that much. Wooden nailing-blocks, though not much better, are frequently used. The block should be the full thickness of the bricks, plus the upper and lower joints. If

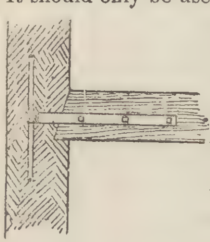


Fig. 74.

there is any mortar over or under the block, the nailing will jar it loose and the block fall out. The best arrangement to secure nailings is to build-in porous terra-cotta blocks or bricks.

Beam ends. Where ends of beams are built into the wall, they should always be cut off to a slant, as shown in Figure 74.

The anchors should be attached to the side, so as to allow the beam to fall out in case it is burned through. If the beams were not cut to a slant, the leverage produced by their weight, when burned through, would be apt to throw the wall; as it is, each beam can fall out easily and the wall, being corbelled over the beam-opening, remains standing. It is desirable to "build-in" the ends of wooden beams as little as possible, to prevent dry-rot; if it can be arranged to circulate air around their ends, it will help preserve them. Beams should always be levelled-up with good-sized pieces of slate, and not with wood-chips, which are liable to crush. The old-fashioned way of corbelling out to receive beams, leaving the wall intact, has much to commend it. A modern practice is to corbel out one course of brick, at each ceiling-level, just sufficient to take the projection of furring-strips; this will stop draughts in case of fire, also rats and mice from ascending. All slots for pipes, etc., should be bricked up solid around the pipes for about one foot at each ceiling-level for the same purposes.

Wooden lintels. Where wooden lintels are used in walls, there should always be a relieving-arch over them, so arranged that it would stand, even if the lintel were burned out or removed; the lintel should have as little bearing as possible, and be shaved off at the ends.

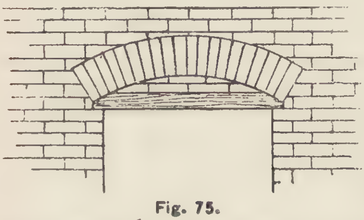


Fig. 75.

Figure 75 shows a wooden lintel correctly built-in. Figure 76 shows a very blundering way of building-in a wooden lintel, but one, nevertheless, frequently met with. It is obvious, however, in the latter case, that if the lintel were removed the abutment to the arch would sink and let the arch down. The relieving-arch, after it has set, should be strong enough to carry the wall, the lintel being then used for nailing only. The rule for lintels is to make their depth about one-tenth of the span.

Bonded arches best. Arches are built of "row-locks" (that is, "headers,") or of "stretchers," or a combination of both, according

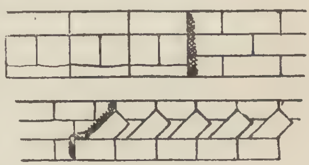
to design. The strongest arch, however, is one which has a combination of both headers and stretchers; that is, one which is bonded on the face, and also bonded into the backing. Straight arches and arches built in circular walls should always be bonded into the backing, or if the design does not allow of this, they should be anchored back. "Straight" arches should be built with a slight "camber" up towards the centre, to allow for settlement and to satisfy the eye. About one-eighth inch rise at the centre fo each foot of span is sufficient. Straight arches should never be built, as shown in Figure 77, and known as the French or Dutch arch, as there is absolutely no strength to them. Fireplaces are frequently arched over in this way, but the practice is a very bad one.

Fig. 76.

Fig. 77.

Brick facings.

Brick facings, as laid up in this country, usually consist of all stretchers. Every fifth or sixth course is bonded into the backing, either by splitting the brick in two, as shown in Figure 78, and using short headers behind it, or by breaking off the rear corners, as shown in Figure 79, and using diagonally-laid bond-brick.



Figs. 78 and 79.

The latter course is the better, but there is no strength in either; particularly as, as a rule, the front brick are so much softer and weaker than those in the backing.

The English bond, in which a course of headers alternates with a course of stretchers, is much to be preferred; or, better yet, the Flemish bond, where in each course a header alternates with a stretcher. Of course, in both English and Flemish bond, if the front brick are thinner than the bricks used in the backing, larger and more unsightly joints will be necessary in front.

Regular work the best. It is best, as a rule, not to count on the front work for strength. We frequently see masons laying up brick walls by first laying a single course of headers or stretchers on the outside of the wall, and then one on the inside, and then filling the balance of wall with bats and all kind of rubbish. This makes a very poor wall. The specification should provide that no bats or broken brick will be allowed, leaving it to the architect's discretion to stop their use, if it is being overdone; of course, some few will have to be used. But, after all, the best wall is that one which is built the most regularly and with the most frequent bonds, and no architect should be talked out of good, regular work, as being *too theoretical*, by so-called "practical" men. The necessity for regularity and bond is easily illustrated by taking a lot of bricks of different sizes, or even toy blocks, and attempting to pile them up without regularity; or, even if piled regularly, without bond. It will quickly be seen that the most regular and most frequently-bonded pile will go the highest. By "bond" is meant alternating headers and stretchers with regularity, and so as to cover and break joints.

Use of bond-stones. The use of "bond-stones" *at intervals only* is bad; they should be carried through the whole surface (width and length) of wall and be of even thickness, or else be omitted. Using bond-stones in one place only tends to concentrate the compression on one part of the wall. Thus, bond-stones built under each other at regular intervals, as shown in Figure 80, are bad, as they give the pressure no chance to spread, but keep concentrating it back onto the part of wall immediately under bond-stones, whereas, in Figure 81, the pressure is allowed to spread gradually over a larger area of the wall. Where, however, a heavy girder, column or other weight comes on a wall, it is distributed by means of a large block, generally granite or stone, or sometimes by a large iron plate.

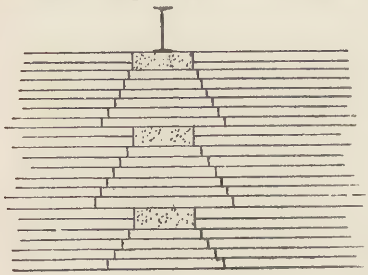


Fig. 80.

The block or plate should have sufficient area not to crush the brick-work directly under it. Where girder-ends are built into walls, it is also desirable to build a block *over* the girder-end as well as under the same. The upper block prevents any part of the wall

from resting on the girder or being affected by its shrinkage, if of wood; if the girder is of iron the upper block will wedge in the girder-end more firmly, and the girder will be able to carry more load. See page 57.

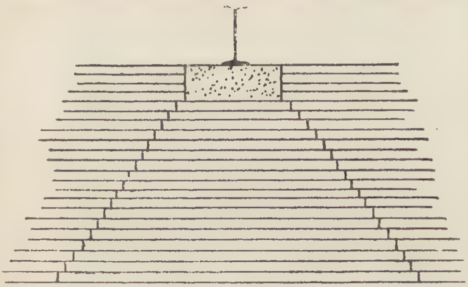


Fig. 81.

Anchors for girders and beams are usually made of iron and of such shape as to allow the girder or beam to fall out in case of fire. Anchors made of iron are not objectionable in inner walls, or where not exposed to dampness; all iron should be thoroughly painted, however, with red lead or metallic paint. Before painting, all rust should be scraped off. Don't believe the "practical" man who says the paint will stick better if you leave the rust on the iron; it will stick better to the rust, no doubt, but not to the iron. For outside work all iron should be galvanized; but it is better to use copper for anchors, dowels, clamps, etc. All copings should be clamped together, the clamps being counter-sunk. Slanting-work and tracery should be dowelled together. Where iron is let into stones, and run with lead or sulphur, the iron is apt to swell with rust or heat and burst the stone. Dampness in walls is one of the worst dangers, both on account of frost and decay.

Metal anchors.

Drip-moulds.

All exterior mouldings and sills should be projected and have "drip-moulds" cut underneath, to cause the water to drop or drip; this will prevent considerable dampness and keep the outside surface of the wall from becoming dirty, as the dust lodging on top of mouldings discolors the rain-water, and the latter, instead of streaking down the wall, will drop off.

Hollow walls. Walls are frequently built hollow to prevent dampness, but this raises many objections. Shall the inner or outer wall be the thicker? If the outer wall, then all beams, etc., have to be that much longer, so as to rest on the stronger part; they are liable to transfer dampness, and then, too, the thicker, and, consequently, greater, part of wall is exposed to dampness. If the inner wall is thicker, the construction, so far as beams, etc., are concerned,

no doubt is better, but the outer part is apt to be destroyed by the frost. Then at windows and doorways both must be connected, and dampness is apt to get through. It is well to ventilate the air-space between walls, at the bottom from inside and at the top from outside. The bottom of spaces should be drained. Tops of arches, or lintels over openings, should be cemented and asphalted (in the air-space), to shed any dampness settling on them. The outer and inner walls should be frequently anchored together. Iron anchors galvanized, or copper anchors are best; they should have a half-twist, as shown, to prevent water running along them.

Fig. 82.

Care must be taken to keep hollow walls free from hanging mortar, which will communicate moisture from one wall to the other.

But hollow walls are not nearly so good as solid walls with porous terra-cotta furrings, or coated on inside with Antihydrine material, if not furred.

Where walls are coped with stone, there should be damp-courses of slate or asphalt under same, and the back side should be flashed, to prevent dampness descending. If gutters are cut in stone cornices, they should be lined with metal, preferably copper, the outer edge being let into a raggle and run in with lead.

Underpinning. When a wall, already built, has to have its foundations carried down lower, it is called "underpinning the wall." Holes are made through the wall at intervals and through these (at right angles to the wall) are placed the "needles," that is, heavy timbers, which carry the weight of the wall. Where the needle comes in contact with the wall, small cross-beams are laid on its upper side, and wedged and filled with mortar, to get a larger and more even bearing against the wall. At the inner and outer ends of the needles heavy upright timbers are placed underneath, running down to the new, lower level. The foot or ground bearing of these timbers is formed by heavy planks crossing each other, to spread the weight over more ground; wedges are driven under the feet of the uprights, till the ends of the needle are forced up, and the centre of the needle shows a decided downward curve of deflection, indicating that the weight of wall is on the needle. Frequently jack-screws are used in place of wedges, to get the weight onto the uprights. As soon as the needles carry the weight of wall, the intermediate portions of wall are torn out and the excavating to the lower level begins. If the soil is loose, sheath-piling must be resorted to on each side of wall. Frequently the feet of the uprights are "cribbed," that is, sheath-piled all around, to hold the ground under

them together and keep it from compressing. The new wall is built up from the lower level between and around the needles. On top of the new wall two layers of dressed-stone are placed filling up between the old and new work. Between these stones iron wedges are driven in opposite pairs, one from the inside and one from the outside. These wedges must be evenly driven from both sides or the wall might tip. These wedges are driven until the weight of the wall is on them and off the needles.

This is readily seen, for the needles straighten out when relieved of the load. The jack-screws are now lowered or the wedges under the uprights eased up; the uprights taken away, needles removed, and the holes filled up. Underpinning operations must be slowly and carefully performed, as they are very risky. If there is any danger of a wall tipping during the operation, grooves are cut into the wall and "shores" or braces placed against it. The feet of the shores rest on cross-planks, same as uprights, and are wedged up to get a secure bearing of the top of the shore against the wall. Where the outside of a wall cannot be got at, "spring needles" are used from the inside. That is, the one end of the needle acts as a lever and supports the wall, while the other, inner end, is chained and anchored down to prevent its tipping up.

Strength of bricks. The strength of a wall depends, of course, largely on the material used. A good, hard-burned brick, well laid in cement-mortar, makes a very strong wall. To tell a good brick, first examine the color; if it is very light, an orange-red, the brick is apt to be soft. If the brick is easily carved with a knife, it is soft. If it can be crushed to powder easily, it is soft. If two bricks are struck together sharply, and the sound is dull, the bricks are poor; if the sound is clear, ringing, metallic, the bricks are good and hard. If a brick shows a neat fracture, it is a good sign; a ragged fracture is generally a poor sign. The fracture also shows the evenness of the burning and fineness of the material. A brick that chips and cannot be cut easily is a good brick. The darker the brick, the harder burned. This, of course, does not hold good for artificially-colored bricks. The straighter and more regular the brick, the softer it is (as a rule), as hard-burning is apt to warp a brick.

What has been said of the strength of bricks holds good of terra-cotta. The latter should be designed to be of same thickness, if possible, in all parts, and any hollows caused thereby must be filled-in solid. It is best to fill-in the hollows with bricks and mortar several

days in advance, and let the filling set, so as to be sure it will not swell up afterwards and burst the terra-cotta.

Strength of stones. To judge of the strength and durability of stones is a more difficult matter. If the stone be fractured, and presents, under a magnifying glass, a bright, clear, sharp surface, it is not likely to crumble from decay; if the surface is dull-appearing and looks earthy, it is likely to decay. Of course, samples can be tested for their crushing and tensile strengths, etc. And we can tell somewhat of the weathering qualities by observing similar stones in old buildings: much, however, depends whether the stones come from the same part of the quarry. Another test is to weigh different samples, when dry; immerse them in water for a given period, say, twenty-four hours, then weigh them again, and the sample absorbing the least amount of water (in proportion to its original weight) is, of course, the best stone.

Another test is to soak the stone in water for two or three days and put it out to freeze; if it does not chip or crack, it will probably weather well. Chemical tests are made sometimes, such as using sulphuric acid, to detect the presence of lime and magnesia; or, soaking the stones in a concentrated boiling solution of sulphate of soda; the stones are then exposed to the air, when the solution crystalizes in the pores and chips off particles of the stone, acting similarly to frost. The stones are weighed before and after the tests, the one showing the least proportional loss of weight being, of course, the better.

If stones are laid on their natural beds, however, little need be feared of the result, if the stone seems at all serviceable. The main dangers to walls are from wet and frost. Very heavy and oft-repeated vibrations may sometimes shake the mortar-joints, but this need not be seriously feared, in most cases; machinery may often cause sufficient vibrations to be unpleasant, or even to endanger wood or iron work, but hardly well-built masonry. Of course, the higher a building is, the greater will be the amount of vibrations and their strength. For this reason it is advisable to place the heaviest machinery on the lowest (ground) floor. The beds of such machinery should be as far as possible from any foundations of walls, columns, etc., and the beds should be independent and isolated from all other masonry. Malo, in *Le Génie Civil*, recommends the use of asphalt for machinery-foundations, as they take up the vibrations and noise, and are as solid as masonry, if properly built. His claim seems well founded, and has been demonstrated practically;

Machinery foundations.

the asphalt foundation not only preventing vibrations but stopping the sound. A wooden form is made, covered inside with well-greased paper; into this are placed slightly conical-shaped wooden bars and boxes, also covered with well-greased paper, which are secured in the places to be occupied by the bolts and bolt-heads, and arranged for easy withdrawal. A layer of melted asphalt a few inches thick is then poured into the mould; over this are dumped heated, perfectly clean, sharp, broken stones and pebbles, rammed solid, the pebbles filling all interstices; then more asphalt is poured in, then another layer of stones and pebbles, etc. It is claimed that this foundation becomes so solid that it will not yield enough to disarrange the smooth running of any machinery, while its slightly-elastic mortar, besides avoiding vibrations and noise, prolongs very much the durability and usefulness of the machinery. Two dangers must be guarded against; viz., the direct contact of oil or heat with the asphalt. Stationary drip-pans guard against the former, while a layer of rubber, wood, cement, or other non-conductive material would accomplish the latter object. Where noise from machinery is to be avoided, a layer about one inch thick, of hard rubber or soft wood, should be placed immediately under the engine-plate. If this layer were bedded in asphalt the precaution would be still more effective. Where this asphalt foundation is of any thickness, it is apt to bulge. In such cases the writer uses cast-iron reinforcing boxes with outside horizontal strengthening ribs and bolted at corners to hold asphalt in. There must, of course, be an upper and lower box, sliding vertically past each other, that is the edges of one inside the other's edges, to allow for vertical vibration in the asphalt concrete.

Quality of asphalts. In all cases where asphalt is used, that with the least proportion of bitumen should be preferred.

Seyssell asphalt, which comes from France, is undoubtedly the best, and next to this comes the Swiss or Neuchâtel asphalt. A very good quality of asphalt is also obtained from the mountains in our Western States.

Trinidad asphalt, which is much used in this country, is much inferior, being softer and containing a larger proportion of bitumen or tar—a great disadvantage in many cases. Coming, as it does from a lake it is not originally solid, as are the other asphalts.

Openings over each other. In all walls try to get all openings immediately over each other. A rule of every architect should be to make an elevation of *every* interior wall, as well as of the exterior walls, to see that openings come over each other.

Tower walls. It is foolish to make chimneys or tower walls unnecessarily thick (and heavy), as they brace and tie themselves together at each corner, and, consequently, are much

stronger than ordinary walls. Tower-walls, however, often require thickening all the way down, to allow for deep splays and jambs at the belfry openings. The chief danger in towers is at the piers on main floor, which are frequently whittled down to dangerous proportions, to make large door openings. In tall towers and chimneys the leverage from wind must be carefully considered.

Considering the importance of ascertaining the exact strength of walls, it is remarkable that so little attention has been paid to the subject by writers and experimenters. The only known rule to the writer is Rondelet's graphical rule, which is as follows:

Rondelet's rule. If A B (Figure 83) be the height of a wall, B C the length of wall without buttress or cross-wall (A B C being a right angle), then draw A C; make A D = to either $\frac{1}{8}$ or $\frac{1}{10}$ or $\frac{1}{12}$ of A B, according to the nature of wall and building it is intended for ($\frac{1}{12}$ for dwellings, $\frac{1}{10}$ for churches and fireproof buildings, and $\frac{1}{8}$ for warehouses); then make A E = A D, and draw E F parallel to A B; then is B F the required thickness of wall. The rule, however, in many cases, gives an absurd result. Gwilt's "*Encyclopædia of Architecture*" gives this rule, and many additional rules, for its modification. There are so many of them, and they are so complex, however, as to be utterly useless in practice. Most cities have the thickness of walls regulated by law, but, as a rule, these thicknesses give the minimum strength that will do, and where they do not regulate the amount and size of flues and openings, frequently allow dangerously-weak spots in the wall.

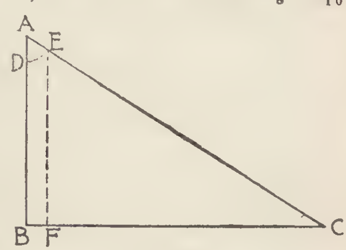


Fig. 83.

Formula for masonry. The writer prefers to use a formula, which he has constructed and based on Rankine's formula for long pillars, see Formula (3), and which allows for every condition of height, load, and shape and quality of masonry. In the case of piers, columns, towers or chimneys, whether square, round, rectangular, solid or hollow, the Formula (3) can be used, just as there given, inserting for ρ^2 its value, as given in the last column of Table I, using, of course, the numbered section corresponding to the cross-section of the pier, column or tower. By taking cross-sections at different points of the height, and using for l the height in inches from each such cross-section to the top, we will readily find how much to offset the wall. Care must be taken, where there are openings, to be sure to get the piers heavy enough to carry the additional load; the extra allowance

for piers should be gotten by calculating the pier first as an isolated pier of the height of opening, and then by taking one of our cross-sections of the whole tower or chimney at the level where the openings are, and using whichever result required the greater strength.

Strength of piers, chimneys and towers. As it would be awkward to use the height l i inches, we can modify the formula to use the heigl. L in feet. Further, we know the value of n for brickwork, from Table II, and can insert this, too; we should then have:

For brick or rubble piers, chimneys and towers, of whatever shape:

$$w = \frac{a.\left(\frac{c}{f}\right)}{1 + 0{,}475.\left(\frac{L^2}{\varrho^2}\right)} \qquad (59)$$

Where $w =$ the *safe total* load on pier, chimney or tower in pounds.

Where $a =$ the area of cross-section of pier, etc., in square inches, at any point of height.

Where $L =$ the height, in feet, from said point to top of masonry.

Where $\left(\frac{c}{f}\right) =$ the safe resistance to crushing, per square inch, as given in Table V. (See page 135.)

Where $\varrho^2 =$ the square of the radius of gyration, of the cross-section, in inches, as given in Table I.

If it is preferred to use *feet and tons* (2000 lbs. each) we should have

$$W = \frac{A.\left(\frac{c}{f}\right)}{14 + 0{,}046.\left(\frac{L^2}{P^2}\right)} \qquad (60)$$

Where $W =$ the *safe total* load on masonry, in tons, of 2000 lbs each.

Where $A =$ the area of cross-section of masonry, at any point of height, in square feet.

Where $L =$ the height, in feet, from said point to top.

Where $\left(\frac{c}{f}\right) =$ the safe resistance to crushing, in lbs., per square inch, as given in Table V. (See page 135.)

Where $P^2 =$ the square of the radius of gyration, as given in last column of Table I, — *but all dimensions to be taken in feet.*

To obtain the load on masonry, include weight of all masonry, floors, roofs, etc., above the point and (if wind is not figured separately) add for wind 15 lbs. for each square foot of outside superficial area of *all* walls above the point.

Where towers, chimneys, or walls, etc., are isolated, that is not

braced, and liable to be blown over by wind, the wind-pressure, must be looked into separately.

In regard to the use of $\left(\frac{c}{f}\right)$ the safe resistance, per square inch, of the material to crushing, it should be taken from Table V. So that for rubble-work we should use:

$$\left(\frac{c}{f}\right) = 100$$

And the same for poor quality brick, laid in lime mortar.

For fair brick in lime and cement (mixed) mortar, we should use:

$$\left(\frac{c}{f}\right) = 150$$

And for the best brickwork in cement mortar, we should use:

$$\left(\frac{c}{f}\right) = 200$$

If, however, a wall (or pier) is over 3 feet thick, and laid in good cement mortar, with the best hard-burned brick, and there are not many flues, etc., in the wall, we can safely use:

$$\left(\frac{c}{f}\right) = 250$$

If the wall (or pier) is over 3 feet thick, and there are no flues or openings, and the best brick and foreign Portland cements are used, it would be perfectly safe to use:

$$\left(\frac{c}{f}\right) = 300.$$

Example.

A tower 16 feet square outside, carries a steeple weighing, including wind-pressure, some 15 tons. The belfry openings, on each side, are central and virtually equal to openings 8 feet wide by 18 feet high each. There are 8 feet of solid wall over openings. What should be the thickness of belfry piers? The masonry is ordinary rubble-work.

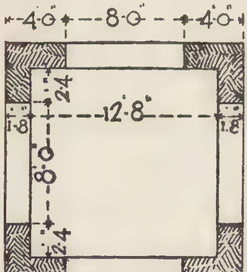


Fig. 84

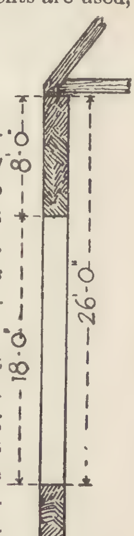


Fig. 85.

Tower Walls. In the first place we will try Formula (60) giving strength of whole tower at base of belfry piers. The load will be: 4.(26.16 — 18.8 — 26.1⅔) = 914 superficial feet of masonry 20″ thick, weighing say 245 lbs. per superficial foot = 223000 lbs. (see Figures

84 and 85) : add to this spire and we have at foot of belfry piers:

Actual load = 253000 lbs. or = 126 tons.

Now P^2 (the square of the radius of gyration) would be $P^2 = \frac{I}{A}$; the area $A = 16^2 - (12\frac{2}{3}^2 + 4.8.1\frac{2}{3}) = 43$; the moment of inertia $I = \frac{1}{12}.(16^4 - 12\frac{2}{3}^4 - 3\frac{1}{3}.8^3 - 8.16^3 + 8.12\frac{2}{3}^3) = 1799$.

Therefore $P^2 = \frac{1799}{43} = 41{,}8$. Now for rubble-work, Table V, $\left(\frac{c}{f}\right) = 100$; and, from Formula (60), the safe load would be:

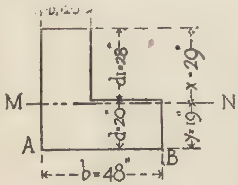


Fig. 86.

$$W = \frac{43.100}{14 + 0{,}046.\frac{26.26}{41{,}8}} = \frac{4300}{14{,}77}$$

= 291 tons; or more than strong enough.

Piers at opening. Now let us examine the strength of each pier by itself, Figure 86. In the first place we must find the distance y of the neutral axis M–N from say the line A B. This from Table I, Section No. 20 is:

$$y = \frac{\frac{48.20^2}{2} + 20.28.\left(20 + \frac{28}{2}\right)}{48.20 + 20.28} = 18,''8;\ \text{or, say, } y = 19''.$$

Now $i = \frac{20.29^3 + 28.1^3 + 48.19^3}{3} = 272347$ (in inches) and $a = 20.48 + 20.28 = 1520$ square inches, therefore $\varrho^2 = \frac{i}{a} = 179$ (in inches).

The length of each pier is 18 feet, or

$L = 18$.

Therefore, from Formula (59) we have the safe load:

$$W = \frac{1520.100}{1 + 0{,}475.\frac{18.18}{179}} = 81940 \text{ pounds,}$$

or say the safe load on each pier would be 41 tons.

The actual load we know is $\frac{126}{4} = 31\frac{1}{2}$ tons, or the pier is more than safe.

Thickness of walls. Now let us see how far down it would be safe to carry the 20″ walls. We use formula (60) and have from Section Number 4, of Table 1:

$$P^2 = \frac{16^2 + 12\frac{2}{3}^2}{12} = 34\frac{2}{3} \text{ (in feet).}$$

The area would be

$$A = 16^2 - 12\tfrac{2}{3}^2 = 96 \text{ square feet.}$$

The load for each additional foot under belfry would be then:

$$96.150 = 14400 \text{ lbs., or } 7,2 \text{ tons.}$$

The whole load from top down for each additional foot would be, in tons:

$$W_1 = 126 + (L - 26).7,2 = 7,2.L - 61$$

While the safe load from Formula (60) would be:

$$W = \frac{96.100}{14 + 0,046.\frac{L^2}{34\frac{2}{3}}}$$

Now trying this for a point 50 feet below spire, we should have the actual load:

$$W_1 = 7,2.50 - 61 = 299 \text{ tons.}$$

and the safe load:

$$W = \frac{96.100}{14 + 0,046.\frac{50.50}{34\frac{2}{3}}} = 554 \text{ tons, or, we can go still}$$

lower with the 20″ work. For 70 feet below spire, we should have actual load:

$$W_1 = 7,2.70 - 61 = 443 \text{ to}$$

while the safe load:

$$W = \frac{96.100}{14 \times 0,046.\frac{70.70}{34\frac{2}{3}}} = 468 \text{ tons, or, 70 feet would be}$$

about the limit of the 20″ work.

If we now thicken the walls to 24″, we should have

$$A = 112 \text{ square feet.}$$

P^2 from Section 4, Table I, $= 33\frac{1}{3}$ (in feet).

The weight per foot would be $112.150 = 16800$ lbs. (or 8,4 tons) additional for every foot in height of 24″ work.

Therefore the actual load would be,

$$443 + (L - 70).8,4 \text{ or}$$

$$W_1 = L.8,4 - 145.$$

Now, for $L = 80$ feet, we should have the actual load:

$W_1 = 527$ tons, while the safe load would be:

$$W = \frac{112.100}{14 + 0,046.\frac{80.80}{33\frac{1}{3}}} = 491 \text{ tons.}$$

This, though a little less than the actual load, might be passed. Rubble stone work, however, should not be built to such height, good

brickwork in cement would be better, as it can be built lighter; for $\left(\frac{c}{f}\right) = 200$, would give larger results, and brickwork weighs less, besides; then, too, we have the additional advantage of saving considerable weight on the foundations.

Thickening the walls of a tower or chimney on the inside does not strengthen them nearly so much as the same material applied to the outside would, either by offsetting the wall outside, or by building piers and buttresses.

It is mainly for this reason, and also to keep the flue uniform, that chimneys have their outside dimensions increased towards the bottom.

Example.

Calculation of chimneys. *A circular brick chimney is to be built 150 feet high, the flue entering about 6 feet from the base; the horse-power of boilers is 1980 HP. What size should the chimney be?*

The formula for size of flue is:

$$A = \frac{0{,}3.\,HP + 10}{\sqrt{L}} \tag{61}$$

Where $A =$ the area of flue, in square feet.

Where $L =$ the length of vertical flue in feet.

Where $HP =$ the total horse-power of boilers.

Size of flue. A circular flue will always give a better draught than any other form, and the nearer the flue is to the circle the better will its shape be.

In our case the flue is circular, so that we will have

$$A = \frac{22}{7} . R^2 \text{ (see Table I, Sec. No. 7) or}$$

$$R = \sqrt{\frac{7.A}{22}}$$

Inserting the value of A from formula (61) we have:

$$R = \sqrt{\frac{7}{22} . \frac{0{,}3.1980 + 10}{\sqrt{144}}} = \sqrt{\frac{7}{22} . 50{,}3} = 4$$

or the radius of flue will be 4 feet (diameter 8 feet).

Now making the walls at top of chimney 8″ thick and adopting the rule of an outside batter of about $\frac{1}{4}$″ to the foot, or say 4″ every 15 feet, we get a section as shown in Figure 87.

Let us examine the strength of the chimney at the five levels A, B, C, D and E.

The thickness of the base of each part is marked on the right-hand section, and the average thickness of the section of the part on the left-hand side.

Take the part above A; the average area is $\left(\frac{22}{7}.5^2\right.$ — flue area) or, $78 - 50 =$ 28 square feet.

This multiplied by the height of the part and the weight of one cubic foot of brickwork (112 lbs.) gives the weight of the whole, or actual load.

$$W_1 = 28.30.112 =$$

94080 lbs., or 47 tons.

The area of the base at A would be:

$$A = \left(\frac{22}{7}.5\tfrac{1}{3}^2 - \text{flue area}\right); \text{ or } A = 89 - 50 =$$

39 square feet.

The height of the part is $L = 30$.

The square of the radius of gyration, in feet, is:

$$P^2 = \frac{5\tfrac{1}{3}^2 + 4^2}{4} = 11{,}11$$

Inserting these values in Formula (60) the safe load at A would be:

$$W = \frac{39.200}{14 + 0{,}046 . \dfrac{30.30}{11{,}11}}$$

= 440 tons, or about nine times the actual load. Now, in examining the joint B we must remember to take the whole load of brickwork to the top as well as whole length L to top (or 60 feet).

The load on B we find is:

$W_1 = 131$ tons, while the safe load is·

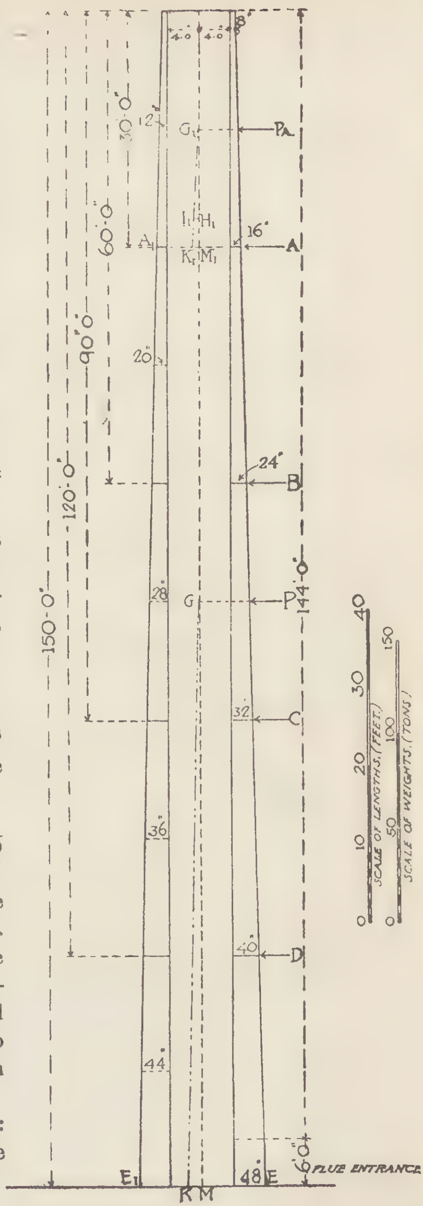


Fig. 87.

$$W = \frac{63.200}{14 + 0{,}046.\frac{60.60}{13}} = 472 \text{ tons.}$$

Similarly, we should find on C the load:

$W_1 = 259$ tons, while the safe load is:

$$W = \frac{89.200}{14 + 0{,}046.\frac{90.90}{15{,}11}} = 461 \text{ tons.}$$

On D we should find the load:

$W_1 = 432$ tons, while the safe load is:

$$W = \frac{119.200}{14 + 0{,}046.\frac{120.120}{17{,}44}} = 448 \text{ tons.}$$

Below D the wall is considerably over three feet thick, and is solid, therefore we can use $\left(\frac{c}{f}\right) = 300$, provided good Portland cement is used and best brick, which should, of course, be the case at the base of such a high chimney. We should have then the load on E:

$W_1 = 657$ tons, while the safe load is:

$$W = \frac{151.300}{14 + 0{,}046.\frac{150.150}{20}} = 690 \text{ tons.}$$

The chimney is, therefore, more than amply safe at all points, the bottom being left too strong to provide for the entrance of flue, which will, of course, weaken it considerably. We might thin the upper parts, but the bricks saved would not amount to very much and the offsets would make very ugly spots, and be bad places for water to lodge. If the chimney had been square it would have been much stronger, though it would have taken considerably more material to build it.

Tops of Chimney Flues.

It is generally best to build the flue of a chimney plumb from top to bottom, and, of course, of same area throughout. Sometimes the flue is gradually enlarged towards the top for some five to ten feet in height, which is not objectionable, and the writer has obtained good results thereby; some writers, though, claim the flue should be diminished at the top, which, however, the writer has never cared to try. Galvanized iron bands should be placed around the chimney at intervals, particularly around the top part, which is exposed very much to the disintegrating effects of the weather and the acids contained in the smoke. No smoke flue should ever be pargetted (plastered) inside, as the acids in the smoke will eat up the lime, crack the plaster, and cause it to fall. The

crevices will fill with soot and be liable to catch fire. The mortar-joints of flues should be of lime and cement, or, better yet, of fire-clay, and should be carefully struck, to avoid being eaten out by the acids.

Calculation of Walls.—Bulging. Where walls are long, without buttresses or cross-walls, such as gable-walls, side-walls of building, etc., we can take a slice of the wall, one running foot in length, and consider it as forced to yield (bulge) inwardly or outwardly, so that for ρ^2 we should use:

$\rho^2 = \frac{d^2}{12}$; where d the thickness of wall in inches. The area or a would then be, in square inches, $a = 12.d$.

Inserting these values in formula (59) we have for

BRICK OR STONE WALLS.

$$w = \frac{d.\left(\frac{c}{f}\right)}{0,0833 + 0,475.\frac{L^2}{d^2}} \qquad (62)$$

Where $w =$ the safe load, in lbs., on each running foot of wall (d'' thick).

Where $d =$ the thickness, in inches, of the wall at any point of its height.

Where $L =$ the height, in feet, from said point to top of wall.

Where $\left(\frac{c}{f}\right) =$ the *safe* resistance to crushing, in lbs., per square inch, as given in Table V. (See page 135.)

If it is preferred to use tons and feet, we insert in formula (60): for $A = D$, where D the thickness of wall, in feet, and we have:

$P^2 = \frac{D^2}{12}$; therefore

$$W = \frac{D.\left(\frac{c}{f}\right)}{14 + 0,552.\frac{L^2}{D^2}} \qquad (63)$$

Where $W =$ the *safe* load, in tons, of 2000 lbs., on each running foot of wall (D feet thick).

Where $D =$ the thickness of wall, in feet, at any point of its height.

Where $L =$ the height, in feet, from said point to the top of wall.

Where $\left(\frac{c}{f}\right) =$ the safe resistance to crushing of the material, in lbs., per square inch, as found in Table V. (See page 135.)

Anchored Walls. Where a wall is thoroughly anchored to each tier

of floor beams, so that it cannot possibly bulge, except between floor-beams, use the height of story (that is, height between anchored

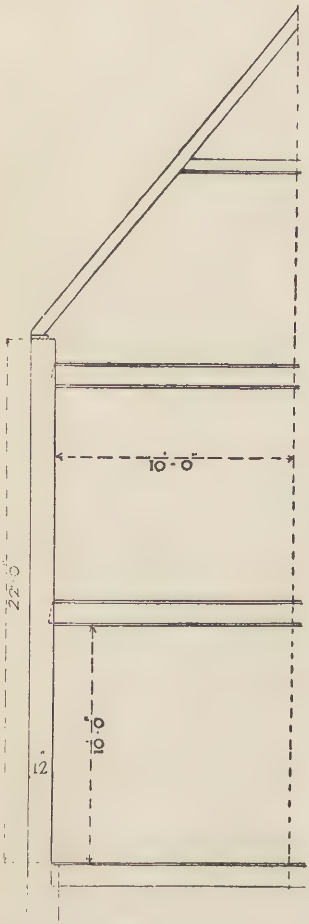


Fig. 88.

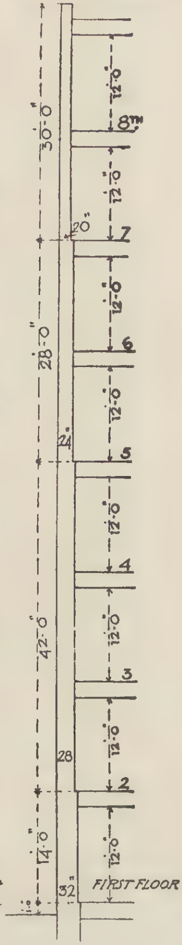


Fig. 89.

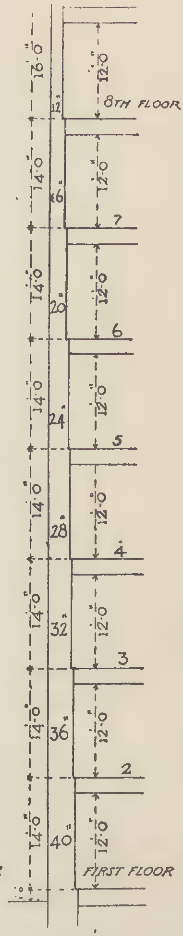


Fig. 90.

beams in feet) in place of L and calculate d or D for the bottom of wall at each story.

The load on a wall consists of the wall itself, from the point at which the thickness is being calculated to the top, plus the weight of

one foot in width by half the span of all the floors, roofs, partitions etc. Where there are openings in a wall, add to pier the proportionate weight which would come over opening; that is, if we find the load per running foot on a wall to be 20000 lbs., and the wall consists of four-foot piers and three-foot openings alternating, the piers will, of course, carry not only 20000 lbs. per running foot, but the 60000 lbs. coming over each opening additional, and as there are four feet of pier we must add to each foot $\frac{60000}{4} = 15000$ lbs.; we therefore calculate the pier part of wall to carry 35000 lbs. per running foot. The actual load on the wall must not exceed the safe load as found by the formula (62) or (63).

Example.

Wall of Country House. *A two-story-and-attic dwelling has brick walls* 12 *inches thick; the walls carry two tiers of beams of* 20 *feet span; is the wall strong enough? The brickwork is good and laid in cement mortar.*

We will calculate the thickness required at first story beam level, Figure 88.

The load is, per running foot of wall:

Wall	$= 22.112 =$	2464 lbs.
Wind	$= 22.\ 15 =$	330 lbs.
Second floor	$= 10.\ 90 =$	900 lbs.
Attic floor	$= 10.\ 70 =$	700 lbs.
Slate roof (incl. wind and snow)	$= 10.\ 50 =$	500 lbs.
Total load	$=$	4894 lbs.

For the quality of brick described we should take from Table V:

$$\left(\frac{c}{f}\right) = 200 \text{ lbs.}$$

The height between floors is 10 feet, or

$$L = 10,$$

therefore, using formula (62) we have:

$$w = \frac{d.\left(\frac{c}{f}\right)}{0{,}0833 + 0{,}475.\frac{L^2}{d^2}} = \frac{12.200}{0{,}0833 + 0{,}475.\frac{10.10}{12.12}} = 5807 \text{ lbs.}$$

So that the wall is amply strong. If the wall were pierced to the extent of one-quarter with openings, the weight per running foot would be increased to 6525 lbs. Over 700 lbs. more than the safe load, still the wall, even then, would be safe enough, as we have allowed some 330 lbs. for wind, which would rarely, if ever, be so strong; and further, some 1200 lbs. for loads on floors, also a very

ample allowance; and even if the two ever did exist together it would only run the compression $\left(\frac{c}{f}\right)$ up to 225 lbs. per inch, and for a *temporary* stress this can be safely allowed.

The writer would state here, that the only fault he finds with formulæ (59), (60), (62) and (63), is that their results are apt to give an excess of strength; still it is better to be in fault on the safe side and be sure.

Example.

Walls of City Warehouse. *The brick walls of a warehouse are* 115 *feet high, the* 8 *stories are each* 14 *feet high from floor to floor, or* 12 *feet in the clear. The load on floors per square foot, including the fire-proof construction, will average* 300 *lbs. What size should the walls be? The span of beams is* 26 *feet on an average.* (See Fig. 89, page 142.)

According to the New York Building Law, the required thicknesses would be: first story, 32″; second, third, and fourth stories, 28″; fifth and sixth stories, 24″; seventh and eighth stories, 20″.

At the seventh story level we have a load, as follows, for each running foot of wall:

Wall	$=30.1\frac{2}{3}.112$	$=$	5600
Wind	$=30.30$	$=$	900
Roof	$=13.120$	$=$	1560
Eighth floor	$=13.300$	$=$	3900
Total		$=$	11960 lbs., or 6 tons.

The safe load on a 20″ wall 12 feet high, from formula (63) is:

$$W=\frac{1\frac{2}{3}.200}{14+0{,}552.\frac{12.12}{1\frac{2}{3}.1\frac{2}{3}}}=\frac{333}{14+0{,}552.51{,}84}=7{,}811 \text{ tons, or}$$

15622 lbs.

If one-quarter of the wall were used up for openings, slots, flues, etc., the load on the balance would be 8 tons per running foot, which is still safe, according to our formula.

At the fifth-story level the load would be:

Load above seventh floor		$=$	11960
Wall	$=28.2.112$	$=$	6272
Wind	$=28.30$	$=$	840
Sixth and seventh floors	$=2.13.300$	$=$	7800
Total		$=$	26872 lbs. or $13\frac{1}{2}$ tons.

The safe load on a 24″ wall, 12 feet high, from formula (63) is:

$$W = \frac{2.200}{14 + 0{,}552.\frac{12.12}{2.2}} = \frac{400}{14 + 0{,}552.36} = 11{,}809 \text{ tons,}$$

or 23618 lbs.

This is about 10 per cent less than the load, and can be passed as safe, but if there were many flues, openings, etc., in wall, it should be thickened.

At the second-story level the load would be:

Load above fifth floor		$= 26872$
Wall	$= 42.\ 2\frac{1}{3}.112$	$= 10976$
Wind	$= 42.\ 30$	$= 1260$
Third, fourth and fifth floors	$= 3.13.300$	$= 11700$
Total		$= 50808$, or

25 tons.

The safe-load on a 28″ wall, 12 feet high, from formula (63) is:

$$W = \frac{2\frac{1}{3}.200}{14 + 0{,}552.\frac{12.12}{2\frac{1}{3}.2\frac{1}{3}}} = \frac{467}{14 + 0{,}552.26{,}45} = 16{,}33 \text{ tons,}$$

or 32660 lbs.

Or, the wall would be dangerously weak at the second-floor level.

At the first-floor level the load would be:

Load above second floor		$= 50808$
Wall	$= 14.\ 2\frac{2}{3}.112$	$= 4181$
Wind	$= 14.30$	$= 420$
Second floor	$= 13.300$	$= 3900$
Total		$= 59309$, or $29\frac{1}{2}$ tons.

The safe-load on a 32″ wall, 12 feet high, from formula (63) is:

$$W = \frac{2\frac{2}{3}.200}{14 + 0{,}552.\frac{12.12}{2\frac{2}{3}.2\frac{2}{3}}} = \frac{533}{14 + 0{,}552.20{,}25} = 21{,}169 \text{ tons,}$$

or 42338 lbs.

The wall would, therefore, be weak at this point, too.

Now while the conditions we have assumed, an eight-story warehouse with all floors heavily loaded, would be very unusual, it answers to show how impossible it is to cover every case by a law, not based on the conditions of load, etc. In reality the arrangements of walls, as required by the law, are foolish. Unnecessary weight is piled on top of the wall by making the top 20″ thick, which wall has nothing to do but to carry the roof. (If the span of beams were increased to 31 feet or more the law compels this top wall to be 24″ thick, if 41 feet, it would have to be 28″ thick, an evident waste of material.) It

would be much better to make the top walls lighter, and add to the bottom; in this case, the writer would suggest that the eighth story be 12″; the seventh story 16″; the sixth story, 20″; the fifth story, 24″; the fourth story, 28″; the third story, 32″; the second story, 36″, and the first story 40″, see Figure 90. (Page 142.)

This would represent *but* $4\frac{2}{3}$ cubic feet of additional brickwork for every running foot of wall; or, if we make the first-story wall 36″ too, as hereafter suggested, the amount of material would be exactly the same as required by the law, and yet the wall would be much better proportioned and stronger as a whole. For we should find (for L = 12 feet),

Actual load at eighth-floor level,	3832
Safe load on a 12″ wall from Formula (62)	4298
Actual load at seventh-floor level,	10243
Safe load on a 16″ wall from Formula (62)	9135
Actual load at sixth-floor level,	17176
Safe load on a 20″ wall from Formula (62)	15729
Actual load at fifth-floor level,	24632
Safe load on a 24″ wall from Formula (62)	23750
Actual load at fourth-floor level,	32611
Safe load on a 28″ wall from Formula (62)	32825
Actual load at third-floor level,	41112
Safe load on a 32″ wall from Formula (62)	42638
Actual load at second-floor level,	50136
Safe load on a 36″ wall from Formula (62)	52902
Actual load at first-floor level,	59683
Safe load on a 40″ wall from Formula (62)	63492

The first-story wall could safely be made 36″ if the brickwork is good, and there are not many flues, etc., in walls, for then we could use $\left(\frac{c}{f}\right) = 250$, which would give a safe load on a 36″ wall = 65127 lbs., or more than enough.

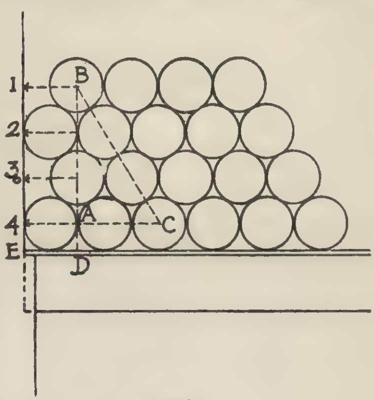


Fig. 91.

The above table shows how very closely the Formula (62) would agree with a practical and common-sense arrangement of exactly the same amount of material, as required by the law.

Thrust of barrels. Now, if the upper floor were laden with barrels, there might be some danger of these thrusting out the wall. We will suppose an extreme case, four layers of flour barrels packed against the wall, leaving a 5-foot aisle in the centre. We should have 20 barrels in each row (Fig. 91), weighing in all $20.196 = 3920$ lbs. These could not well be placed closer than 3 feet from end to end, or, say, 1307 lbs., per running foot of wall; of this amount only one-half will thrust against wall, or say, 650 pounds. The diameter of the barrel is about 20″. If Figure 92 represents three of the barrels, and we make $A\,B = w_1 = \frac{1}{2}$ the load of the flour barrels, per running foot of walls, it is evident that D B will represent the horizontal thrust on wall, per running foot. As D B is the radius, and as we know that $A\,D = 2\,D\,B$ or $=$ 2 radii, we can easily find A B, for:

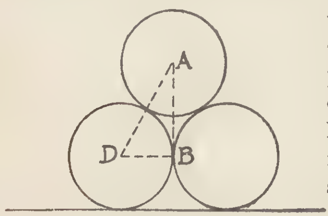


Fig. 92.

$$A\,D^2 - D\,B^2 = A\,B^2 \text{ or } 4.\,D\,B^2 - D\,B^2 = w_1^2 \text{ or}$$

$$D\,B^2 = \frac{w_1^2}{3} \text{ or } D\,B = \frac{w_1}{\sqrt{3}} = \frac{w_1}{1{,}73} = 0{,}578.w_1$$

$$\text{Or, } h = 0{,}578.\,w_1 \qquad (64)$$

Where $h =$ the horizontal thrust, in lbs., against each running foot of wall, $w_1 =$ one-half the total load, in lbs., of barrels coming on one foot of floor in width, and half the span. In our case we should have:

$$h = 0{,}578.650 = 375 \text{ lbs.}$$

Now, to find the height at which this thrust would be applied, we see, from Figure 91, that at point 1 the thrust would be from one line of barrels; at point 2, from two lines; at point 3, from three lines, etc.; therefore, the average thrust will be at the centre of gravity of the triangle A B C, this we know would be at one-third the height A B from its base A C.

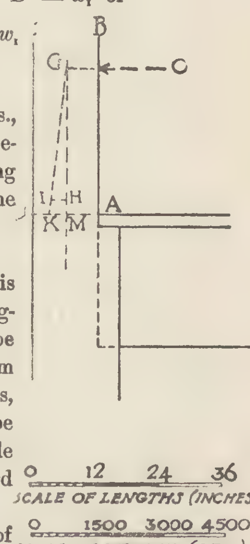


Fig. 93.

Now B C is equal to $6r$ or six radii of the barrels; further, $A\,C = 3r$, therefore:

$$A\,B^2 = 36.r^2 - 9r^2 = 25.r^2, \text{ and } A\,B = 5r; \text{ therefore, } \frac{AB}{3} = 1\tfrac{2}{3}r.$$

To this must be added the radius A D (below A) so that the central point of thrust, O in this case, would be above the beam a distance

$$y = 2\tfrac{2}{3}.r.$$

Where y = the height, in inches, above floor at which the average thrust takes place. Where r = radius of barrels in inches.

Our radius is 10″, therefore :

$$y = 26\tfrac{2}{3}''$$

Now, in Figure 93 let A B C D be the 12″ wall, A the floor level, G M the central axis of wall, and A O = $26\frac{2}{3}$; draw O G horizontally ; make G H at any scale equal to the permanent load on A D, which, in this case, would be the former load less the wind and snow allowances on wall and roof, or

$$3832 - (16.30 + 13.30) = 2962, \text{ or, say } 3000 \text{ lbs.}$$

Therefore, make G H = 3000 lbs., at any scale ; draw H I = h = 375 lbs., at same scale, and draw and prolong G I till it intersects D A at K. The pressure at K will be p = G I = 3023.

We find the distance M K measures

$$\text{M K} = x = 3\tfrac{1}{3}''.$$

Therefore, from formula (44) the stress at D will be :

$$v = \frac{3023}{144} + 6.\frac{3\frac{1}{3}.3023}{12.144} = +\ 56 \text{ lbs. (or compression)}$$

While at A the stress would be, from formula (45) :

$$v = \frac{3023}{144} - 6.\frac{3\frac{1}{3}.3023}{12.144} = -\ 14 \text{ lbs. (or tension), so that}$$

the wall would be safe.

The writer has given this example so fully because, in a recent case, where an old building fell in New York, it was claimed that the walls had been thrust outwardly by flour barrels piled against them.

Narrow Piers. Where piers between openings are narrower than they are thick, calculate them, as for isolated piers, using for d (in place of thickness of wall) the width of pier between openings ; and in place of L the height of opening. The load on the pier will consist, besides its own weight, of all walls, girders, floors, etc., coming on the wall above, from centre to centre of openings.

Wind-pressure. To calculate wind-pressure, assume it to be normal to the wall, then if A B C D, Figure 94, be the section of the whole wall above ground (there being no beams or braces against wall).

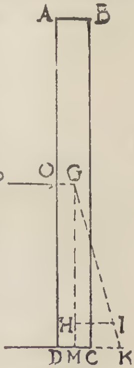


Fig. 94.

Make $O\ D = \frac{1}{2}.\ A\ D = \frac{L}{2}$; draw G H, the vertical neutral axis of the whole mass of wall, make G H, at any convenient scale, equal to the whole weight of wall; draw H I horizontally equal to the total amount of wind-pressure.

This wind-pressure on vertical surfaces is usually assumed as being equal to 30 lbs. per square foot of the surface, provided the surface is flat and normal, that is, at right angles to the wind. If the wind strikes the surface at an angle of 45° the pressure can be assumed at 15 lbs. per foot.

It will readily be seen, therefore, that the greatest danger from wind, to rectangular, or square towers, or chimneys, is when the wind strikes at right angles to the widest side, and not at right angles to the diagonal. In the latter case the exposed surface is larger, but the pressure is much smaller, and then, too, the resistance of such a structure diagonally is much greater than directly across its smaller side.

In circular structures multiply the average outside diameter by the height, to obtain the area, and assume the pressure at 15 lbs. per square foot. In the examples already given we have used 15 lbs., where the building was low, or where the allowance was made on all sides at once. Where the wall was high and supposed to be normal to wind we used 30 lbs. Referring again to Figure 94, continue by drawing G I, and prolonging it till it intersects D C, or its prolongation at K. Use formulæ (44) and (45) to calculate the actual pressure on the wall at D C, remembering that, $x = $ M K; where M the centre of D C, also that,

$$p = \text{G I; measured at same scale as G H.}$$

Remember to use and measure everything uniformly, that is, all feet and tons, or else all inches and pounds. The wind-pressure on an isolated chimney or tower is calculated similarly, except that the neutral axis is central between the walls, instead of being on the wall itself; the following example will fully illustrate this.[1]

Example.

Wind-pressure on Chimney. *Is the chimney, Figure 87, safe against wind-pressure?*

We need examine the joints A and E only, for if these are safe the intermediate ones certainly will be safe too, where the thickening of walls is so symmetrical as it is here.

The load on A we know is 47 tons, while that on E is 657 tons.

[1] Many cities require an additional vertical load for wind pressure to be added to actual weight of walls when calculating lintels, girders, foundations, etc., carrying outside walls. See "Dead Weight of Wind Pressure on Walls," p. 84.

Now the wind-pressure down to A is:

$$P_a = 10.30.15 = 4500 \text{ lbs., or } = 2\tfrac{1}{4} \text{ tons.}$$

On base joint E, the wind-pressure is:

$$P = 12\tfrac{2}{3}.150.15 = 28500 \text{ lbs., or } = 14\tfrac{1}{4} \text{ tons.}$$

We can readily see that the wind can have no appreciable effect, but continue for the sake of illustration. Draw P_a horizontally at half the height of top part A till it intersects the central axis G_1; make $G_1 H_1$ at any convenient scale $= 47$ tons, the load of the top part; draw $H_1 I_1$ horizontally, and (at same scale) $= 2\tfrac{1}{4}$ tons $=$ the wind-pressure on top part; draw $G\ I_1$; then will this represent the total pressure (from load and wind) at K_1 on joint $A\ A_1$.

Use Formula (44) to get the stress at A_1 where $x = K_1 M_1 = 9''$, or $\frac{3}{4}$ feet; and $p = G_1 I_1$, which we find scales but little over 47 tons; and Formula (45) for stress at A. For d the width of joint we have of course the diameter of base, or $10\tfrac{2}{3}$ feet. Therefore

Stress at $A_1 = \frac{47}{39} + 6.\frac{47.\frac{3}{4}}{39.10\frac{2}{3}} = +\ 1{,}71$ tons, per square foot, or $\frac{1{,}71.2000}{144} = +\ 24$ lbs. (compression), per square inch, and stress at $A = \frac{47}{39} - 6.\frac{47.\frac{3}{4}}{39.10\frac{2}{3}} = +\ 0{,}7$ tons, per square foot, or $\frac{0{,}7.2000}{144} = +\ 10$ lbs. (compression), per square inch.

To find the pressure on base $E\ E_1$; draw P G horizontally at half the whole height; make $G\ M = 657$ tons (or the whole load)[1] and draw M K horizontally, and $= 14\tfrac{1}{4}$ tons (or the whole wind-pressure). Draw G K and (if necessary) prolong till it intersects $E\ E_1$ at K. From formulæ (44) and (45) we get the stresses at E_1 and E: p being $= G\ K = 658$ tons; and $x = M\ K = 20''$, or $1\tfrac{2}{3}$ feet. For d the width of base we have the total diameter, or 16 feet. Therefore stress at $E_1 = \frac{658}{151} + 6.\frac{658.1\frac{2}{3}}{151.16} = +\ 7$ tons per sq. ft., or $\frac{7.2000}{144} = +97$ lbs. (compression), per square inch, and stress at $E = \frac{658}{151} - 6.\frac{658.1\frac{2}{3}}{151.16} = +\ 1\tfrac{2}{3}$ tons (compression), per square foot, or $\frac{1\frac{2}{3}.2000}{144} = +\ 23$ lbs. (compression) per square inch.

There is, therefore, absolutely no danger from wind.

Strength of Corbels. Corbels carrying overhanging parts of the walls, etc., should be calculated in two ways, first, to see

[1] Scale of weights, Fig. 87, applies to $G_1 H_1$, etc., but not to G M.

whether the corbel itself is strong enough. We consider the corbel as a lever, and use either Formula (25), (26) or (27); according to how the overhang is distributed on the corbel, usually it will be (25). Secondly, to avoid crushing the wall immediately under the corbel, or possible tipping of the wall. Where there is danger of the latter, long iron beams or stone-blocks must be used *on top* of the back or wall side of corbel, so as to bring the weight of more of the wall to bear on the back of corbel.

To avoid the former (crushing under corbel) find the neutral axis G H of the whole mass, above corbel, Figure 95; continue G H till it intersects A B at K, and use Formulæ (44) and (45).

If M be the centre of A B, then use $x =$ K M, and $p =$ weight of corbel and mass above; remembering to use and measure all parts alike; that is, either, all tons and feet, or all pounds and inches.

CHAPTER V.

ARCHES.

THE manner of laying arches has been described in the previous chapter, while in the first chapter was given the theory for calculating their strength; all that will be necessary, therefore, in this chapter will be a few practical examples. Before giving these, however, it will be of great assistance if we first explain the method of obtaining *graphically* the neutral axis of several surfaces, for which the arithmetical method has already been given (p. 7.).

Neutral axis found graphically.

To find the vertical neutral axis of two plane surfaces A B E F and B C D E, proceed as follows: Find the centres of gravity G_{II} of the former surface and G_{I} of the latter surface. Through these centres draw G_{II} H_{II} and G_{I} H_{I} vertically.

Draw a vertical line a c anywhere, and make ab at any scale equal to surface A B E F, and at same scale make b c = B C D E. From any point o, draw o a, ob and oc. From the point of intersection g_{I} of o c with H_{I} G_{I}, draw g_{I} g_{II} parallel o b till it intersects H_{II} G_{II} at g_{II}; from g_{II} draw g_{II} g parallel with o a till it intersects o c at g; through g draw g G vertically and this is the desired vertical neutral axis of the whole mass.

Where there are many parts the same method is used.

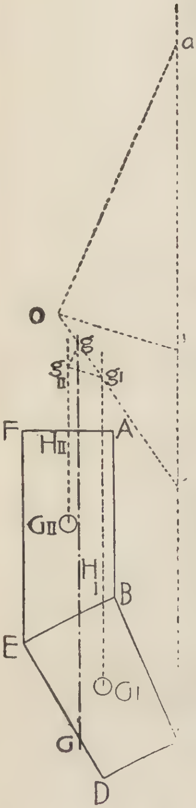


Fig. 98.

We will assume a segmental arch divided into five equal parts. Calling part A B C D = No. I; part D C E F = No. II, etc., and the vertical neutral axis of each part, No. I, No. II. etc.

Draw $a\ f$ vertically and at any convenient scale, make

$a\ b =$ No. I or A B C D

$b\ c =$ No. II or D C E F,

$c\ d =$ No. III

$d\ e =$ No. IV and

$e\ f =$ No. V

From any point o draw oa, ob, oc, od, oe and of.

From point of intersection 1 of vertical axis No. I and oa, draw 1 g parallel with $o\ b$ till it intersects vertical axis No. II at g; then draw $g\ h$ parallel with $o\ c$ till it intersects vertical axis No. III at h; similarly draw $h\ i$ parallel with $o\ d$ to axis No. IV and $i\ j$ parallel with $o\ e$ to axis No. V, and, finally, draw j 5 parallel with $o\ f$ till it intersects $a\ o$ (or its prolongation) at 5. A vertical axis through 5 is the vertical neutral axis of the whole mass. Prolong $j\ i$ till it intersects $o\ a$ at 4, $i\ h$ till it intersects $o\ a$ at 3, and $h\ g$ till it intersects $o\ a$ at 2.

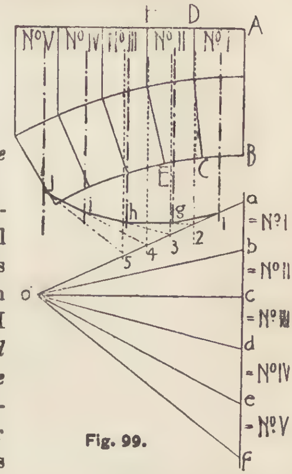


Fig. 99.

A vertical axis through 4 will then be the vertical neutral axis of the mass of Nos. I, II, III and IV; a vertical axis through 3 will be the vertical neutral axis of the mass of Nos. I, II and III; and through 2 the vertical neutral axis of the mass of Nos. I and II. Of course the axis through 1 is the vertical neutral axis of No. I.

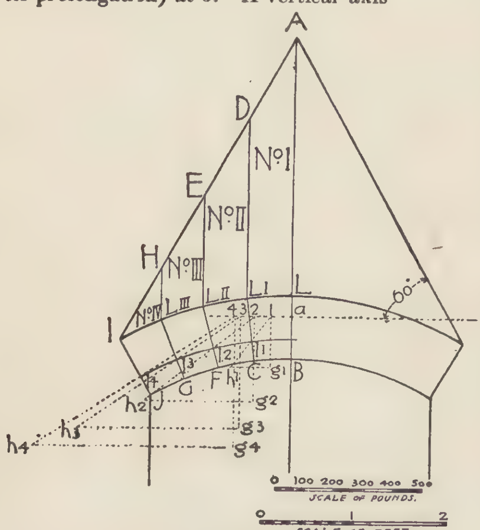


Fig 100.

We will now take a few practical examples.

Example I.

In a solid brick wall an opening 3 feet wide is bricked over with an 8-inch arch. Is this strong enough?[1]

The thickness of the wall or arch, of course, does not matter, *where the wall is solid*, and we need only assume the wall and arch to be one foot thick. If the wall were thicker, the arch would be correspondingly thicker and stronger, so that in all cases where a

Arch in solid wall.

load is evenly distributed over an arch we will consider both always as one foot thick. If a wall is hollow, or there are uneven loads, we can either take the full actual thickness of the arch, or we can proportion to one foot of thickness of the arch its proportionate share of the load.

In our example we assume everything as one foot thick. The load coming on the half-arch B J I L Fig. 100, will be enclosed by the

Voussoirs arbitrary.

lines A L and I A at 60° with the horizon. We divide the arch into, say, four equal voussoirs B C = C F = F G = G J. (The manner of dividing might, of course, have been arbitrary as well as equal, had we preferred.) Draw the radiating lines through C, F and G, and from their upper points draw the vertical lines to D, E and H.

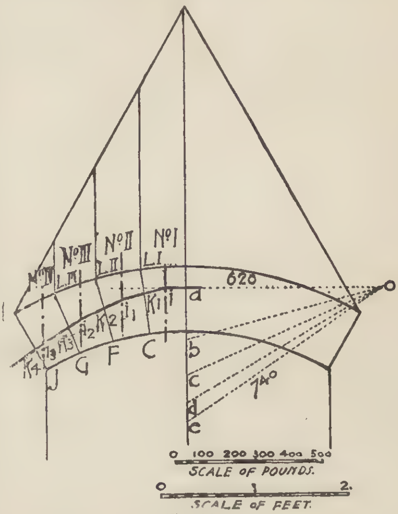


Fig. 101.

Now find the weight of each slice, *remembering* always to include the weight of the voussoir in each slice. We have, then, approximately,

No. I	(A B C D) =	$3' \times \frac{1}{2}' \times 112$ pounds	= 168	pounds
No. II	(D C F E) =	$2\frac{1}{8}' \times \frac{1}{2}' \times 112$ "	= 119	"
No. III	(E F G H) =	$1\frac{1}{2}' \times \frac{7}{16}' \times 112$ "	= 73	"
No. IV	(H G J I) =	$\frac{7}{8}' \times \frac{7}{16}' \times 112$ "	= 43	"
		Total	= 403	"

[1] For convenience, most of the figures are duplicated: the first, showing manner of obtaining the horizontal thrust; the second, the manner of obtaining the line of pressure.

As the arch is evidently heavily loaded at the centre, we assume the point a at one-third the height of B L from the top, or $L\ a = \frac{LB}{3} = 2\frac{2}{3}''$ and draw the horizontal line $a\ 4$.

As previously explained, find the neutral axes:

Horizontal pressures.

1 g_1 of part No. I,
2 g_2 of parts No. I plus No. II,
3 g_3 of parts Nos. I, II and III, and
4 g_4 of the whole half arch.

Now make at any scale $1\ g_1 = 168$ units = No. I; similarly at same scale $2\ g_2 = 168 + 119$ units $= 287$ = No I and No. II, and

$3\ g_3 = 287 + 73$ units $= 360$ = No. I + No. II + No. III.

$4\ g_4 = 360 + 43$ units $= 403$ = weight of half arch and its load.

Now make: $C\ l_1 = \frac{1}{3}\ C\ L_1 = 2\frac{2}{3}''$.

Similarly, $F\ l_2 = \frac{1}{3}\ F\ L_2 = 2\frac{2}{3}''$; also, $G\ l_3 = \frac{1}{3}\ G\ L_3 = 2\frac{2}{3}''$,

And, $J\ l_4 = \frac{1}{3}\ J\ I = 2\frac{2}{3}''$.

Through g_1, g_2, g_3 and g_4 draw horizontal lines, and draw the lines $1\ l_1$, $2\ l_2$, $3\ l_3$ and $4\ l_4$ till they intersect the horizontal lines at h_1, h_2, h_3 and h_4; then will $g_1\ h_1$ measured at same scale as $1\ g_1$ represent the horizontal thrust of A B C D; $g_2\ h_2$ the horizontal thrust of A B F E; $g_3\ h_3$ the horizontal thrust of A B G H and $g_4\ h_4$ the horizontal thrust of the half arch and its load. In this case it happens that the latter is the greatest, so that we select it as our horizontal pressure, and make (in Fig. 101) $a\ o = g_4\ h_4 = 620$ pounds, at any convenient scale.

Now (in Fig. 101) make $a\ b = 168$ pounds = No. I.

$b\ c = 119$ " = No. II.

$c\ d = 73$ " = No. III.

$d\ e = 43$ " = No. IV.

Draw *ob*, *oc*, *od* and *oe*.

Curve of pressure. Now begin at a, draw $a\ 1$ parallel with $o\ a$ till it intersects axis No. I at 1; from 1 draw $1\ i_1$, parallel with $o\ b$ till it intersects axis No. II at i_1; from i_1 draw $i_1\ i_2$ parallel with $o\ c$ till it intersects axis No. III at i_2; from i_2 draw $i_2\ i_3$ parallel with $o\ d$ till it intersects axis No. IV at i_3; from i_3 draw $i_3\ K_4$ parallel with $o\ e$. A curve through the points a, K_1, K_2, K_3 and K_4 (where the former lines intersect the voussoir joint lines) and tangent to the line $a\ 1\ i_1\ i_2\ i_3\ K_4$ would be the real curve of pressure. The amount of pressure on joint $C\ L_1$ would be concentrated at K_1 and would be equal to $o\ b$ (measured at same scale as $a\ b$, etc.). The pressure on joint $F\ L_2$ would be concentrated at K_2 and be equal to $o\ c$. The pressure on

joint G L_3 would be concentrated at K_3 and equal to *o d*. The pressure on the skew-back joint J I would be concentrated at K_4 and be equal to *o e*. The latter joint evidently suffers the most, for not only has it got the greatest pressure to bear, but the curve of pressure is farther from the centre than at any other joint. We need calculate this joint only, therefore, for if it is safe, the others certainly are so, too. By scale we find that J K_4 measures $2\frac{1}{2}''$, or K_4 is $(x =)\ 1\frac{1}{2}''$ from the centre of joint; we find further that *o e* scales 740 units, therefore $(p =)\ o\ e =$ 740 pounds.

Stress at extrados and intrados.

The width of joint is, of course, 8″, and the area = 8 x 12 = 96 square inches; therefore, from Formula (44)

$$\text{Stress at edge J} = \frac{740}{96} + 6.\frac{740.\ 1\frac{1}{2}}{96.\ 8} = +16{,}26$$

and from (45)

$$\text{Stress at edge I} = \frac{740}{96} - 6.\frac{740.\ 1\frac{1}{2}}{96.\ 8} = -0{,}85$$

Or the edge J would be subject to the slight compression of $16\frac{1}{4}$ pounds, and edge at I to a tension of a little less than one pound per square inch. The arch, therefore, is more than safe.

Example II.

A four-inch rowlock brick arch is built between two iron beams, of five feet span, the radius of arch being five feet. The arch is loaded at the rate of 150 *pounds per square foot. Is it safe?*

In this case we will divide the top of arch A D into five equal sections and assume that each section carries 75 pounds — (which, of course, is not quite correct). We find the horizontal pressures (Fig. 102) $g_1 h_1$, $g_2 h_2$, etc., as before, and find

Brick floor-arch.

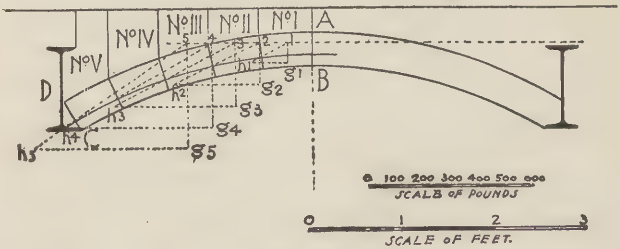


Fig. 102.

again $g_5\ h_5$ the largest and equal to 575 pounds. We now make (Fig. 103) at any convenient scale, $o\ a = g_5\ h_5 = 575$ pounds, and $a\ b =$

$b\,c = c\,d = d\,e = e\,f =$ 75 pounds and draw $o\,a$, $o\,b$, $o\,c$, etc. We now find the broken line $a\ 1\ i_1\ i_2\ i_3\ i_4\ K_5$ where:

$a\ 1$ is parallel with $o\,a$		$1\ i_1$ is parallel with $o\,b$	
$i_1\,i_2$ "	" $o\,c$	$i_2\,i_3$ "	" $o\,d$
$i_3\,i_4$ "	" $o\,e$	$i_4 K_5$ "	" $o\,f$

In this case again evidently the greatest stress is on the skew-back joint C D, for it not only has the greatest pressure $o\,f$, but the curve of

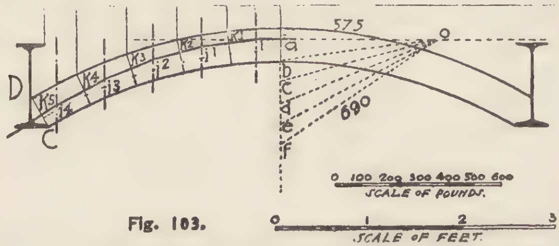


Fig. 103.

pressure passes farther from the centre than at any other joint. We find that C K_5 scales $1\frac{1}{4}$ inches, therefore the distance of K_5 from the centre is $(x =)\ \frac{3}{4}''$. We scale $o\,f$ and find it scales 690 units, or $(p) =$ 690 pounds. The joint is 4″ wide and its area $= 4 \times 12 = 48$ square inches.

From Formulæ (44) and (45) we have then:

$$\text{Stress at C} = \frac{690}{48} + 6.\frac{690.\ \frac{3}{4}}{48.\ 4} = +\ 30{,}6 \text{ pounds and}$$

$$\text{Stress at D} = \frac{690}{48} - 6.\frac{690.\ \frac{3}{4}}{48.\ 4} = -\ 1{,}8 \text{ pounds.}$$

The arch, therefore, is perfectly safe.

Example III.

Two iron beams, five feet apart, same as before, but filled with a straight 7″ hollow fire-clay arch. The load per foot to be assumed at 140 pounds. Is the arch safe?

Fireproof floor-arch. Of the 350 pounds on the half arch we will assume 80 pounds to come on each of the blocks and 30

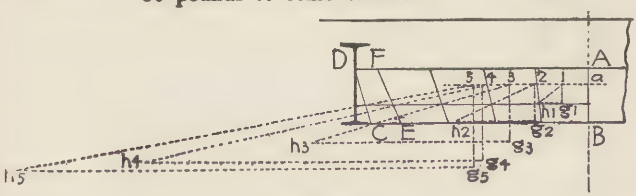


Fig. 104.

pounds on the skew-back. We then (in Fig. 104) find, as before the horizontal pressures, $g_1 h_1$, $g_2 h_2$, etc. Again we find the largest pressure to be $g_5 h_5$, and as it scales 2040 units, we make (in Fig. 105) at any convenient scale and place $o\ a = g_5 h_5 = 2040$ pounds. We also make $a\ b = b\ c = c\ d = d\ e = 80$ units and $e\ f = 30$ units.

Draw oa, ob, oc, etc. Drawing the lines parallel thereto, beginning at a we get the line $a\ 1\ i_1\ i_2\ i_3\ i_4\ K_5$, same as before Imagining a joint

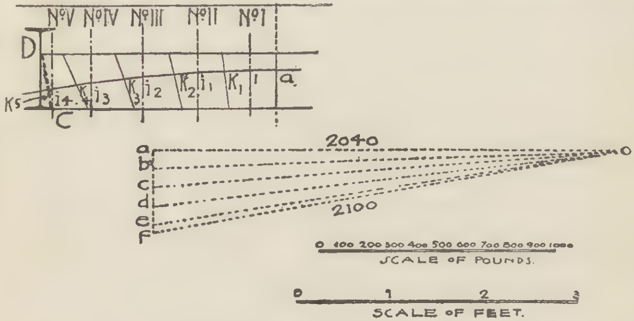


Fig. 105.

at C D this would evidently be the joint with greatest stress, for the same reasons mentioned before. We find C K_5 scales $2\frac{5}{8}''$, and as C D scales $7\frac{1}{4}''$ the point K_5 is distant from the centre of joint.

$$(x =)\ 3\tfrac{5}{8} - 2\tfrac{5}{8} = 1''$$

as $o\ f$ scales 2100 units or pounds, and the joint is $7\frac{1}{4}''$ deep with area $= 7\frac{1}{4}.\ 12 = 87$ square inches, we have:

$$\text{Stress at C} = \frac{2100}{87} + 6.\ \frac{2100.\ 1}{87.\ 7\frac{1}{4}} = +\ 44{,}14 \text{ pounds and}$$

$$\text{Stress at D} = \frac{2100}{87} - 6.\ \frac{2100.\ 1}{87.\ 7\frac{1}{4}} = +\ 4{,}14 \text{ pounds.}$$

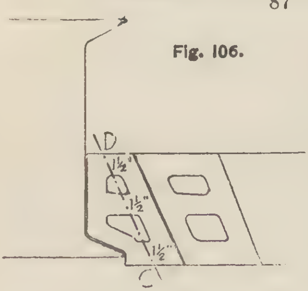


Fig. 106.

The arch, therefore, would seem perfectly safe. But the blocks are not solid; let us assume a section through the skew-back joint C D to be as per Fig. 106. We should have in Formulæ (44) and (45) x, p, and the depth of joint same as before, but for the area we should use $a = 3.1\frac{1}{2}.12 = 54$ square inches, or only the area of solid parts of block. Therefore we should have:

$$\text{Stress at C} = \frac{2100}{54} + 6. \frac{2100.\ 1}{54.\ 7\frac{1}{4}} = +\ 71 \text{ pounds, and}$$

$$\text{Stress at D} = \frac{2100}{54} - 6. \frac{2100.\ 1}{54.\ 7\frac{1}{4}} = +\ 6{,}71 \text{ pounds.}$$

There need, therefore, be no doubt about the safety of the arch.

Example IV.

Over a 20-inch brick arch of 8 feet clear span is a centre pier 16′ wide, carrying some two tons weight. On each side of pier is a window opening 2½ feet wide, and beyond, piers similarly loaded. Is the arch safe?

Arch in front wall concentrated loads. We divide the half arch into five equal voussoirs. The amounts and neutral axes of the different voussoirs, and loads coming over each, are indicated in circles and by arrows; thus, on the top voussoir E B (Fig. 107) we have a load of 2100 pounds, another of 62 pounds, and the weight of voussoir or 228 pounds. The neutral axis of the three is the vertical through G_I (Fig. 108). Again on voussoir E F (Fig. 107) we

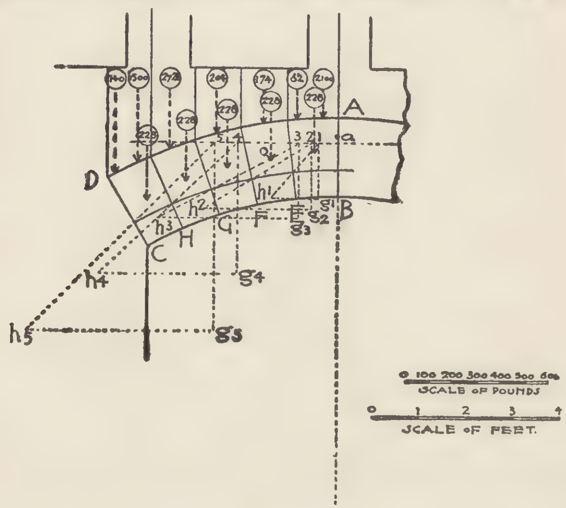


Fig. 107.

have the load 174 pounds, and weight of voussoir 228 pounds; the vertical neutral axis of the two being through G_{II} (Fig. 108). Similarly we get the neutral axes G_{III}, G_{IV} and G_V (Fig. 108) for each of the other voussoirs. Now remembering that 1 g_I (Fig. 107) is the

neutral axis of and equal to the voussoir B E and its load; 2 g_2 the neutral axis of and equal to the sum of the voussoirs B E and E F and their loads, etc., we find the horizontal thrusts $g_1\,h_1$, $g_2\,h_2$, $g_3\,h_3$, etc. The last $g_5\,h_5$ is again the largest, and we find it scales 7850 units or pounds.

The arch being heavily loaded we selected a at one-third from the top of A B. We now make (Fig. 108) $a\,o = 7850$ pounds or units at any scale; and at same scale make $a\,b = 2390$ pounds; $b\,c = 402$ pounds; $c\,d = 432$ pounds; $d\,e = 2956$ pounds, and $e\,f = 1868$ pounds. Draw $o\,b$, $o\,c$, $o\,d$, etc. Now draw as before a 1 parallel with $o\,a$ to axis G_{I}; also 1 i_1 parallel with $o\,b$ to axis G_{II}; $i_1\,i_2$ parallel with $o\,c$ to G_{III}, etc. We then again have the points a, K_1, K_2, K_3, K_4

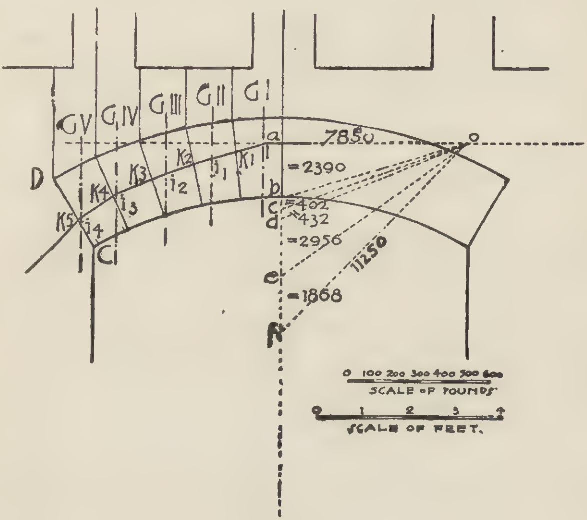


Fig. 108.

and K_5 of the curve of pressure. As K_5 is the point farthest from the centre of arch-ring and at the same time sustains the greatest pressure ($o\,f$) we need examine but the joint C D; for if this is safe so are the others. We insert, then, in Formulæ (44) and (45) for

$p = o\,f = 11250$ pounds, and as K_5 C measures $6\frac{1}{2}''$,

$x = 10'' - 6\frac{1}{2}'' = 3\frac{1}{2}''$; also as the joint is 20″ wide,

$a = 12.\ 20 = 240$ square inches.

Therefore

$$\text{Stress at C} = \frac{11250}{240} + 6.\,\frac{11250.\,3\frac{1}{2}}{240.\,20} = +\ 96 \text{ pounds, and}$$

$$\text{Stress at D} = \frac{11250}{240} - 6.\,\frac{11250.\,3\frac{1}{2}}{240.\,20} = -\ 3 \text{ pounds.}$$

The arch is, therefore, safe.

Example V.

A 12-inch brick semi-circular arch has 12 foot span. A solid brick wall is built over the arch to a level with one foot above the keystone. The abutment piers are 5 feet high to the spring of arch and are each 3 feet wide, including, of course, the width of skew-backs. Are the arch and piers safe?

Arch in fence-wall with abutment.

As before, we will assume arch, pier, and wall over arch, each one foot thick. We will divide the load over arch into seven equally wide slices. This will make uneven voussoirs, but this does not matter, as our joint lines (and voussoirs) are only imaginary anyhow, and not necessarily of the shape of the actual voussoirs, which in brick would, of course, be represented by each single brick. The amount of the sums of each voussoir and its load, and the vertical neutral axes of the different sets are given by the arrows and lines G_1, G_2, G_3, etc. (in Fig. 110). When considering the safety of the abutment we treat it exactly the same as the voussoirs (and loads) of the arch; that is, we

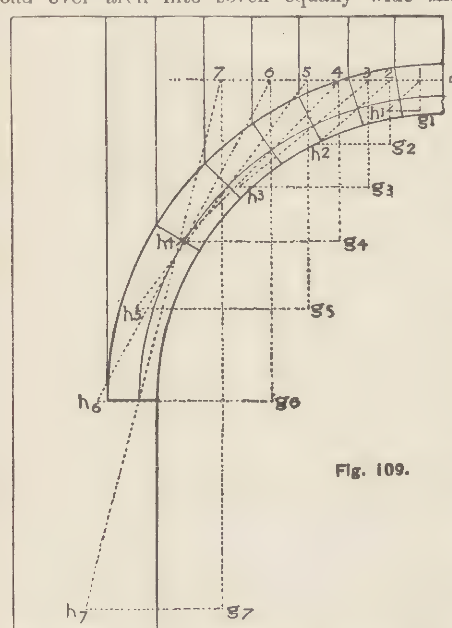


Fig. 109.

take the whole weight of the abutment, viz., C D E F I H C and find its neutral axis G_8.

Returning now to the arch, we go through the same process as before. We find the horizontal pressures (Fig. 109) $g_1 h_1$, $g_2 h_2$, etc. In this case we find that the last pressure $g_7 h_7$ is not as large as $g_6 h_6$; therefore we adopt the latter; it scales 1425 units or pounds. We now make (Fig. 110) $a\ o = 1425$ pounds; and $a\ b = 251$ pounds; $b\ c = 280$ pounds; $c\ d = 373$ pounds, etc.; $g\ h$ is equal to the last section of arch or 1782 pounds. We continue, however, and make $h\ i = 4600$ pounds $=$ the weight of abutment. Draw $o\ a$, $o\ b$, $o\ c$, etc., to $o\ i$. Then get the tangents to the curve of pressure, as before, viz.: $a\ 1\ i_1\ i_2\ i_3\ i_4\ i_5\ i_6\ K_7$; we now continue $i_6\ K_7$, which is parallel with $o\ h$ till it intersects the vertical axis G_8 of the abutment at i_7 and from thence we draw $i_7\ K_8$ parallel with $o\ i$.

Thrust on abutment. We will now examine the base joint I H of pier. I K_8 scales $10\frac{1}{4}''$, and as the pier is $36''$ wide, K_8 is $7\frac{3}{4}''$ from the centre of joint. The area is $a = 12.36 = 432$ and the pressure is $p = o\ i = 9100$ pounds.

Therefore,

$$\text{Stress at I} = \frac{9100}{432} + 6.\frac{9100.7\frac{3}{4}}{432.36} = +48 \text{ pounds,}$$

and

$$\text{Stress at H} = \frac{9100}{432} - 6.\frac{9100.7\frac{3}{4}}{432.36} = -6 \text{ pounds.}$$

There is, therefore, a slight tendency for the pier to revolve around the point I, raising itself at H; still the tendency is so small, only 6 pounds per square inch, that we can safely pass the pier, so far as danger from thrust is concerned.

Joint C D at the spring of the arch looks rather dangerous, however, as $i_6\ i_7$ cuts it so near its edge D. Let us examine it. D K_7 measures $1\frac{1}{2}''$, therefore K_7 is $4\frac{1}{2}''$ from the centre of joint, which is $12''$ wide. The area is, of course, $a = 12.\ 12 = 144$ and the pressure $p = o\ h = 4600$ pounds. Therefore,

$$\text{Stress at D} = \frac{4600}{144} + 6.\frac{4600.4\frac{1}{2}}{144.12} = +104 \text{ pounds,}$$

and

$$\text{Stress at C} = \frac{4600}{144} - 6.\frac{4600.4\frac{1}{2}}{144.12} = -40 \text{ pounds.}$$

It is evident, therefore, that the arch itself is not safe, and it should be designed deeper; that is, the joints should be made deeper (say, $16''$), and a new calculation made.

Tie-rods to arches. If, instead of an abutment-pier, we had used an iron tie-rod, its sectional area would have to be sufficient to resist a tension equal to the greatest horizontal thrust *o a*;

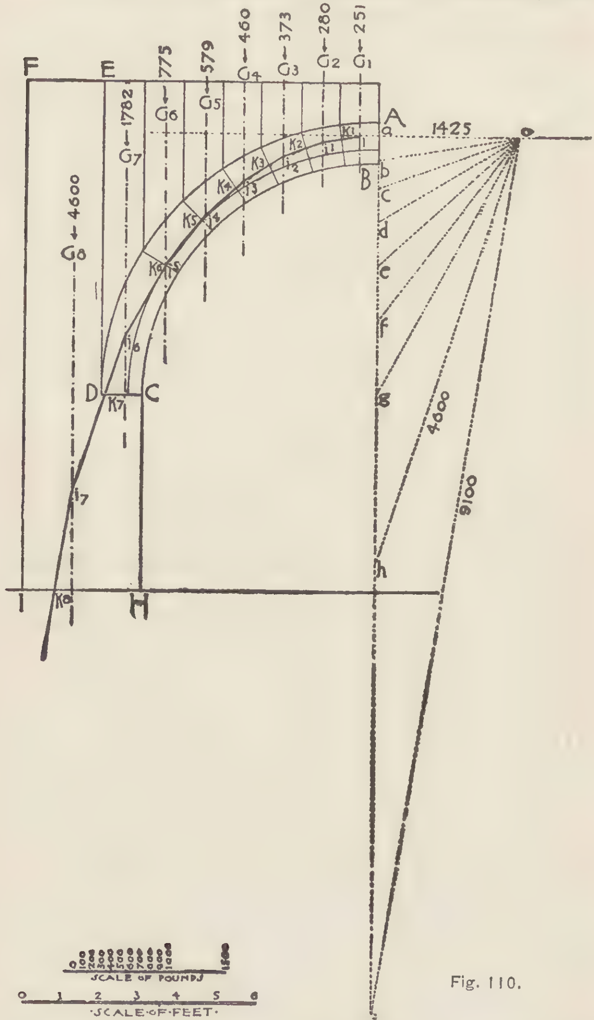


Fig. 110.

and care should be taken to proportion the washers at each end, large

enough that they may have sufficient bearing-surface so as not to crush the material of the skew-backs.

Thus, in Example III, Fig. 105, if we should place the iron tie-rods to the beams 5 feet apart, they would resist a tension equal to five times the horizontal thrust $o\ a$, which, of course, was calculated for 1 foot only, or

$$t = 5.\ 2040 = 10200 \text{ pounds.}$$

The safe resistance of wrought-iron to tension is from Table IV, 12000 pounds per square inch; we need, therefore:

$$\frac{10200}{12000} = 0{,}85 \text{ square inches}$$

of area in the rod; or the rod should be $1\frac{1}{16}''$ diameter. A 1″ or even $\frac{7}{8}''$ rod would probably be strong enough, however, as such small iron is apt to be better welded, and, consequently, stronger, and the load on the arch would probably be a "dead" one.

As the end of rod will bear directly against the iron beam, the washers need have but about $\frac{1}{4}''$ bearing all around the end of the rod, so that the nut would probably be large enough, and no washer be needed.

Example VI.

A pier 28″ *wide and* 10′ *high supports two abutting semi-circular arches; the right one a* 20″ *brick arch of* 8′ *span; the left one an* 8″ *brick arch of* 3′ *span. The loads on the arches are indicated in the Figure* **111.** *Is the pier safe?*

Uneven arches with central pier. The loads are so heavy compared to the weight of the voussoirs, that we will neglect the latter, in this case, and consider the vertical neutral axis of and the amount of each load as covering the voussoirs also; except in the case of the lower two voussoirs, where the axes are considerably affected. We find the curve of pressure of each arch as before.

For the large arch we would have the curve through a and i, for the small one through a_1 and i_1; the points i and i_1 being the intersections of the curves with the last vertical neutral axis of each arch.

Now from i draw $i\ x$ parallel with $o\ f$, and from i_1 draw $i_1\ x$ (backwards), but parallel with $o_1\ f_1$ till the two lines intersect at x. Now make $f\ g$ parallel with and $= o_1\ f_1$, and draw $g\ h$ vertically $= 2600$ pounds $=$ the weight of the pier from the springing line to the base (1′ thick). Draw $o\ g$ and $o\ h$. Now returning to x, draw $x\ y$ parallel with $g\ o$ till it intersects the neutral axis of the pier at y, and from y draw $y\ z$ parallel with $o\ h$ till it intersects the base joint C D of

pier (or its prolongation) at K_1. Continue also $x\ y$ till it intersects the springing joint A B at K. Now, then, to get the stresses at

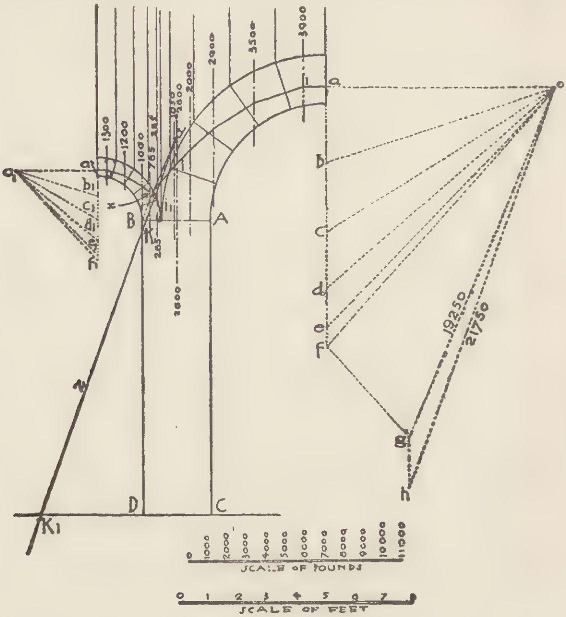


Fig. 111.

joint A B we know that the width of joint is 28″, therefore $a = 28.12 = 336$; further $p = o\ g = 19250$ pounds, and as $B\ K$ scales one inch, K is distant 13″ from the centre of joint, therefore

$$\text{Stress at B} = \frac{19250}{336} + 6.\frac{19250.\ 13}{336.\ 28} = +\ 217 \text{ pounds.}$$

and

$$\text{Stress at A} = \frac{19250}{336} - 6.\frac{19250.\ 13}{336.\ 28} = -\ 102 \text{ pounds.}$$

Thrust on central pier. The arch, therefore, cannot safely carry such heavy loads. The pier we shall naturally expect to find still more unsafe, and in effect have, remembering that joint D C is 28″ wide, therefore, area 336 square inches, and as K_1 distant 54″ from centre of joint, and $p = o\ h = 21750$ pounds.

$$\text{Stress at D} = \frac{21750}{336} + 6.\frac{21750.\ 54}{336.\ 28} = +\ 813 \text{ pounds.}$$

and

$$\text{Stress at C} = \frac{21750}{336} - 6.\frac{21750.\ 54}{336.\ 28} = -684 \text{ pounds.}$$

Relief through iron-work. The construction, therefore, must be radically changed, if the loads cannot be altered. If the arches are needed as ornamental features, they should be constructed to carry their own weight only, and iron-work overhead should carry the loads, and bear either nearer to, or directly over, the piers, as farther trials and calculations might call for. If this is done the wall should be left hollow under all but the ends of iron-work until it gets its "set"; that is, until it has taken its full load and deflection; and then the wall should be pointed with soft "putty" mortar.

To avoid cracks.

Example VII.

The foundations of a building rest on brick piers 6′ wide and 18′ apart. The piers are joined by 32″ brick inverted arches and tied together 8′ above the spring of the arch. Piers and arches are 3′ thick. Load on central piers is 72 tons; on end pier 60 tons. Is this construction safe?

Inverted arches. We will first examine the inner or left pier. The pier being 3′ thick and the load 72 tons, each 1′ of thickness will, of course, carry $\frac{72}{3} = 24$ tons. The width from centre to centre of piers is just 24′, so that each running foot of wall under

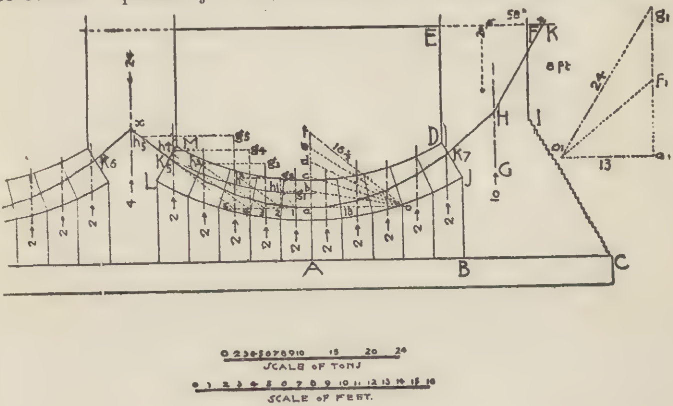


Fig. 112.

arch will receive a pressure of one ton. Now all we need to do is to imagine this pressure as the load on the arch. We can either draw

the arch upside down with a load of one ton per foot, or we can make the drawing with the arch in correct position and the weights pressing upward, as shown in Fig. 112. We divide the load into five equal

Strength of arch.

slices, each about 2′ wide, therefore = 2 tons each. We make $a\ b = 2$ tons; $b\ c = 2$ tons, etc., and find the horizontal pressures $g_1\ h_1$, $g_2\ h_2$, etc., same as before. Again, $g_5\ h_5$ is the largest, measuring 13 units or tons. Now make $o\ a = 13$ tons, and draw $o\ b$, $o\ c$, etc. Construct the line $a\ 1\ K_5$ or curve of pressure same as before. Joint L M is evidently the most strained one. We find M K_5 measures 11″, and as the arch is 32″ (= M L), of course, K_5 is 5″ from the centre of joint. The area of joint is $a = 32.\ 12 = 384$. We scale $o\ f$, the pressure at K_5, and find that it measures $16\frac{1}{2}$ units, or 33000 pounds, therefore

$$\text{Stress at M} = \frac{33000}{384} + 6.\frac{33000.\ 5}{384.\ 32} = +\ 168 \text{ pounds}$$

$$\text{Stress at L} = \frac{33000}{384} - 6.\frac{33000.\ 5}{384.\ 32} = +\ 6 \text{ pounds.}$$

Central pier.

The arch, therefore, is perfectly safe. Of course, the left pier is safe, for being an inner pier the resistance $K_6\ x$ of the adjoining arch to the left will just counter-balance the thrust of our arch, or $K_5\ x$. But at the end (right) pier this is different. We, of course, proportion the length of foundation A C, to get same pressure as on rest of wall under arch. The end pier carries 60 tons, or $\frac{60}{3} = 20$ tons per foot thick. The pressure per running foot on wall under arch we found to be one ton, therefore A C should

Thrust on end pier.

be 20 feet long. The half-arch will take the pressure of 10 feet (from A to B) or 10 tons, and the balance (10 tons) will come on B C. This will act through its central axis G H, which, at the arch skew-back, will be half-way between the end of arch J and outside pier-line I F. This will, of course, deflect somewhat the abutment (or last pressure) line K_7 H of the arch. At any convenient place draw $a_1\ f_1$ vertically equal 10 tons, and $a_1\ o_1$ horizontally equal 13 tons, the already known horizontal pressure. Then K_7 H is, of course, parallel with and equal $o_1\ f_1$. Now make $f_1\ g_1 =$ 10 tons and draw $o_1\ g_1$. Now on the pier draw H K parallel with $o_1\ g_1$; H being the point of intersection of G H and K_7 H. As the pier is tied back 8′ above the arch, we take our joint-line at E F, being 8′ above D. We find, by scale, that K is 58″ from the centre of E F, the latter being 72″ wide. The area is $a = 72.\ 12 = 864$. And as $o_1\ g_1$ scales 24 units, the pressure is, of course, 24 tons or 48000 pounds,

therefore:

$$\text{Stress at F} = \frac{48000}{864} + 6. \frac{48000.\ 58}{864.\ 72} = + 315 \text{ pounds.}$$

and

$$\text{Stress at E} = \frac{48000}{864} - 6. \frac{48000.\ 58}{864.\ 72} = - 207 \text{ pounds.}$$

Use of buttress. There is, therefore, no doubt of the insecurity of the end pier. Two courses for safety are now open. Either we can build a buttress sufficiently heavy to resist the thrust of the end arch, or we can tie the pier back. The former case is easily calculated; we simply include the mass of the buttress in the resistance and shift the axis G H to the centre of gravity of the area of pier and buttress up to joint E F. Of course, the buttress should be carried up to the joint-line E F, but it can taper away from there.

Use of tie-rods. If we tie back with iron we need sufficient area to resist a tension equal to the horizontal pressure $a\ o$, which in this case is 13 tons or 26000 pounds per foot thick of wall. As the wall is 3′ thick, the total horizontal thrust is $3.\ 26000 = 78000$ pounds. The tensional stress of wrought-iron being 12000 pounds per square inch, we need $\frac{78000}{12000} = 6\frac{1}{2}$ square inches area, or, say, two wrought-iron straps, 4″ x $\frac{7}{8}$″, one each side of pier. By this method the inner or left arch becomes practically the end arch. For the last two piers and the right arch become one solid mass; and not only is their entire weight thrown against the second or inner arch, but the centre of gravity of the whole mass shifts to the centre line of end arch, or in our case 9′ *inside* of the end pier; so that there is no possible doubt of the strength of the abutment. There is one element of weakness, however, in the small bearing the pier has on the skew-back of the arch, the danger being of the pier cracking upwards and settling past and under the arch. This can be avoided, as already explained, by building a large bond-stone across the entire pier, forming skew-backs for the arch to bear against.

Example VIII.

A semi-circular dome, circular in plan, is 40′ inside diameter. The shell is 5′ thick at the spring and 2′ at the crown. The dome is of cut-stone. Will it stand?

Calculation of dome. We draw a section (Fig. 114) of the half-dome and treat it exactly the same as any other arch. The only difference is in the assumption of the weights. Instead of assuming the arch 1′ thick, we take with each voussoir its entire weight around

one-half of the dome. Thus, in our case, we divide the section into six imaginary voussoirs. The weight on each voussoir will act through its centre of gravity. Now weight No. I will be equal to the area of the top voussoir multiplied by the circumference of a semi-circle with A B as radius. Similarly, No. II will be equal to the area of the second voussoir multiplied by the circumference of a semi-circle with A C as radius; No. III will be equal to the area of voussoir 3, multiplied by the circumference of a semi-circle with A D as radius, etc. The vertical neutral axes Nos. I, II, III, etc., act, of course, through the centres of gravity of their respective voussoirs.

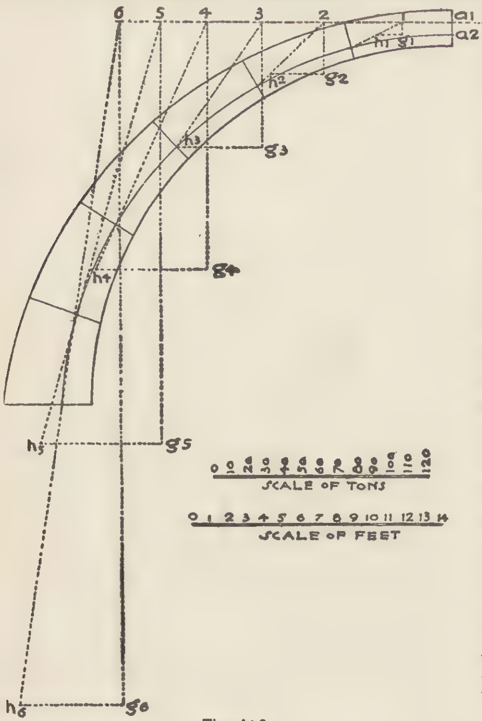


Fig. 113.

Now the top voussoir measures 5′ 6″ by an average thickness of 2′ 1″ or $5\frac{1}{2}.\ 2\frac{1}{12} = 11{,}46$ or, say, $11\frac{1}{2}$ square feet. As A B (the radius) measures 2′ 9″, the circumference of its semi-circle would be 8′,6. Taking the weight of the stone at 160 pounds per cubic foot, we should then have the weight of No. I = 11,5. 8, 6. 160 = 15824 pounds, or, say, 8 tons.

Similarly we should have:

No. II = 12,4. 25,1. 160 =	49798	pounds, or, say,	25 tons.
No. III = 14,2. 40. 160 =	90880	" "	45 "
No. IV = 17,4. 52,7. 160 =	146716	" "	73 "
No. V = 21,2. 62,8. 160 =	213017	" "	106 "
No. VI = 28,7. 69,1. 160 =	317307	" "	158 "

We now make $a\ b = 8$ tons, $b\ c = 25$ tons, $c\ d = 45$ tons, $d\ e = 73$ tons, $e\ f = 106$ tons and $f\ g = 158$ tons. We find the horizontal pressures $g_1\ h_1$, $g_2\ h_2$, etc. (in Fig. 113), same as before. In this case

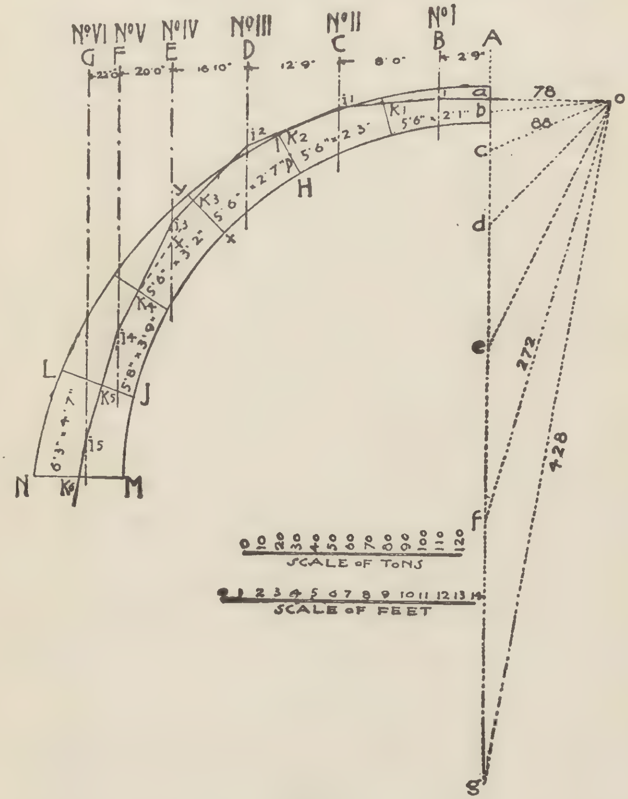


Fig. 114.

we find that the largest pressure is not the last one, but $g_5\ h_5$ which measures 78 units; we therefore select the latter and (in Fig. 114) make $a\ o = g_5\ h_5 = 78$ tons. Draw $o\ b$, $o\ c$, $o\ d$, etc., and construct the line $a\ 1\ i_1\ i_2\ i_3\ i_4\ i_5\ K_6$, same as before. In this case we cannot tell at a glance which is the most strained voussoir joint, for at joint H I the pressure is not very great, but the line is farthest from the centre of joint. Again, while joint J L has not so much pressure as

the bottom joint M N, the line is farther from the centre. We must, therefore, examine all three joints.

We will take H I first. The width of joint is 2′ 5″ or 29″. The pressure is $o\,c$, which scales 88 tons or 176000 pounds. The distance of K_2 from the centre of joint P is 16″. The area of the joint is, of course, the full area of the joint around one-half of the dome, or equal to H I multiplied by the circumference of a semi-circle with the distance of P from $a\,g$ as radius. The latter is 10′ 6″, therefore area $= 2\frac{5}{12}.\ 33 = 80$ square feet or 11520 square inches, therefore:

$$\text{Stress at I} = \frac{176000}{11520} + 6.\frac{176000.\ 16}{11520.\ 29} = +\ 65{,}9$$

$$\text{Stress at H} = \frac{176000}{11520} - 6.\frac{176000.\ 16}{11520.\ 29} = -\ 35{,}3.$$

For joint J L we should have: the width of joint $= 50''$. The pressure $= o\,f = 272$ tons or 544000 pounds. The distance of K_5 from the centre of joint is 8″, while for the area we have 50. $66\frac{2}{3}$. 12 $= 40000$ square inches, therefore:

$$\text{Stress at J} = \frac{544000}{40000} + 6.\frac{540000.\ 8}{40000.\ 50} = +\ 26{,}6 \text{ pounds,}$$

and

$$\text{Stress at L} = \frac{544000}{40000} - 6.\frac{540000.\ 8}{40000.\ 50} = +\ 0{,}5 \text{ pound.}$$

For the bottom joint M N we should have the width of joint $= 60''$. The pressure $= o\ g = 428$ tons or 856000 pounds. The distance of K_6 from the centre of joint is 4″, while for the area we have 60. $70\frac{2}{3}$. 12 $= 50880$ square inches, therefore:

$$\text{Stress at M} = \frac{856000}{50880} + 6.\frac{856000.\ 4}{50880.\ 60} = +\ 23{,}5 \text{ pounds,}$$

and

$$\text{Stress at N} = \frac{856000}{50880} - 6.\frac{856000.\ 4}{50880.\ 60} = +\ 10 \text{ pounds.}$$

The arch, therefore, would seem perfectly safe except at the joint H I, where there is a tendency of the joint to open at H. Had we, however, remembered that this is an arch, lightly loaded, and started our line at the lower third of the crown joint, instead of at the upper third, the line would have been quite different and undoubtedly safe.

The dangerous point K_2 in that case would be much nearer the centre of joint, while the other lines and joints would not vary enough to call for a new calculation, and we can safely pass the arch. One thing must be noted, however, in making the new figure, and that is that the horizontal pressures $g_1\,h_1$, $g_2\,h_2$, etc. (in Fig. 113), would have

to be changed, too, and would become somewhat larger than before, as the line a_1 6 would now drop to the level of a_2. A trial will show that the largest would again be $g_5\ h_5$, and would scale 80 units or tons, which should be used (in Fig. 114) in place of $a\ o$.

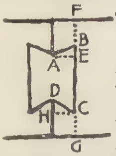


Fig. 115.

Thrust of dome. As regards abutments, there usually are none in a dome; it becomes necessary, therefore, to take up the horizontal thrust, either by metal bands around the entire dome, or by dovetailing the joints of each horizontal course. We must, of course, take the horizontal thrust existing at each joint. Thus, if we were considering the second joint H I the horizontal thrust would be $g_2\ h_2$; or, if we were considering the fifth joint J L the horizontal thrust would be $g_5\ h_5$. For the lower joint M N we might take its own horizontal thrust $g_6\ h_6$, which is smaller than $g_5\ h_5$, provided we take care of the joint J L separately; if not, we should take the larger thrust.

Metal bands. If we use a metal band its area manifestly should be strong enough to resist *one-half* the thrust, as there will be a section in tension at each end of the semi-circle, or

$$a = \frac{h}{2.\left(\frac{t}{f}\right)} \qquad (65)$$

Where a = area, in square inches, of metal bands around domes, at any joint.

Where h = the horizontal thrust at joint, in pounds.

Where $\left(\frac{t}{f}\right)$ = the safe resistance of the metal to tension, per square inch.

In our case, then, for the two lower joints, if the bands are wrought-iron, we should have, as h = 80 tons = 160000 pounds.

$$a = \frac{160000}{2.\ 12000} = 6\tfrac{2}{3}$$

Or we would use a band, say, 5″ x 1⅓″.

Strength of dowels. If dovetailed dowels of stone are used, as shown in Fig. 115, there should be one, of course, in every vertical joint. The dowels should be large enough not to tear apart at A D, nor to shear off at A E, nor to crush A B. Similarly, care should be taken that the area of C G + B F is sufficient to resist the tension, and of H C to resist the shearing.

The strain on A D will, of course, be tension and equal to one-half the horizontal thrust; the same formula holds good, therefore, as for

metal bands, except, of course, that we use for $\left(\frac{t}{f}\right)$ its value for whatever stone we select. Supposing the dome to be built of average marble, we should have from Table V,

$$\left(\frac{t}{f}\right) = 70, \text{ therefore,}$$

$$a = \frac{160000}{2.\ 70} = 1143$$

Now if the dome is built in courses 2′ 6″ or 30″ high, the width of dovetail at its neck A D would need to be:

$$\text{A D} = \frac{1143}{30} = 38 \text{ inches.}$$

As this would evidently not leave sufficient area at G C and B F we must make A D smaller, and shall be compelled to use either a metal dowel, or some stronger stone. By reference to Table V we find that for bluestone,

$$\left(\frac{t}{f}\right) = 140, \text{ therefore:}$$

$$a = \frac{160000}{2.\ 140} = 571 \text{ and,}$$

$$\text{A D} = \frac{571}{30} = 19 \text{ inches.}$$

This would almost do for the lower joint, which is 60″ wide, for if we made: G C + B F = 38″ and
A D = 19″ it would leave
60 — 38 + 19 = 3″ or, say, 1½″ splay; that is, D H = 1½″. Had we used iron, or even slate, however, there would be no trouble.

The shearing strain on either A E or H C will, of course, be equal to one-quarter of the horizontal thrust or

$$b = \frac{h}{4.\ d.\left(\frac{g}{f}\right)} \tag{66}$$

Where b = the width of one-half the dowel, in inches (A E)
Where h = the horizontal thrust at the joint, in pounds
Where d = the height of the course, in inches.

Where $\left(\frac{g}{f}\right)$ = the safe resistance to shearing, per square inch, of either dowel or stone voussoir (whichever is weaker).

When the shearing stress of a stone is not known, we can take the tension instead. Thus in our case as the marble is the weaker, we

should use $\left(\frac{g}{f}\right) = 70$, therefore

$$A\,E = H\,C = \frac{160000}{4.\ 30.\ 70} = 19 \text{ inches.}$$

The compression on A B need not be figured, of course, for while the strain is the same as on A E, the area of A B is somewhat larger, and all stones resist compression better than shearing.

Cambered plank arch. When a plank is "cambered," sprung up, and the ends securely confined, it becomes much stronger transversely than when lying flat. The reason is very simple, as it now acts as an arch, and forces the abutments to do part of its work. Such a plank can be calculated the same as any other arch.

Spanish tile arches. Quite a curiosity in construction, somewhat in the above line, has been recently introduced in New York by a Spanish architect. He builds floor-arches but 3″ thick, of 3 successive layers of 1″-thick tiles, up to 20′ span, and more. His arches have withstood safely test-loads of 700 pounds a square foot. The secret of the strength of his arches consists in their following closely the curve of pressure, thus avoiding tension in the voussoirs, as far as possible. But even were this to exist, it could not open a joint without bodily tearing off several tiles and opening many joints, as shown in Fig. 116, owing to the fact that each course is thoroughly bonded and breaks joints with the course below; besides this, each upper layer is attached to its lower layer by Portland-cement mortar. Specimens of these tiles have been tested for the writer and were found to be as follows:

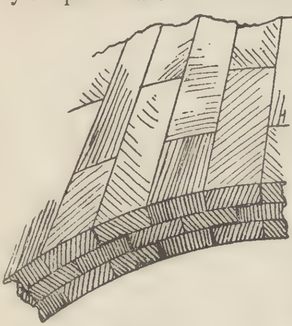


Fig. 116.

Compression (12 days old) $c = 2911$ pounds

or say $\left(\frac{c}{f}\right) = 300$ pounds.

Tension (10 days old) $t = 287$ pounds

or say $\left(\frac{t}{f}\right) = 30$ pounds.

The shearing stress of the mortar-joints is evidently greater than the tension, as samples tested tore across the tile and could not be sheared off.

The modulus of rupture (about 10 days old) was

$k = 91$ pounds

or, say, $\left(\frac{k}{f}\right) = 10$ pounds.

Of course older specimens would prove much stronger.

There is only one valid objection that the writer has heard so far against these arches, and this is, that in case of any uneven settlement they might prove dangerous; as, of course, the margin in which the curve of pressure can safely shift in case of changed surroundings is very small. The writer does not think the objection very great, however, as settlements are apt to ruin any arch and should be carefully provided against in every case. The arches have some very great advantages. The principal one, of course, is their lightness of construction and saving of weight on the floors, walls and foundations. Then, too, in most cases iron beams can be entirely dispensed with, the arches resting directly on the brick walls; of course, there must be weight enough on the wall to resist the horizontal thrust, or else iron tie-rods must be resorted to. The former is calculated as already explained for retaining walls. If tie-rods are used, they are calculated as explained above for the example of the 7″ flat floor arch. An example of these 3″-tile arches may be of interest.

Example IX.

A segmental 3″ tile arch, built as explained above, has 20′ clear span with a rise at the centre of 20″. The floor is loaded at the rate of 150 *pounds per square foot. Is the arch safe?*

Example of tile arch. We divide the arch into five parts or voussoirs, each voussoir carrying 2′ of floor = 300 pounds; we make (Fig. 118) ab = 300 pounds, bc = 300 pounds, etc., and find

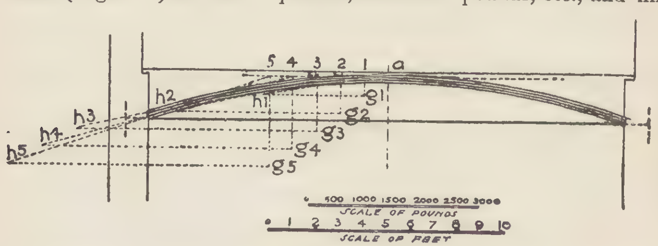


Fig. 117.

the horizontal pressures (Fig. 117) $g_1 h_1$, $g_2 h_2$, etc. The last one $g_5 h_5$ is the largest and scales 4500 units or pounds. We now make (Fig.

118) $a\,o = g_5\,h_5 = 4500$ pounds and draw *ob*, *oc*, *od*, etc. We next construct the curve of pressure $a\,K$ and find that it coincides as closely as possible with the centre line of the arch. This means that

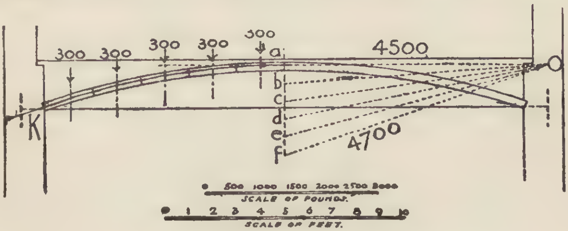


Fig. 118.

the pressure on each joint will be uniformly distributed. That on the lower joint is, of course, the largest and is $= o\,f$, which scales 4700 pounds. The area of the joint is, of course, $a = 3.\ 12 = 36$ square inches, therefore the greatest stress per square inch will be

$$\frac{4700}{36} = 130 \text{ pounds compression.}$$

As the tests gave us 300 pounds compression, per square inch, as safe stress of a sample only twelve days old, the arch is, of course, perfectly safe.

If, however, instead of a uniform load, we had to provide for a very heavy concentrated load, or heavy-moving loads, or vibrations, it would not be advisable to use these arches.

Danger of sliding. So far we have simply considered the danger of compression or tension at the joints of an arch; there is, however, another element of danger, though one that does not arise frequently, viz.: the danger of one voussoir sliding past the other. Where strong and quick-setting cements are used this danger is, of course, not very great. But in other cases, and particularly in large arches, it must be guarded against. The angle of friction of brick against brick (or stone against stone) being generally assumed at 30°, care must be taken that the angle formed by the curve of pressure at the joint, with a normal to the joint (at the point of intersection K) does not form an angle greater than 30°. If the angle be greater than 30° there is danger of sliding; if smaller, there is, of course, no danger. Thus, if in Fig. 114 we erect through K_4 a normal $K_4\,X$ to the joint, the angle $X\,K_4\,i_3$ should not exceed 30°.

Danger of shearing. In arches with heavy-concentrated loads at single points, there might, in rare cases, be danger of the

load shearing right through the arch. The resistance to shearing would, of course, be directly as the vertical area of cross-section of the arch, and in such cases this area must be made large enough to resist any tendency to shear.

Depth at crown. Arches are frequently built shallower at the crown and increasing gradually in depth towards the spring, the amount being regulated, of course, by the curve of pressure and Formulæ (44) and (45).

To establish the first experimental thickness at the crown of an arch, many engineers use the empyrical formula:

$$x = y.\sqrt{r} \qquad (67)$$

Where $x =$ the depth of arch, at crown, in inches.
Where $r =$ the radius at crown, in inches.
Where $y =$ a constant, as follows:

For cut stone, in blocks: $y = 0{,}3$
For brickwork $y = 0{,}4$
For rubblework $y = 0{,}45$

When Portland cement is used, a somewhat lower value may be assumed for y. The depth thus established for crown is only expermental, of course, and must be varied by calculation of curve of pressure, etc.

Approximate rule. In cases where the architect does not feel the necessity for such a close calculation of the arch, it will be sufficient to find the curve of pressure. If this curve of pressure comes *within* the inner third of arch-ring, at every point, the arch is safe, provided the thrust on each joint, divided by the area of joint, does not exceed *one-half* of the safe compressive stress of the material, or:

$$\frac{p}{a} = \frac{1}{2}.\left(\frac{c}{f}\right) \qquad (68)$$

Where $p =$ the thrust on joint, in pounds.
Where $a =$ the area of joint, in square inches.
Where $\left(\frac{c}{f}\right) =$ the safe resistance to compression, per square inch, of the material.

Vaulted and Groined Arches. Vaulted and groined arches are calculated on the same principles as ordinary arches and domes; though, in groined arches, as a rule, the ribs do all the work, the spandrels between the ribs being filled with stone slabs or other light material; in some European examples even flower-pots, plastered on the under side, have been used for this purpose.

CHAPTER VI.

FLOOR BEAMS AND GIRDERS.

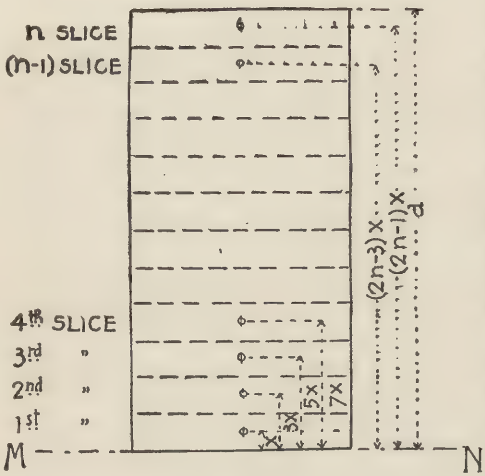


Fig. 119.

Moment of Inertia. THE writer has so often been asked for more information as to the meaning of the term *Moment of Inertia* that a few more words on this subject may not be out of place.

All matter, if once set in motion, will continue in motion unless stopped by gravity, resistance of the atmosphere, friction or some other force; similarly, matter, if once at rest, will so remain unless started into motion by some external force. Formerly it was believed, however, that all matter had a certain repugnance to being moved, which had to be first overcome, before a body could be moved. Probably in connection with some such theory the term arose.

In reality matter is perfectly indifferent whether it be in motion or in a state of rest, and this indifference is termed "Inertia." As used to-day, however, the term Moment of Inertia is simply a symbol or name for a certain part of the formula by which is calculated the force necessary to move a body around a certain axis with a given

velocity in a certain space of time; or, what amounts to the same thing, the resistance necessary to stop a body so moving.

In making the above calculation the "sum of the product of the weight of each particle of the body into the square of its distance from the axis" has to be taken into consideration, and is part of the formula; and, as this sum will, of course, vary as the size of the body varies, or as the location or direction of the axis varies, it would be difficult to express it so as to cover every case, and therefore it is called the "Moment of Inertia." Hence the general law or formula given covers every case, as it contains the Moment of Inertia, which varies, and has to be calculated for each case from the known size and weight of the body and the location and direction of the axis.

In plane figures, which, of course, have no thickness or weight, the *area* of each particle is taken in place of its weight; hence in all plane figures the Moment of Inertia is equal to the "sum of the products of the area of each particle of the figure multiplied by the square of its distance from the axis."

Calculation of Moment of Inertia. Thus if we had a rectangular figure (119) b inches wide and d inches deep revolving around an axis M–N, we would divide it into many thin slices of equal height, say n slices each of a height $= 2.\ X.$

The distance of the centre of gravity of the first slice from the axis M–N will, of course be $= \frac{1}{2}.\ 2.\ X. = 1.\ X$

The distance of the centre of gravity of the second slice will be $=$ $3.\ X$,

that of the third slice will be $= 5.\ X$,

that of the fourth slice will be $= 7.\ X$,

that of the last slice but one will be $= (2\,n - 3).\ X.$

and that of the last slice will be $= (2\,n - 1).\ X$

The area of each slice will, of course, be $= 2.\ X.\ b$; therefore the Moment of Inertia of the whole section around the axis M–N will be (see p. 8),

$$i = 2.\ X.\ b.\ (1.\ X)^2 + 2.\ X.\ b.\ (3.\ X)^2 + 2.\ X.\ b.\ (5.\ X)^2 + 2.\ X.\ b.\ (7.\ X)^2 + \text{etc.} \ldots\ldots + 2.\ X.\ b.\ [(2n-3).X]^2 + 2.\ X.\ b.[(2n-1).X]^2 \text{ or,}$$

$$i = 2.X.^3\,b.\ [1^2 + 3^2 + 5^2 + 7^2 + \text{etc.} \ldots\ldots + (2n-3)^2 + (2n-1)^2]$$

now the larger n is, that is the thinner we make our slices, the nearer will the above approximate:

$$i = 2.\ X^3.\ b.\left[\frac{4}{3}.\ n^3\right]$$

$$= 8.\ X^3.\ n^3.\ \frac{b}{3}$$

Therefore, as: 2. X. $n = d$ we have, by cubing,

8. $X^3.\ n^3 = d^3$; inserting this in above, we have:

$$i = \frac{d^3.\ b.}{3} \text{ or } \frac{b.\ d^3}{3}$$

The same value as given for i in Table I, section No. 29. Of course it would be very tedious to calculate the Moment of Inertia in every case; besides, unless the slices were assumed to be very thin, the result would be inaccurate; the writer has therefore given in Table I, the exact Moments of Inertia of every section likely to arise in practice.

Moment of Resistance. The Moment of Inertia applies to the whole section, the "Moment of Resistance," however, applies only to each individual fibre, and varies for each; it being equal to the Moment of Inertia of the whole section divided by the distance of the fibre from the axis.

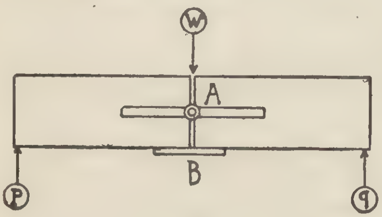


Fig. 120.

Now to show the connection of the Moments of Inertia and Resistance with transverse strains, let us consider the effect of a weight on a beam (supported at both ends).

If we consider the beam as cut in two and hinged at the point A (where the weight is applied), Fig. 120; further, if we consider a piece of rubber nailed to the bottom of each side of the beam, it is evident that the effect of the weight will be, as per Fig. 121.

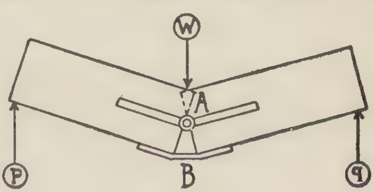


Fig. 121.

Effect of load on beam. Examining this closer we find that the corners of the beams above A (or their fibres) will crush each other, while those below A, are separated farther from each other, and the piece of rubber at B greatly stretched. It is evident, therefore, that the fibres nearest A experience the least change, and

that the amount of change of all the fibres is directly proportionate to their distance from A (as the length of all lines drawn parallel to the base of a triangle, are proportionate to their distance from the apex); further, that the fibres at A experience no change whatever. Now, if instead of considering the effect of a load on a hinged beam we

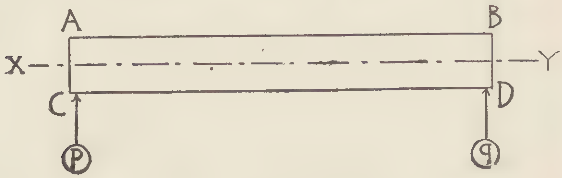


Fig. 122.

took an unbroken beam, the effect would be similar, but, instead of being concentrated at one point, it would be distributed along the entire beam; thus the beam A B D C (Fig. 122) which is not loaded, becomes when loaded, the slightly curved beam (A B D C) Fig. 123.

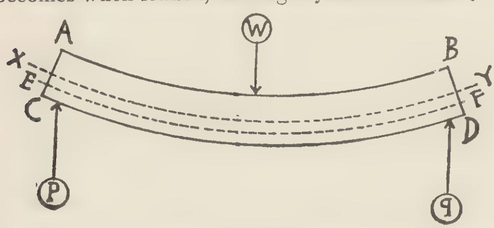


Fig. 123.

It is evident that the fibres along the upper edge are compressed or A B is shorter than before; on the other hand the fibres along C D are elongated, or in tension, and C D is longer than before; if we now take any other layer of fibres as E F, they — being below the neutral (and central)[1] axis X–Y — are evidently elongated; but not so much so, as C D: and a little thought will clearly show that their elongation is proportioned to the elongation of the fibres C D, directly as their respective distances from the neutral axis X–Y. It is further evident that the neutral axis X–Y is the same length as before, or its fibres are not strained; it is, therefore, at this point that the strain changes from one of tension to one of compression.

In Fig. 124 we have an isometrical view of a loaded beam.

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[1] As a rule the neutral axis can be safely assumed to be central, but it is not necessarily so. In materials, such as cast-iron, stone, etc., where the resistance of the fibres to compression and tension varies greatly, the axis will be far from the centre, nearer the weaker fibres.

Rotation around neutral axis. Let us now consider an infinitesimally thin (cross) section of fibres A B C D in reference to their own neutral axis M–N. It is evident that if we were to double the load on the beam, so as to bend it still more, that the fibres along A B would be compressed towards or would *move* towards the centre of the beam; the fibres along D C on the contrary would be elongated or would *move* away from the centre of the beam.

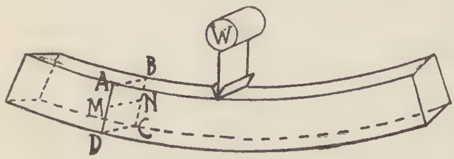


Fig. 124.

The fibres along M–N, being neither stretched nor compressed, would remain stationary.

The fibres between M–N and A B would all move towards the centre of the beam, the amount of motion being proportionate to their distance from M–N; the fibres between M–N and D C on the contrary would move away from the centre of the beam the amount of motion being proportionate to their distance from M–N; a little thought, therefore, shows clearly that the section A B C D turns or rotates on its neutral axis M–N, whenever additional weight is imposed on the beam.

This is why we consider in the calculations the moment of *Inertia* or the moment of resistance of a *cross-section* as *rotating* on its neutral axis.

Now let us take the additional weight off the beam and it will spring back to its former shape, and, of course, the fibres of the infinitesimally thin section A B C D will resume their normal shape; that is, those that were compressed will stretch themselves again, while those that were stretched will compress themselves back to their former shape and position, and those along the neutral axis will remain constant; or, in other words, this thin layer of fibres A B C D can be considered as a double wedge-shaped figure $A\ B\ A_1\ B_1\ M\ N\ D\ C$

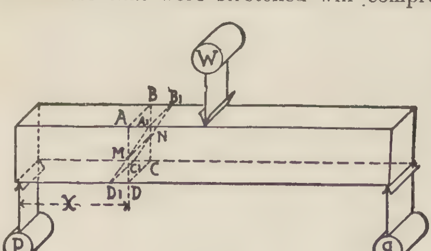


Fig. 125.

$D_1 C_1$ (Fig. 125) the base of the wedges becoming larger or smaller as the weight on the beam is varied.

Resistance of Wedge. Now to proceed to the calculation of the resistance of this wedge. It is evident that whatever may be the external strain on the beam at the section A B C D, the beam will owe whatever resistance it has at that point to the resistances of the fibres of the section or wedge to compression and tension.

Now considering the right-hand side of the beam as rigid, and the section A B C D as the point of fulcrum of the external forces, we have only one external force p, tending to turn the left-hand side of the beam upwards around the section A B C D, its total tendency, effect or moment m at A B C D, we know is $m = p.\,x$ (law of the lever).

Now to resist this we have the opposition of the fibres in the wedge A B A_1 B_1 M N to compression and the opposition to tension of the fibres in the wedge D C D_1 C_1 M N. For the sake of convenience, we will still consider these wedges, as wedges but so infinitesimally thin that we can safely put down the amount of their contents as equal to the area of their sides, so that — if A B $= b$ (the width of beam) and A D $= d$ (the depth of beam) — we can safely call *each* wedge as equal to $b.\,\frac{d}{2}$.

Now as the centre of gravity of a wedge is at $\frac{1}{3}$ of the height from its base, or $\frac{2}{3}$ of the height from its apex (and as the height of each wedge is $= \frac{d}{2}$) it would be $= \frac{2}{3}\cdot\frac{d}{2} = \frac{d}{3}$ from axis M–N. The moment of a wedge at any axis M–N is equal to the contents of the wedge multiplied by the distance of its centre of gravity from the axis, the whole multiplied by the stress of the fibres, (that is their resistance to tension or compression). Now the contents of each wedge being $= b.\,\frac{d}{2}$, the distance of centre of gravity from M–N $= \frac{d}{3}$, and the stress being say $= s$, we have for the resistance of *each* wedge

$$= b.\,\frac{d}{2}\cdot\frac{d}{3}\cdot s$$

$$= \frac{b.\,d^2}{6}\cdot s$$

Now if the stress on the fibres along the extreme upper or lower edges $= k$ (or the modulus of rupture), it is evident that the *average* stress on the fibres in either wedge will $= \frac{k}{2}$, or $s = \frac{k}{2}$ (for the

stress on each fibre being directly proportionate to the distance from the neutral axis the stress on the average will be equal to half that on the base). Now inserting $\frac{k}{2}$ for s in the above formula, and multiplying also by 2, (as there are two wedges resisting), we have the total resistance to rupture or bending of the section A B C D ($A_1\ B_1\ C_1\ D_1$)

$$= \frac{b.\,d^2}{6} \cdot \frac{k}{2} \cdot 2$$

$$= \frac{b.\,d^2}{6} \cdot\ k$$

Now, by reference to Table I, section No. 2, we find that $\frac{b.d^2}{6} =$ Moment of Resistance for the section A B C D; therefore, we have proved the rule, that when the beam is at the point of rupture at any point of its length the bending moment at that point is equal to the moment of resistance of its cross-section at said point multiplied by the modulus of rupture.

Where girders or beams are of wood, it becomes of the highest importance that they should be sound and perfectly dry. The former that they may have sufficient strength, the latter that they may resist decay for the longest period possible.

Formation of Wood. Every architect, therefore, should study thoroughly the different kinds of timber in use in his locality, so as to be able to distinguish their different qualities. The strength of wood depends, as we know, on the resistance of its fibres to separation. It stands to reason that the young or newly formed parts of a tree will offer less resistance than the older or more thoroughly set parts. The formation of wood in trees is in circular layers, around the entire tree, just inside of the bark. As a rule one layer of wood is formed every year, and these layers are known, therefore, as the "annular rings," which can be distinctly seen when the trunk is sawed across. These rings are formed by the (returning) sap, which, in the spring, flows upwards between the bark and wood, supplies the leaves, and returning in the fall is arrested in its altered state, between the bark and last annular ring of wood. Here it hardens, forming the new annular ring. As subsequent rings form around it, their tendency in hardening is to shrink or compress and harden still more the inner rings, which hardening (by compression) is also assisted by the shrinkage of the bark. In a sound tree, therefore, the strongest wood is at the heart or centre of growth. The

heart, however, is rarely at the exact centre of the trunk, as the sap flows more freely on the side exposed to the effects of the sun and wind; and, of course, the rings on this side are thicker, thus leaving the heart constantly, relatively, nearer to the unexposed side.

Heart-Wood. From the above it will be readily seen that timber should be selected from the region of the heart, or it should be what is known as "heart-wood." The outer layers should be rejected, as they are not only softer and weaker, but, being full of sap, are liable to rapid decay. To tell whether or no the timber is "heart-wood" one need but look at the end, and see whether it contains the centre of the rings. No bark should be allowed on timber, for not only has it no strength itself, but the more recent annular rings near it, are about as valueless.

Medullary Rays. In some timbers, notably oak, distinct rays are noticed, crossing the annular rings and radiating from the centre. These are the "medullary rays," and are elements of weakness. Care should be taken that they do not cross the end of the timber horizontally, as shown at A in Fig. 126, but as near vertically as possible, see B in Fig. 127. The beautiful appearance of quartered oak and other woods is obtained by cutting the planks so that their surfaces will show slanting cuts through these medullary rays.

Seasoning cracks. All timber cracks more or less in seasoning, nor need these cracks cause much worry, unless they are very deep and long. They are, to a certain extent, signs of the amount of seasoning the timber has had. They should be avoided, as much as possible, near the centre of the timber, if regularly loaded, or near the point of greatest bending moment, where the

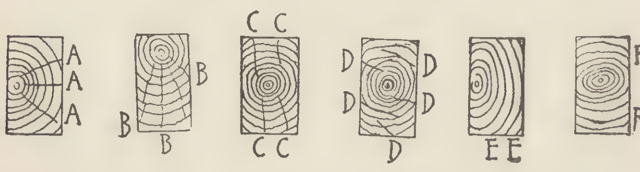


Fig. 126. Fig. 127. Fig. 128. Fig. 129. Fig. 130. Fig. 131.

loads are irregular. If timber without serious cracks cannot be obtained, allowance should be made for these, by increasing its size.

Vertical, or nearly vertical cracks (as C, Fig. 128) are not objectionable, and do not weaken the timber. But horizontal cracks (as D, Fig. 129), are decidedly so, and should not be allowed.

Knots. Knots in timber are another element of weakness. They are the hearts, where branches grow out of the trunk. If they are of nearly the same color as the wood, and their rings gradually die out into it, they need not be seriously feared. If, however, they are very dark or black, they are sure to shrink and fall out in time, leaving, of course, a hole and weakness at that place. Dead knots, — that is, loose knots, — in a piece of timber, mean, as a rule, that the heart is decaying. Knots should be avoided at the centre of a beam, regularly loaded, and at the point of greatest bending moment, where the loads are irregular. The farther the knots (and cracks) are from these points the better.

Wind-shakes. Timber with "wind-shakes" should be entirely avoided, as it has no strength. These are caused by the wind shaking tall trees, loosening the rings from each other, so that when the timber is sawed, the wood is full of small, almost separate pieces or splinters at these points.

A timber with wind-shakes should be condemned as unsound.

A timber with the rings at the end showing nearly vertical (E Fig. 130) will be much stronger than one showing them nearly horizontal. (F Fig. 131.)

Signs of sound Timber. To tell sound timber, Lord Bacon recommended to speak through it to a friend from end to end. If the voice is distinctly heard at the other end it is sound. If the voice comes abruptly or indistinctly it is knotty, imperfect at the heart, or decayed. More recent authorities recommend listening to the ticking of a watch at the other end, or the scratching of a pin on its surface. If, in sawing across a piece it makes a clean cut, it is neither too green nor decayed. The same if the section looks bright and smells sweet. If the section is soft or splinters up badly it is decayed. If it wets the saw it is full of sap and green. If a blow on timber rings out clearly it is sound; if it sounds soft, subdued, or dull, it is very green or else decayed. The color at freshly-sawed spots should be uniform throughout; timbers of darker cross-section are generally stronger than those of lighter color (of the same kind of wood.)

The annular rings should be perfectly regular. The closer they are, the stronger the wood. Their direction should be parallel to the axis throughout the length of the timber, or it will surely twist in time, and is, besides, much weaker. Where the rings at both ends are not in the same direction the timber has either twisted in

growing, or has a "wandering heart,"—that is, a crooked one. Such timber should be condemned. Besides looking at the rings at the end, a longitudinal cut near the heart will show whether it has grown regularly and straight, or whether it has twisted or wandered.

The weight of timber is important in judging its quality. If specimens of a wood are much heavier than the well-known weight of that wood, when seasoned, they may be condemned as green and full of sap. If they are much lighter than thoroughly seasoned specimens of the same wood, they are very probably decayed.

Methods of Seasoning. Tredgold claims that timber is "seasoned" when it has lost one-fifth of its original weight (when green); and "dry" when it has lost one-third. Some timbers, however, lose nearly one-half of their original weight in drying. Many methods are used to season or dry timber quickly.

The best method, however, is to stack the timber on dry ground (in as dry an atmosphere as possible) and in such a position that the air can circulate, as freely as possible around each piece. Sheds are built over the timber to protect it from the sun, rain, and also from severe winds as far as possible.

Timber dried slowly, in this manner, is the best. It will crack somewhat, but not so much so as hastily dried timber. Many processes are used to keep it from cracking, the most effective being to bore the timber from end to end, at the centre, where the loss of material does not weaken it much, while the hole greatly relieves the strain from shrinkage. Some authorities claim that two years' exposure is sufficient, though formerly timber was kept very much longer. But even two years is rarely granted with our modern conditions, and most of the seasoning is done after the timber is in the building. Hence its frequent decay. There are many artificial methods for drying timber, but they are expensive. The best known is to place it in a kiln and force a rapid current of heated air past it, this is known as "kiln-drying." It is very apt to badly "check" or crack the wood. To preserve timber, besides charring, the "creosoting" process is most effective. The timber is placed in an iron chamber, from which the air is exhausted; after which creosote is forced in under a high pressure, filling, of course, all the pores which have been forced open by the suction of the departing air. Creosoted wood, however, cannot be used in dwellings, as the least application of warmed air to it, causes a strong odor, and would render the building untenantable.

Effect of Drying. Timber properly dried can be raised to four times (400% of) the strength of its green state (notably spruce, see U. S. Dept. of Agriculture Circular 108, Aug. 26th, 1807). But as it absorbs more or less moisture from the air, from 8% to 16% of its dry weight, say 12% average, the actual gain in strength by drying is only 240%. Again wood, in drying, checks and is thus weakened, so that no greater strength should be allowed for than it shows in its green state.

Timber is so uncertain, and affected in so many ways by local conditions, that a large factor of safety should always be used.

Fire-proofing. Now-a-days wood is fire-proofed, that is made unburnable. There are many different processes; some—treatment by electricity; others—chemical baths; again—application of fire-proof chemicals by pressure, filling the pores; or direct application or washes of chemicals.

Woods so treated are very much heavier, and are very much hardened, greatly increasing the expense, not only on account of the cost of treatment, but on account of the greatly increased labor necessary to work the wood, and the destruction of edges of and of tools on account of the hardened character of the wood.

Manner of Shrinkage. In shrinking, the distance *between* rings remains constant, and it is for this reason that the finest floors are made from quartered stuff; for (besides their greater beauty), the rings being all on end, no horizontal shrinkage will take place; the width of boards remaining constant, and the shrinkage being only in their thickness; neither will timber shrink on end or in its length. Figures 132 and 133 show how timber will shrink. The first from a quartered log, the other from one with parallel cuts. The dotted part shows the shrinkage. The side-pieces G in Fig. 133 will curl, as shown, besides shrinking. By observing the directions of the annular rings, therefore, the future behavior

Fig. 132.

Fig. 133.

of the timber can be readily predicted. Of course, the figures are greatly exaggerated to show the effect more clearly.

Decay of Timber. If the heart is not straight its entire length, the piece will twist lengthwise. Shrinkage is a serious danger, but the chief danger in the use of timber lies in its decay. All timber will decay in time, but if it is properly dried, before being built in, and all sap-wood discarded, and then so placed that no moisture can get to the timber, while fresh air has access to all parts of it, it will last for a very long time; some woods even for many centuries. In proportion as we neglect the above rules, will its life be short-lived. There are two kinds of decay, *wet* and *dry* rot. The wet rot is caused by alternating exposures to dampness and dryness; or by exposure to moisture and heat; the dry rot, by confining the timber in an air-tight place. In wet rot there is "an excess of evaporation;" in dry rot there is an "imperfect evaporation." Beams with ends built solidly into walls are apt to rot; also beams surrounded solidly with fireproof materials; beams in damp, close, and imperfectly ventilated cellars; sleepers bedded solidly in damp mortar or concrete, and covered with impervious papers or other materials; also timbers exposed only at intervals to water or dampness, or timbers in "solid" timbered floors.

Dry rot is like a contagious disease, and will gradually not only eat up the entire timber, but will attack all adjoining sound woodwork. Where rotted woodwork is removed, all adjoining woodwork, masonry, etc., should be thoroughly scraped and washed with strong acids.

Ventilation necessary. Where wood has, of necessity, to be surrounded with fireproof materials, a system of pipes or other arrangements, should be made to force air to same through holes, either in the floors or ceilings, but in no case connecting two floors; the holes can then be made small enough not to allow the passage of fire. Where the air is forced in under pressure it would be advisable at times to force in disinfectants, such as steam containing evaporated carbolic acid, fumes of sulphur, etc.

Coating woodwork with paint or other preparations will only rot the wood, unless it has been first thoroughly dried and every particle of sap removed. When wood is painted, it should be on

all surfaces, or the unpainted surface will absorb moisture, ultimately causing rot.

Cross-bridging. Timber must not be used too thin, or it will be apt to twist. For this reason floor-beams should not be used thinner than three inches. To avoid twisting and curling, cross-bridging is resorted to. That is, strips usually $2'' \times 3''$ are cut between the beams, from the bottom of one to the top of the next one, the ends being cut (in a mitre-box), so as to fit accurately against the sides of beams, and each end nailed with at least two strong nails. The strips are always placed in double courses, across the beams, the courses crossing each other like the letter x between each pair of beams.

This is known as "herring-bone" cross-bridging. Care should be taken that all the parallel pieces in each course are in the same line or plane. The lines of cross-bridging can be placed as frequently as desired, for the more there are, the stiffer will be the floor. About six feet between the lines is a good average. Sometimes solid blocks are used between the beams, in place of the herring-bone bridging. Cross-bridging is also of great help to a floor by relieving an individual beam from any great weight accidentally placed on it (such as one foot of a safe, or one end of a book-case), and distributing the weight to the adjoining beams. Unequal settlements of the individual beams are thus avoided. Where a floor shows signs of weakness, or lacks stiffness, or where it is desirable to force old beams, that cannot be well removed, to do more work, two lines of slightly wedge-shaped blocks are driven tightly between the beams, in place of the cross-bridging. The beams are then bored, and an iron rod is run between the lines of wedges, from the outer beam at one end to the outer beam at the other, and, of course, at right angles to all. At one end the rod has a thread and nut, and by screwing up the latter the beams are all forced upwards, "cambered," and the entire floor arched. It will be found much stronger and stiffer; but, of course, will need leveling for both floor and ceiling. Under the head and nut at ends of rod, there must be ample washers, or the sides of end beams will be crushed in, and the effect of the rod destroyed.

Stiffening weak floors.

Girders, which cannot be stiffened sideways, should be, at least, half as thick as they are deep, to avoid lateral flexure.

Framing of Beams. In using wooden beams and girders, much framing has to be resorted to. The used joints between timbers are numerous, but only a very few need special mention here. Beams should not rest on girders, if it can be avoided, on account of the additional dropping caused by the sum of the shrinkage of both, where one is over the other. If framing is too expensive, bolt a wide piece to the under side of the girder, sufficiently wider than the girder to allow the beams to rest on it, each side. If this is not practicable bolt pieces onto each side of the girder, at the bottom, and notch out the beams to rest against and over these pieces. The bearing of a beam should always be as near its bottom as possible. If a beam is notched so as to bear near its centre, it will split longitudinally. Where a notch of more than one-third the height of beam, from the bottom, is necessary, a wrought-iron strap or belt should be secured around the end of beam, to keep it from splitting lengthwise.[1]

If framing can be used, the best method is the "tusk and tenon" joint, as shown in Figs. 134 and 135. In the one case the tenon goes through the girder and is secured by a wooden wedge on the other side; in the other it goes in only about a length equal to twice its depth, and is spiked from the top of girder. The latter is the most used. By both methods the girder is weakened but very little, the principal cut being near its neutral axis, while the beam gets bearing near its bottom, and its tenon is thoroughly strengthened to prevent its shearing off. The dimensions given in the figures are all in parts of the height of beams. Headers and trimmers at fire-places and other openings are frequently framed together, though it would be more advisable to use "stirrup-irons." The short tail-beams, however, can be safely tenoned into the header.

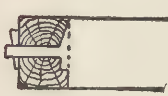


Fig. 134.

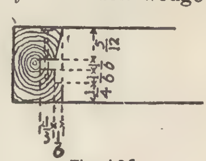


Fig. 135.

[1] Where beams come against the sides of girders, the top of beams should be from one to four inches above the top of girder as these beams shrink much quicker and more than thick girders, and thus the long humps or ridges in floors over girders will be avoided.

In calculating the strength of framed timber, the point where the mortise, etc., are cut, should be carefully calculated by itself, as the cutting frequently renders it dangerously weak, at this point, if not allowed for. For the same reason plumbers should not be allowed to cut timbers. As a rule, however, cuts near the wall are not dangerous, as the beam being of uniform size throughout, there is usually an excess of strength near the wall.

Stirrup-irons. Stirrup-irons are made of wrought-iron; they are secured to one timber in order to provide a resting-place for another timber, usually at right angles to and carried by the former. They should always lap over the farther side of the carrying timber, to prevent slipping, as shown in Fig. 136.

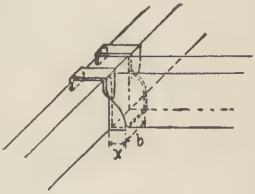


Fig. 136.

The iron should be sufficiently wide not to crush the beam, where resting on it; the section of iron must be sufficient not to shear off each side of beams. The twist must not be too sudden, or it will straighten out and let the carried timber down. To put the above in formulæ we should have for the width of stirrup-iron (x)

Width of Stirrup-irons.

$$x = \frac{s}{b.\left(\frac{c}{f}\right)}. \tag{69}$$

Where $x =$ the width of stirrup-iron, in inches.

Where $s =$ the shearing strain, in lbs., on end of beam, being carried.

Where $b =$ the width of beam being carried, in inches.

Where $\left(\frac{c}{f}\right) =$ the safe resistance, in pounds, to compression, across the fibres, of the beam, being carried.

For the thickness of stirrup-iron we should have:

$$y = \frac{s}{2.x.\left(\frac{g}{f}\right)} \tag{70}$$

Which for wrought-iron (Table IV.) becomes,

Thickness of Stirrup-iron.

$$y = \frac{s}{16000.\,x} \tag{71}$$

Where $y =$ the thickness of stirrup-iron, in inches.

Where $s =$ the shearing strain on end of beam, in lbs.

Where $x =$ is found by formula (69).

Providing, however, that y should never be *less* than one-quarter inch thick.

Example.

A girder carries the end of a beam, on which there is a uniform load of two thousand pounds. The beam is four inches thick, and of Georgia pine. What size must the stirrup-iron be?

Example stirrup-irons. The shearing strain at each end of the beam will, of course, be one thousand pounds, which will be the load on stirrup-irons. (See Table VII). From Table IV we find for Georgia pine, *across* the fibres, $\left(\frac{c}{f}\right) = 200$, we have, therefore. for the width of stirrup-iron from Formula (69)

$$x = \frac{1000}{4.200} = 1\tfrac{1}{4}''$$

Therefore the thickness of iron from Formula (71) should be

$$y = \frac{1000}{16000.1\frac{1}{4}} = \frac{1}{20}''$$

we must make the iron however at least $\frac{1}{4}''$ thick and therefore use a section of $1\frac{1}{4} \times \frac{1}{4}''$.

In calculating ordinary floor-beams the shearing strain can be overlooked, as a rule; for, in calculating transverse strength we allow only the safe stress on the fibres of the upper and lower edges, while the intermediate fibres are less and less strained, those at the neutral axis not at all. The reserve strength of these only partially used fibres will generally be found quite ample to take up the shearing strain.

Rectangular beams. The formulæ for transverse strength are quite complicated, but for rectangular sections (wooden beams) they can be very much simplified *provided we are calculating for strength only and not taking deflection into account.*

Remembering that the moment of resistance of a rectangular section is (Table I) $= \frac{b.d^2}{6}$ and inserting into Formula (18) the value for m according to the manner of loading and taken from (Table VII), we should have:

For uniform load on beam.

Transverse strength of rectangular beams.

$$u = \frac{b.d^2}{9.L} \cdot \left(\frac{k}{f}\right) \tag{72}$$

For centre load on beam.

$$w = \frac{b.d^2}{18.L} \cdot \left(\frac{k}{f}\right) \tag{73}$$

For load at any point of beam.

$$w_1 = \frac{b.d^2.L}{72.M.N} \cdot \left(\frac{k}{f}\right) \qquad (74)$$

For uniform load on cantilever.

$$u = \frac{b.d^2}{36.L} \cdot \left(\frac{k}{f}\right) \qquad (75)$$

For load concentrated at end of cantilever.

$$w = \frac{b.d^2.}{72.L} \cdot \left(\frac{k}{f}\right) \qquad (76)$$

For load at any point of cantilever.

$$w_1 = \frac{b.d^2}{72.Y} \cdot \left(\frac{k}{f}\right) \qquad (77)$$

Where $u =$ safe uniform load, in pounds.

Where $w =$ safe centre load on beam, in pounds; or safe load at end of cantilever, in pounds.

Where $w_1 =$ safe concentrated load, in pounds, at any point.

Where $Y =$ length, in feet, from wall to concentrated load (in cantilever).

Where M and $N =$ the respective lengths, in feet, from concentrated load on beam to each support.

Where $L =$ the length, in feet, of span of beam, or length of cantilever.

Where $b =$ the breadth of beam, in inches.

Where $d =$ the depth of beam, in inches.

Where $\left(\frac{k}{f}\right) =$ the safe modulus of rupture, per square inch, of the material of beam or cantilever (see Table IV).

The above formulæ are for rectangular wooden beams supported against lateral flexure (or yielding sideways). Where beams or girders are not supported sideways the thickness should be equal to at least half of the depth.

No allowance for deflection. *The above formulæ make no allowance for deflection*, and except in cases, such as factories, etc., where strength only need be considered and not the danger of cracking plastering, or getting floors too uneven for machinery, are really of but little value. They are so easily understood that the simplest example will answer:

Example.

Take a 3″ × 10″ hemlock timber and 9 feet long (clear span), loaded in different ways, what will it safely carry? taking no account of deflection.

The safe modulus of rupture $\left(\frac{k}{f}\right)$ for hemlock from Table IV is

$$= 750 \text{ pounds.}$$

If both ends are supported and the load is uniformly distributed the beam will safely carry, (Formula 72):

$$u = \frac{3.10^2}{9.9} \cdot 750 = 2778 \text{ pounds.}$$

If both ends are supported and the load concentrated at the centre, the beam will safely carry, (Formula 73):

$$w = \frac{3.10^2}{18.9} \cdot 750 = 1389 \text{ pounds.}$$

If both ends are supported and the load is concentrated at a point I, distant four feet from one support (and five feet from the other) the beam will safely carry, (Formula 74):

$$w_1 = \frac{3.10^2.9}{72.4.5} \cdot 750 = 1406 \text{ pounds.}$$

If one end of the timber is built in and the other end free and the load uniformly distributed, the cantilever will safely carry, (Formula 75):

$$u = \frac{3.10^2}{36.9} \cdot 750 = 694 \text{ pounds.}$$

If one end is built in and the other end free, and the load concentrated at the free end, the cantilever will safely carry, (Formula 76):

$$w = \frac{3.10^2}{72.9} \cdot 750 = 347 \text{ pounds.}$$

If one end is built in and the other end free, and the load concentrated at a point I, which is 5 feet from the built-in end, the cantilever will safely carry, (Formula 77):

$$w_1 = \frac{3.10^2}{72.5} \cdot 750 = 625 \text{ pounds.}$$

Where, however, the span of the beam, in feet, greatly exceeds the depth in inches (see Table VIII), and regard must be had to deflection, the formulæ (28) and (29), also (37) to (42) should always be used, inserting for i its value from Table I, section No. 2, or:—

$$i = \frac{b.d^3}{12}$$

Where b = the thickness of timber, in inches.

Where d = the depth of timber, in inches.

Where i = the moment of inertia of the cross-section, in inches.

Table IX, however, gives a much easier method of calculating wooden beams, allowing for both rupture and deflection, and Formulæ (72) to (77) have only been given here, as they are often erro-

neously given in text-books, as the only calculations necessary for beams.

Basis of Tables XII and XIII. To still further simplify to the architect the labor of calculating wooden beams or girders, the writer has constructed Tables XII and XIII.

Table XII is calculated for floor beams of dwellings, offices, churches, etc., at 90 pounds per square foot, including weight of construction. The beams are supposed to be cross-bridged.

Table XIII is for isolated girders, or lintels, uniformly loaded, and supported sideways. When not supported sideways decrease the load, or else use timber at least half as thick as it is deep. In no case will beams or girders (with the loads given) deflect sufficiently to crack plastering. For convenience Table XII has been divided into two parts, the first part giving beams of from 5′ 0″ to 15′ 0″ span, the second part of from 15′ 0″ to 29′ 0″ span.

How to use Table XII. The use of the table is very simple and enables us to select the most economical beam in each case. For instance, we have say a span of 21′ 6″. We use the second part of Table XII. The vertical dotted line between 21′ 0″ and 22′ 0″ is, of course, our line for 21′ 6″. We pass our finger down this line till we strike the curve. To the left opposite the point at which we struck the curve, we read:

21.6 spruce, W. P. 56 — 4-14-14 or:

at 21′ 6″ span we can use spruce or white-pine floor beams, of 56 inches sectional area each, viz.: 4″ thick, 14″ deep and 14″ from centres. Of course we can use any other beam *below* this point, as they are all stronger and stiffer, but we must not use any other beam *above* this point. Now then, is a 4″ $\times$ 14″ beam of spruce or white pine, and 14″ from centres the most economical beam. We pass to the columns at the right of the curve and there read in the first column 48″. This means that while the sectional area of the beam is 56 square inches, it is equal to only 48 square inches *per square foot of floor*, as the beams are more than one foot from centres. In this column the areas are all reduced to the "area per square foot of floor," so that we can see at a glance if there is any cheaper beam *below* our point. We find below it, in fact, many cheaper beams, the smallest area (per square foot of floor) being, of course, the most economical. The smallest area we find is 36, 0 or 36 square inches of section per square foot of floor (this we find three times, in the sixteenth, twenty-ninth and thirty-first lines from the bottom). Passing to the left we find they represent, respectively, a Georgia pine

beam, 3″ thick, 16″ deep and 16″ from centres; or a Georgia pine beam 3″ thick, 14″ deep and 14″ from centres; or a white oak beam 3″ thick, 16″ deep and 16″ from centres. If therefore, we do not consider depth, or distance from centres, it would simply be a question, which is cheaper, 48 inches (or four feet "board-measure") of white pine or spruce, or 36 inches (or three feet "board-measure") of either white oak or Georgia pine. The four other columns on the right-hand side, are for the same purpose, only the figures for each kind of wood are in a column by themselves; so that, if we are limited to any kind of wood we can examine the figures for that wood by themselves. Take our last case and suppose we are limited to the use of hemlock; now from the point where our vertical line (21′ 6″) first struck the curve, we pass to the right-hand side of Table, to the second column, which is headed "Hemlock." From this point we seek the smallest figure *below* this level, but in the same column; we find, that the first figure we strike, viz: 41, 2 is the smallest, so we use this; passing along its level to the left we find it represents a hemlock beam of 48 square inches cross-section, or 3″ thick, 16″ deep and 14″ from centres.

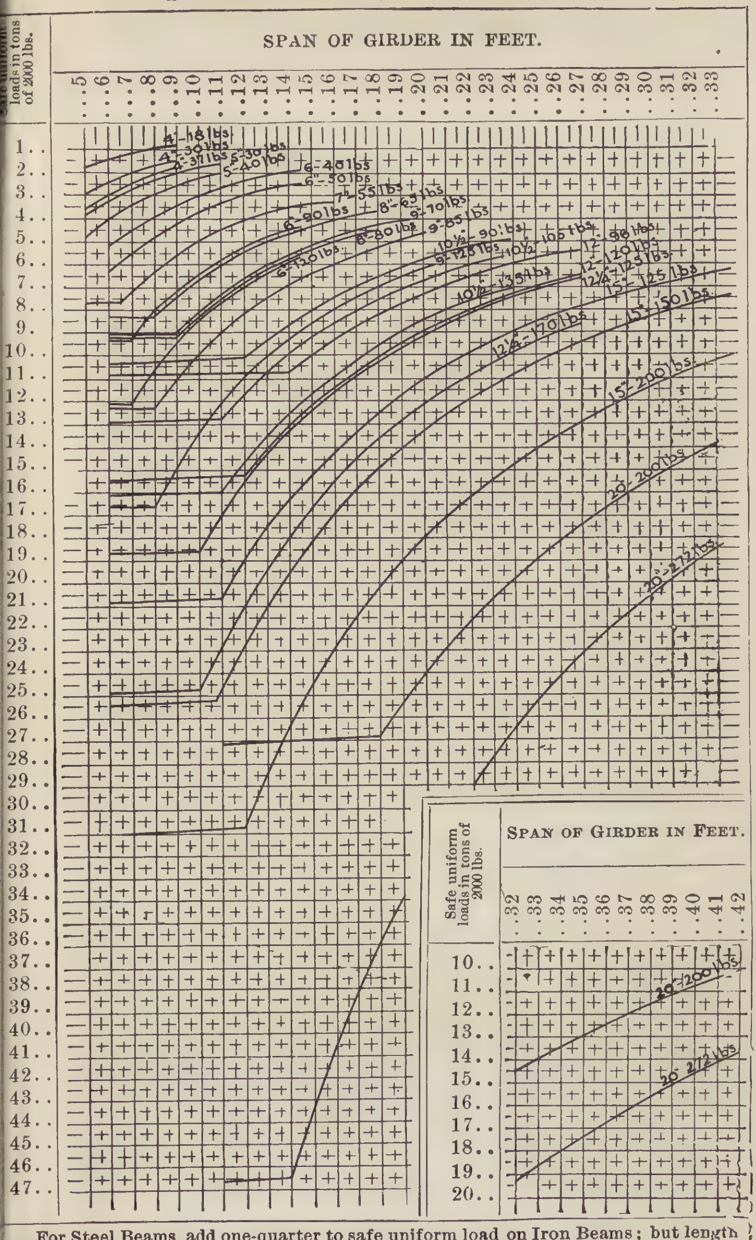
In case the size of the beam is known, its safe span can, of course, be found by reversing the above procedure, or if the depth of beam and span is settled, we can find the necessary thickness and distance between centres; in this way the Table, of course, covers every problem.

Table XIII is calculated for wooden girders of all sizes. Any thickness not given in the table can be obtained by taking the line for a girder of same depth, but one inch thick and multiplying by the thickness. For very short spans, look out for danger of horizontal shearing (see formula 13); where this danger exists, pass vertical bolts through ends of girder, or bolt thin iron plates, or straps, or even boards with vertical grain, to each side of girder, at ends.

How to use Table XIII. The use of this table is very simple. The vertical columns to the left give the safe uniform loads on girders (sufficiently stiff not to crack plastering) for different woods: these apply to the dotted parts of curves. The columns on the right-hand side give the same, but apply to the parts of curves drawn in full lines.

If we have a 6″ × 16″ Georgia pine beam of 20 feet span and want to know what it will carry, we select the curve marked at its upper end 6 × 16 = 96; we follow this curve till it intersects the vertical line 20′ 0″; as this is in the part of curve drawn full, we

IRON I-BEAM GIRDERS, BRACED SIDEWAYS.



For Steel Beams, add one-quarter to safe uniform load on Iron Beams; but length of span in feet must not exceed twice the depth of beam in inches, or deflection will be too great for plastering.

pass horizontally to the right and find under the column marked "Georgia Pine," 7980, which is the safe, uniform load in pounds. Supposing, however, we had simply settled the span, say 8 feet, and load, say 7000 pounds, and wished to select the most economical girder, being, we will say, limited to the use of white pine: the span not being great we will expect to strike the dotted part of curve, and therefore select the fourth (white pine) column *to the left.* We pass down to the nearest figure to 7000 and then pass horizontally to the right till we meet the vertical 8 feet line; this we find is, as we expected, at the dotted part, and therefore our selection of the left column was right. We follow the curve to its upper end and find it requires a girder $4'' \times 12'' = 48$ square inches. Now, can we use a cheaper girder? of course, all the lines *under* and *to the right* of our curve are stronger, so that if either has a smaller sectional area, we will use it. The next curve we find is a $6'' \times 10'' = 60''$; then comes a $4'' \times 14'' = 56''$; then an $8'' \times 10'' = 80''$; then a $6'' \times 12'' = 72''$ and so on; as none has a smaller area we will stick to our $4'' \times 12''$ girder, provided it is braced or supported sideways. If not, to avoid twisting or lateral flexure, we must select the next cheapest section, where the thickness is at least equal to half the depth;[1] the cheapest section beyond our curve that corresponds to this, we find is the $6'' \times 10''$ girder, which we should use if not braced sideways.

In the smaller sections of girders where the difference between the loads given from line to line is proportionally great, a safe load should be assumed between the two, according to the proximity to either line at which the curve cuts the vertical. The point where the curve cuts the *bottom* horizontal line of each part is the length of span for which the safe load opposite the line is calculated.

Heavier Floors. Where a different load than 90 pounds per square foot, must be provided for, we can either increase the thickness of beams as found in Table XII, or decrease their distance from centres, either in proportion to the additional amount of load. Or, if we wish to be more economical, we can calculate the safe uniform load on *each* floor beam, and consider it as a separate girder, supported sideways, using of course, Table XIII.

Basis of Tables XIV and XV. The Tables XIV and XV are very similar to the foregoing, but calculated for wrought-iron I-beams.[2] Table XIV gives the size of beams and distance from centres re-

[1] The rule for calculating the exact thickness will be found later, Formula (78).

[2] To save the great additional cost of this book to the *student*, if entirely new tables were used for the present sizes, weights and shapes of rolled sections, the tables from "Safe Building" have been retained as they

quired to carry different loads per square foot of floor, 150 pounds per square foot of floor (including the weight of construction), however, being the usual load allowed for in churches, office-buildings, public halls, etc., where the space between beams is filled with arched brickwork, or straight hollow-brick arches, and then covered over with concrete. A careful estimate, however, should be made of the exact weight of construction per square foot, including the ironwork, and to this should be added 70 pounds per square foot, which is the greatest load likely ever to be produced if packed solidly with people. Furniture rarely weighs as much, though heavy safes should be provided for separately. The load on roofs should be 30 pounds additional to the weight of construction, to provide for the weight of snow or wind. Look out for tanks, etc., on roofs. Plastered ceilings hanging from roofs add about 10 pounds per square foot, and slate about the same. Where a different load than given in the Table must be provided for, the distance between centres of beams can be reduced, proportionally from the next greater load; or the weight on each beam can be figured and the beam treated as a girder, supported sideways, in that case using Table XV. Both tables are calculated for the beams not to deflect sufficiently to crack plastering.

How to use Table XIV. The use of Table XIV is very simple. Supposing we have a span of 24 feet and a load of 150 pounds per square foot. We pass down the vertical line 24′ 0″ and strike first the 12″—96 pounds beam, which (for 150 pounds) is opposite (and three-quarters way between) 3′ 0″ and 3′ 4″ therefore 3′ 3″ from centres. The next beam is the 12″—120 pounds beam 4′ 0″ from centres; then the 12″—125 pounds beam 4′ 1″ from centres; then the 15″—125 pounds beam 5′ 0″ from centres and so on. It is simply a question, therefore, which "distance from centres" is most desirable and as a rule in fireproof buildings it is desirable to keep these as near alike as possible, so as not to have too many different spans of beam arches and centres. If economy is the only question, we divide the weight of beam by its distance from centres, and the curve giving the smallest result is, of course, the cheapest. Supposing, however, that we desire all distances from centres alike, say 5 feet. In that case we pass down the 150-pound column to and then along the horizontal line 5′ 0″ till we strike the vertical (span) line, in this case 24′ 0″, and then take the cheapest beam to the

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sufficiently teach the student the theory and method of using all sections.
But little, if any, structural iron is rolled now-a-days, and the shapes of steel are varied so often, it would anyhow be impossible to cover all of them permanently in a book of this general character. Fortunately the steel companies all issue elaborate hand-books from time to time, and these should always be used in actual practice.

right of the point of intersection. Thus, in our case the nearest beam would be 15″—125 pounds; next comes 12¼″—170 pounds; then 15″—150 pounds, etc. As the nearest beam is the lightest in this case, we should select it. The weight of a beam is always given per yard of length. The reason for this is that a square inch of wrought-iron, one yard long, weighs exactly 10 pounds. Therefore if we know the weight per yard in pounds we divide it by ten to obtain the exact area of cross-section in square inches; or if we know the area, we multiply by ten and obtain the exact weight per yard.

How to use Table XV. The use of Table XV, is very similar to that of Table XIII, but that the safe uniform load is given (in the first column) in tons of 2000 pounds each. The continuation of the two 20″ beams up to 42 feet span is given in the separate table, in the lower right-hand corner. To illustrate the Table: if we have a span of say 21 feet we pass down its vertical line; the first curve we strike is the 10½″—90 pounds beam, which is three-quarters space beyond the horizontal line 5 (tons); therefore a 10½″—90 pounds beam at 21 feet span will carry safely 5¾ tons uniform load, and will not deflect sufficiently to crack plaster. (Each full horizontal space represents one ton). The next beam at 21 feet span is 10½″—105 pounds, which will safely carry 6½ tons. Then comes the 12″—96 pounds beam, which will safely carry 7 tons, and so on down to the 20″—272 pounds beam, which will safely carry 33¾ tons.

If we know the span (say 17 feet) and uniform load (say 7½ tons) to be carried, we pass down the span line 17′ 0″ and then horizontally along the load line 7½ till they meet, which in our case is at the 9″—125 pounds beam; we can use this beam or any cheaper beam, whose curve is under it. We pass over the different curves *under* it, and find the cheapest to be the 12″—96 pounds beam, which we, of course, use.

Iron beams must be scraped clean of rust and be well painted. They should not be exposed to dampness, nor to salt air, or they will deteriorate and lose strength rapidly. The best method is to have all rolled iron or steel cleaned and inspected at the mill, then coated with a heavy coat of linseed oil before shipment, then a coat of paint or preservative on arrival, and a final coat after setting. Before each of the three coatings remove all scales, rust, sand and dirt.

Steel beams. Steel beams are used almost exclusively to-day, as they are cheaper and stronger, though not so reliable as wrought-iron. They are cheaper to manufacture than iron beams, as they are made directly from the pig and practically in one process; while with iron beams the ore is first converted into cast-iron, then puddled into the muck-bar, re-heated, and then rolled. Steel beams, however, are not apt to be of uniform quality. Some may

be even very brittle; they are, however, very much stronger than iron (fully 25 per cent. stronger), but as their deflection is only about 7, 3 per cent. less than that of iron beams, there is not so much relative economy of material possible in their use. If steel beams are used they can be spaced one quarter distance (between centres) farther apart than given in Table XIV for iron beams; or they will safely carry one-quarter more load than given in Table XV; but in no case, where full load is allowed, must the span in feet, (of steel beams), exceed twice the depth in inches. With full safe loads the deflection of steel beams will always be greater than that of iron beams (about ⅛ larger). Where, therefore, it is desirable not to have a greater deflection than with iron beams, add only 7½ per cent. to the distances between centres or "safe loads" as given in Tables for iron beams, instead of 25 per cent.

The strength and consistency of steel beams is variable. It has been found in some cases that steel beams broke suddenly when jarred, (that is, were very brittle,) though test pieces off the ends of these same beams gave very satisfactory results. If steel is used, not only should samples of each rolling be carefully tested, for tenacity, ductility, elasticity, elongation, etc., but now and then the whole beam itself should be tested by actual loading. It will be readily seen that the expense of such tests would bar the use of steel, but no architect can afford to take any chances in such an important part of his building.

Many writers even claim, that, "within the elastic limit," the additional stiffness of steel over iron does not appear; and that it is only beyond this limit that steel is somewhat stiffer than iron. Nevertheless, better or not, steel is here, and here to stay, while wrought-iron is practically unobtainable.

Lateral Flexure in Beams. In using iron and steel beams it is very important that they be supported sideways, so as not to yield to lateral flexure. Where the beams are isolated and unsupported sideways, the safe load must be diminished. Just how much to diminish this load is the question. The practice amongst iron workers is to consider the top flange as a column of the full length of the span, obliged to yield sideways, and with a load equal to the greatest strain on the flange. Modifying, therefore, Formula (3) to meet this view, we should have:

Beams not braced sideways.

$$w_1 = \frac{w}{1 + \frac{y \cdot L^2}{b^2}} \tag{78}$$

Where $w_1 =$ the safe load, in pounds, on a beam, girder, lintel or straight arch, etc., unsupported sideways.

Where $w =$ the safe load, in pounds, on a beam, lintel or straight arch supported sideways.

Where L = the length of clear span, in feet, that beam, etc., is unsupported sideways.

Where b = the least breadth in inches of top flange, or least thickness of beam, lintel or arch.

Where y = a constant, as found in Table XVI.

(In place of w we can use r = the moment of resistance of beam supported sideways, and in place of w_1 we use r_1 = the moment of resistance of beam *not* supported sideways.)

The above practice, however, would seem to diminish the weight unnecessarily, particularly where the beam, girder, etc., is of uniform section throughout; for while the beam in that case, would, be equally strong at all points, it would be strained to the maximum compression only at the point of greatest bending-moment, the strain diminishing towards each support, where the compression would

TABLE XVI.

VALUE OF Y IN FORMULA (78).

Material of beam, girder, lintel, straight arch, etc.	Value of y for girders, beams, etc., of *variable* cross-sections.	Value of y for beams, girders, lintels, straight arches, etc., of *uniform* cross-section throughout.
Cast-Iron	0,5184	0,2304
Wrought-Iron	0,0432	0,0192
Steel	0,0346	0,0154
Wood	0,5702	0,2534
Stone	3,4560	1,5360
Brick	5,7024	2,5344

cease entirely. To consider, therefore, the whole as a long column carrying a weight equal to this maximum compressive strain, seems unreasonable. Box has shown, however, that the maximum tendency to deflect laterally is when we consider the top flange (or top half in rectangular beams, lintels and straight arches) as a column equal to two thirds of the span (unsupported sideways) loaded with a weight equal to one-third of the greatest compressive strain at any point. This greatest compressive strain is always at the point of greatest bending moment (usually the centre of span), and is equal to the area of top flange, multiplied by $\left(\frac{c}{f}\right)$[1]. In case of plate girders the angle-irons and part of web between angle-irons should be included in the area. Box's theory is given in Formula (5); if then we take

[1] This is not quite correct. The greatest compressive strain is really a little less, as will be explained in Vol. II.

one-third of this "maximum tendency to deflect" as safe, we should have the same Formula as (78) but with a smaller value for y. The writer would recommend using the larger value for y, where, as in plate girders, trusses, etc., the section of top flange or chord is diminished, varying according to the compressive strain at each point; and using the smaller value for y, where the section of beam, girder or top chord is uniform throughout.

Use of Table XVI.

Thus the $10\frac{1}{2}''-90$ pounds beam at 20 feet span will safely carry (if supported sideways) a uniform load of 5,9 tons or 11800 pounds (see Table XV.) The width of flange being $4\frac{1}{2}''$, and this width and its thickness, of course, being uniform throughout the entire length of beam, we use the smaller value for y (second column) and have for the actual safe uniform load, if the beam is not secured against lateral flexure:

$$w_1 = \frac{11800}{1 + 0{,}0192.\ \frac{20^2}{4\frac{1}{2}^2}} = \frac{11800}{1 + 0{,}379}$$

$$= \frac{11800}{1{,}379} = 8557 \text{ pounds, or } 4{,}28 \text{ tons.}$$

Had we used the larger value for $y = 0{,}0432$ we should have had

$$w_1 = \frac{11800}{1 + 0{,}854} = \frac{11800}{1{,}854} = 6365 \text{ pounds, or } 3{,}18 \text{ tons,}$$

which closely resembles the value (3,29) given in the Iron Companies' hand-books, but is an excessive reduction under the circumstances.

Doubled Beams.

Where two or more beams are used to carry the same load, as girders for instance, or as lintels in a wall, they should be firmly bolted together, with cast-iron separators between. In this case use for b in Formula (78) the total width, from outside to outside of all flanges, and including in b the spaces between. The separators are made to fit exactly between the inner sides of webs and top and bottom flanges. The separator is swelled out for the bolt to pass through. Sometimes there are two bolts to each separator, but it is better (weakening the beam less) to have but one at the centre of web. The size of separators and bolts vary, of course, to suit the different sizes of beams. They should be placed apart about as frequently as twenty times the width of flange of a single beam. Where beams are placed in a floor, the floor arches usually provide the side bracing. But in order to avoid unequal deflections, and possible cracks in the arches, (from unequal or moving loads or from vibrations) and also to take up the thrust on the end beams of each floor, it is necessary to place

Tie-rods.

lines of tie-rods across the entire line of beams. The size of these rods can be calculated as already explained in the Chapter on Arches (p. 169); they are usually made, however, from ⅝″ to ⅞″ diameter. Each rod extends from the outside web of one beam to the outside web of the next beam. The next rod is a little to one side of it, so that the rods do not really form one straight line, but every other rod falls in the same line. Care must be taken not to get the rods too long, or there will have to be several washers under the head and nut, making a very unsightly job, to say the least. Contractors will do this, however, for the sake of the convenience of ordering the rods all of one or two lengths. Where, therefore, the beams are not spaced evenly the contractor should be warned against this. One end of the rod has a "head" welded on, the other has a "screw-end," which need not be "up-set;" the nut is screwed along this end, thus forcing both nut and head to bear against the beams solidly. The distance between lines of tie-rods, would depend somewhat on our calculation, if made; the usual practice, however, is to place them apart a distance equal to about twenty times the width of flange of a single beam.

Flitch-plate Girder. Sometimes where wooden girders have heavier loads to carry than they are capable of doing, and yet iron girders cannot be afforded, a sheet of plate-iron is bolted between two wooden girders. In this case care must be taken to so proportion the iron, that in taking its share of the load, it will *deflect* equally with the wooden girders, otherwise the bolts would surely shear off, or else crush and tear the wood.

We consider the two wooden girders as one girder and calculate (or read from Table XIII) their safe load, taking care not to exceed 0,03 inches of deflection per foot of span. We then, from Table VII or Formulæ (37) to (41) obtain the exact amount of their deflection under this load. We now calculate the iron plate, for deflection only, inserting the above amount of deflection, and for the load the balance to be borne by the iron-work. An example will best illustrate this:

Example.

A Flitch-plate girder of 20-*foot span consists of two Georgia pine beams each* 6″ × 16″ *with a sheet of plate-iron* 16″ *deep bolted between them. The girder carries a load of* 13000 *pounds at its centre; of what thickness should the plate be? The girder supports a plastered ceiling.*

Strength of wooden part. From Table XIII we find that a Georgia pine beam 6″ × 16″ of 20-foot span will safely carry without cracking plaster 7980 pounds uniform load, or 3990 pounds at its centre (See Case (6) Table VII,) so that the two wooden beams together carry 7980 pounds of the load, leaving a balance of 5020 pounds for the iron plate to carry. The deflection of a 20-foot span Georgia pine beam 6″ × 16″ with 3990 pounds centre load will be, Formula (40)

$$\delta = \frac{1}{48} \cdot \frac{3990.\ 240^3}{e.\ \ i.}$$

e for Georgia pine (Table IV) is = 1200000 and

$$i = \frac{b.d^3}{12} \text{ (Table I. section No. 2), or}$$

$$i = \frac{6.16^3}{12} = 2048, \text{ therefore}$$

$$\delta = \frac{1}{48} \cdot \frac{3990.\ \ 240^3}{1200000.\ 2048} = 0{,}47''$$

Size of Iron Plate. We now have a wrought-iron plate which must carry 5020 pounds centre load, of a span of 20 feet, 16″ deep, and must deflect under this load only 0,47″.

Inserting these values in Formula (40) we have:

$$0{,}47 = \frac{1}{48} \cdot \frac{5020.240^3}{e.\ \ i.}$$

From Table IV we have for wrought-iron

$$e = 27000000$$

While for *i*, we have (Table I. Section No. 2)

$$i = \frac{b.\ d^3}{12} = \frac{b.\ 16^3}{12} = 341.b$$

Inserting these values and transposing we have:

$$b = \frac{5020.\ \ \ 240^3}{48.\ 27000000.\ 341.\ 0{,}47} = 0{,}33$$

Or the plate would have to be $\frac{1}{3}''$ × 16″. Now to make sure that this deflection does not cause too great fibre strains in the iron, we can calculate these from Formulæ (18) and (22). The bending moment at the centre will be (22)

$$m = \frac{5020.\ 240}{4} = 301200$$

The moment of resistance will be (Table I. Section No. 2)

$$r = \frac{b.\ d^2}{6} = \frac{0{,}33.16^2}{6} = 14$$

And from (18) $\dfrac{m}{\left(\dfrac{k}{f}\right)} = r$, or transposing and inserting values,

$$\left(\frac{k}{f}\right) = \frac{301200}{14} = 21514 \text{ pounds.}$$

As the safe modulus of rupture of wrought-iron is only 12000 pounds (Table IV) we must increase the thickness of our plate. Let us call the plate $\frac{5}{8}'' \times 16''$, we should then have

$$r = \frac{5.\ 16^2}{8.\ 6} = 26{,}67 \text{ and}$$

$$\left(\frac{k}{f}\right) = \frac{301200}{26{,}67} = 11256.$$

So that the plate would be a trifle too strong. This would mean that both plate and beams would deflect less. The exact amount might be obtained by experimenting, allowing the beams to carry a little less and the plate a little more, until their deflections were the same, but such a calculation would have no practical value. We know that the deflection will be less than 0, 47″ and further, that plastering would not crack, unless the deflection exceeded $\frac{5}{8}$ of an inch (Formula 28) as

$$20.0{,}03 = 0,\ 6''$$

Size of Bolts. In regard to the bolts, the best position for them would, of course, be along the neutral axis, that is, at half the height of the beam. For here there would be no strain on them. But to place them with sufficient frequency along this line would tend to weaken it too much, encouraging the destruction of the beam from

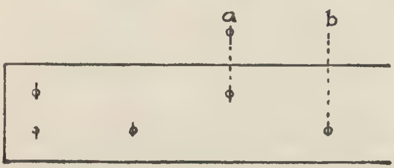


Fig. 137.

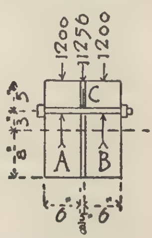


Fig. 138.

longitudinal shearing along this line. For this reason the bolts are placed, alternating, above and below the line, forming two lines of bolts, as shown in Fig. (137). The end bolts are doubled as shown; the horizontal distance, $a-b$, between two bolts should be about equal to the depth of the beam. If we place the bolts in our exam-

ple, say 3″ above and 3″ below the neutral axis, we can readily calculate the size required. Take a cross-section of the beam (Fig. 138) showing one of the upper bolts. Now the fibre strains along the upper edge of the girder, we know are $\left(\frac{k}{f}\right)$ or 1200 pounds per square inch, for the wood, and we just found the balance of the load coming on the iron would strain this on the extreme upper edge $= 11256$ pounds per square inch. As the centre line of the bolt is only 3″ from the neutral axis or $\frac{3}{8}$ of the distance from neutral axis to the extreme upper fibres, the strains on the fibres along this line will be, of course, on the wood $\frac{3}{8}$ of 1200, or 450 pounds per square inch: and on the iron $\frac{3}{8}$ of $11256 = 4221$ pounds per square inch. Now, supposing the bolt to be 1″ in diameter. It then presses on each side against a surface of wood $= 1'' \times 6''$ or $=$ six square inches. The fibre strain being 450 pounds per square inch, the total pressure on the bolt from the wood, each side, is:

$$6.450 = 2700 \text{ pounds.}$$

On the iron we have a surface of $1'' \times \frac{5}{8}'' = \frac{5}{8}$ square inches. And as the fibre strain at the bolt is 4221, the total strain on the bolt from the iron is $= \frac{5}{8}.\ 4221 = 2638$ pounds. Or, our bolt virtually becomes a beam of wrought-iron, circular and of 1″ diameter in cross-section, supported at the points A and B, which are $6\frac{5}{8}''$ apart, and loaded on its centre C with a weight of 2638 pounds.

Therefore we have, at centre, bending-moment (Formula 22)

$$m = \frac{2638.\ 6\frac{5}{8}}{4} = 4369.$$

From Table I, Section No. 7, we know that for a circular section, the moment of resistance is,

$$r = \frac{11}{14} \cdot \mathfrak{r}^3 = \frac{11}{14} \cdot \left(\frac{1}{2}\right)^3 = 0{,}098$$

Now for solid circular bolts, and which are acted on really along their whole length it is customary to take $\left(\frac{k}{f}\right)$ the safe modulus of rupture rather higher than for beams. Where the bolts or pins have heads and nuts at their ends firmly holding together the parts acting across them they are taken at 18000 pounds for steel and at 15000 pounds for iron. We have therefore transposing (Formula 18) for the required moment of resistance

$$r = \frac{4369}{15000} = 0{,}291.$$ Inserting this value for r in the

above we have for the radius of bolt,

$$\frac{11}{14} \cdot r^3 = 0{,}291 \text{ and}$$

$$r = \sqrt[3]{\frac{14}{11} \cdot 0{,}291} = \sqrt[3]{0{,}3704}$$

$$= 0{,}718''$$

Or the diameter of bolt should be 1,436″ or say 1 7-16″. But 1″ will be quite ample, as we must remember that the strains calculated will come only on the one bolt at the centre of span of beam; and that, as the beam remains of same cross section its whole length the extreme fibre strains decrease rapidly towards the supports, and therefore also the strains on the bolts. The end bolts are doubled however, to resist the starting there of a tendency to longitudinal shearing. We might further calculate the danger of the bolt crushing the iron plate at its bearing against it; or crushing the wood each side; or the danger of the iron bolt being sheared off by the iron plate between the wooden beams; or the danger of the iron bolt shearing off the wood in front of it, that is tearing its way out through the wood; but the strains are so small, that we can readily see that none of these dangers exist.

Continuous Girders. When girders run over three or more supports in one piece, that is, are not cut apart or jointed over the supports, the existing strains and reactions of ordinary girders, are very much altered. These are known as "continuous girders." If we have (Fig. 143) three supports, and run a continuous girder over them in one piece and load the girder on each side it will act

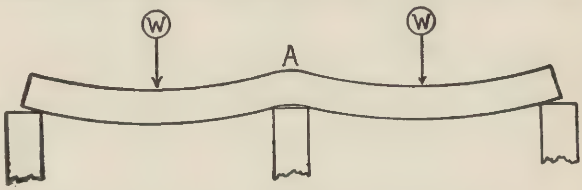


Fig. 143.

as shown in Fig. 143; if the girder is cut it will act as shown in Fig. 144. Very little thought will show that the fibres at A not being able to separate in the first case, though they want to, must cause considerable tension in the upper fibres at A. This tension, of course, takes up or counterbalances part of the compression existing there, and the result is that the first or continuous girder (Fig. 143) is considerably stronger, that is, it is less strained and considerably stiffer, than the sectional or jointed girder (Fig. 144). Again we can readily see that the great tension and conflict of the opposite stresses at A would tend to cause more pressure on the central post in Fig. 143, than on the central post in Fig. 144, and this, in fact, is the case.

In Table XVII, pages 218 and 219, are given the various formulæ for reactions, greatest bending moments and deflections, for the most usual cases of continuous girders. The architect can, if

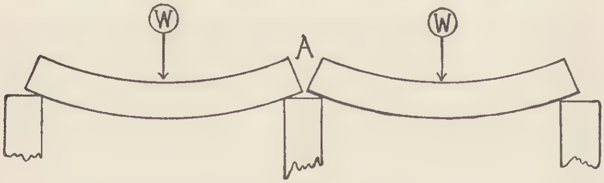


Fig. 144.

he wishes, neglect to allow for the additional strength and stiffness of continuous girders, as both are on the safe side. But he must never overlook the fact *that the central reactions are much greater*, or in other words, that the end supports carry *less*, and the central supports carry *more*, than when the girders are cut.

Variation of Reactions.

Bending moments can be figured, at any desired point along a continuous girder, as usual, subtracting from the sum of the reactions on one side multiplied by their respective distances from the point, the sum of all the weights on the same side, multiplied by their respective distances from the point. Sometimes the result will be negative, which means a reversal of the usual stresses and strains. Otherwise the rules and formulæ hold good, the same as for other girders or beams. Table XVII gives all necessary information at a glance.

Strength is frequently added to a girder or beam by trussing it, as shown in Table XVIII, pages 220 and 221. One or two struts are placed against the lower edge of a beam and a rod passed over them and secured to each end of the beam; by stretching this rod the beam becomes the compression chord of a truss and also a continuous girder running over one or two supports. There must therefore be enough material in the beam to stand the compression, and *in addition* to this enough to stand the transverse strains on the continuous girder. If the loads are concentrated immediately over the braces, there will be no transverse strain whatever, but the braces will be compressed the full amount of the respective loads on each. In the case of uniform loads, transverse strains cannot be avoided, of course, but where loads are concentrated the struts should always be placed immediately under them. Even where loads are placed very unevenly, it is better to have the panels of the truss irregular, thus avoiding cross or transverse strains. This same rule holds good in designing trusses of any kind.

Trussed Beams.

Struts placed under load.

Necessary Conditions. Table XVIII shows very clearly the amount and kind of strains in each part of trussed beams. Where there are two struts and they are of any length care must be taken by diagonal braces or otherwise, to keep the lower ends of braces from tipping towards each other. Theoretically they cannot tip, but practically, sometimes, they do. Care must be taken that the beam is braced sideways, or else it must be figured for its safety against lateral flexure (Formula 5.) Then it must have material enough not to shear off at supports, nor to crush its under side where lying on support. The ends of rods must have sufficient bearing not to crush the wood. Iron shoes are sometimes used, but if very large are apt to rot the wood. In that case it is well to have a few small holes in the shoes, to allow ventilation to end of timber. If iron straps and bolts are used at the end, care must be taken that the strap does not tear apart at bolt holes; that it does not crush itself against bolts; that it does not shear off the bolts, and that it does not crush in the end of timber. Care must also be taken to have enough bolts, so that they do not crush the wood before them, and to keep the bolts from shearing out, that is tearing out the wood before them.

Importance of Joints. In all trusses and trussed works the joints must be carefully designed to cover all these points. Many architects give tremendous sizes for timbers and rods in trusses, thus adding unnecessary weight, but when it comes to the joint, they overlook it, and then are surprised when the truss gives out. The next time they add more timber and more iron, till they learn the lesson. It must be remembered that the strength of a truss is only equal to the strength of its weakest part, be that part a member or only a part of a joint. This subject will be fully dealt with in the chapter on Trusses.

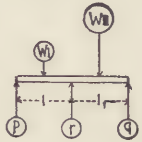
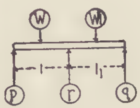
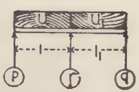
Depth Desirable. The deeper the truss is made, that is, the further we separate the top and bottom chords, the stronger will it be; besides additional depth adds very much to the stiffness of a truss.

Deflection of Girders and Beams. All trussed beams, and all trusses should be "cambered up," that is, built up above their natural lines sufficiently to allow for settling back into their correct lines, when loaded. The amount of the camber should equal the calculated deflection. For all beams, girders, etc., of uniform cross-section throughout, the deflection can be calculated from Formulæ (37) to (42) according to the manner of loading. For wrought-iron beams and plate-girders of uniform cross-section throughout, the deflection can be calculated from the same formulæ; where, however, the load is uniform and it is desired to simplify the calculation, the deflection can be quite closely calculated from the following Formula:

Uniform Cross-section and Load.

$$\delta = \frac{L^2}{75 . d} \qquad (79)$$

TABLE XVII. — CONTINUOUS GIRDERS.

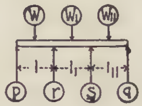
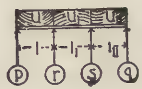
Illustration.	Description.	Amount of Reactions.	Amount of Greatest Bending Moments.	Amount of Greatest Deflections.
	Two equal spans each carrying a central load but loads not equal. $w_{I} \lesseqgtr w_{II}$ $l = l_{I}$	*Left reaction.* $p = \frac{13 \cdot w_{I} - 3 \cdot w_{II}}{32}$ *Centre reaction.* $r = \frac{11}{16} \cdot (w_{I} + w_{II})$ *Right reaction.* $q = \frac{13 \cdot w_{II} - 3 \cdot w_{I}}{32}$	*Located at r* $m = \frac{3}{32} \cdot l \cdot (w_{I} + w_{II})$	*Deflection in left span l* $\delta_{I} = \frac{23 \cdot w_{I} - 9 \cdot w_{II}}{1536 \cdot e \cdot i} \cdot l^3$ *Deflection in right span l_{I}* $\delta_{II} = \frac{23 \cdot w_{II} - 9 \cdot w_{I}}{1536 \cdot e \cdot i} \cdot l^3$
	Two equal spans each carrying a central load $= w$, loads equal. $w = w_{I}$ $l = l_{I}$	*End reactions.* $p = q = \frac{5}{16} \cdot w$ *or* $= \frac{5}{32} \cdot (w + w_{I})$ *Centre reaction.* $r = \frac{11}{8} \cdot w$ *or* $= \frac{11}{16} \cdot (w + w_{I})$	*Located at r* $m = \frac{3}{16} \cdot l \cdot w$ *or* $= \frac{3}{32} \cdot l \cdot (w + w_{I})$	*Deflection in either span* $\delta = \frac{w \cdot l^3}{110 \cdot e \cdot i}$ *or* $= \frac{l^3}{220 \cdot e \cdot i} \cdot (w + w_{I})$
	Two equal spans each loaded with a	*End reactions.* $p = q = \frac{3}{8} \cdot u$ *or* $= \frac{3}{16} \cdot (u + u_{I})$	*Located at r* $m = \frac{u \cdot l}{8}$ *or*	*Deflection in either span* $\delta = \frac{u \cdot l^3}{185 \cdot e \cdot i}$ *or*

Where $\delta, \delta_{I}, \delta_{II}$ = the amount of deflection in inches, if girder of uniform cross-section throughout.

" e = the modulus of elasticity of the material, in pounds-inch, (see Table IV).

" i = the moment of inertia of the cross-section, in inches, (see Table I,

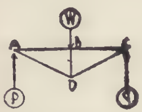
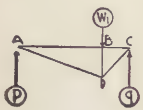
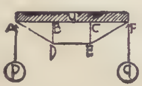
[Table XVII continued.]

	uniform load $= u$ $u = u_{\text{I}}$ $l = l_{\text{I}}$	*Centre reaction.* $r = \frac{5}{4} \cdot u$ *or* $= \frac{5}{8} \cdot (u + u_{\text{I}})$	$= \frac{l}{16} \cdot (u + u_{\text{I}})$	$= \frac{l^3}{370 \cdot e \cdot i} \cdot (u + u_{\text{I}})$
	Three equal spans each carrying a central load $= w$, all loads equal $w = w_{\text{I}} = w_{\text{II}}$ $l = l_{\text{I}} = l_{\text{II}}$	*End reactions.* $p = q = \frac{7}{20} \cdot w$ *or* $= \frac{7}{60} \cdot (w + w_{\text{I}} + w_{\text{II}})$ *Central reactions.* $r = s = \frac{23}{20} \cdot w$ *or* $= \frac{23}{60} \cdot (w + w_{\text{I}} + w_{\text{II}})$	*Located at r or s* $m = \frac{3 \cdot w \cdot l}{20}$ *or* $= \frac{l}{20} \cdot (w + w_{\text{I}} + w_{\text{II}})$	*Deflection in central span* $\delta = \frac{w \cdot l^3}{480 \cdot e \cdot i}$ *or* $= \frac{l^3}{1440 \cdot e \cdot i} \cdot (w + w_{\text{I}} + w_{\text{II}})$ *Deflection in either end span* $\delta = \frac{w \cdot l^3}{87 \cdot e \cdot i}$ *or* $= \frac{l^3}{261 \cdot e \cdot i} \cdot (w + w_{\text{I}} + w_{\text{II}})$
	Three equal spans each loaded with a uniform load $= u$ $u = u_{\text{I}} = u_{\text{II}}$ $l = l_{\text{I}} = l_{\text{II}}$	*End reactions.* $p = q = \frac{2}{5} \cdot u$ *or* $= \frac{2}{15} \cdot (u + u_{\text{I}} + u_{\text{II}})$ *Central reactions.* $r = s = \frac{11}{10} \cdot u$ *or* $= \frac{11}{30} \cdot (u + u_{\text{I}} + u_{\text{II}})$	*Located at r or s* $m = \frac{u \cdot l}{10}$ *or* $= \frac{l}{30} \cdot (u + u_{\text{I}} + u_{\text{II}})$	*Deflection in central span* $\delta = \frac{u \cdot l^3}{1920 \cdot e \cdot i}$ *or* $= \frac{l^3}{5760 \cdot e \cdot i} \cdot (u + u_{\text{I}} + u_{\text{II}})$ *Deflection in either end span* $\delta = \frac{u \cdot l^3}{145 \cdot e \cdot i}$ *or* $= \frac{l^3}{435 \cdot e \cdot i} \cdot (u + u_{\text{I}} + u_{\text{II}})$

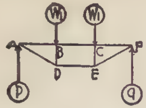
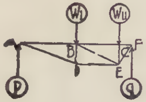
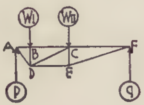
Where $w, w_{\text{I}}, w_{\text{II}} =$ central concentrated loads in pounds, on either span, being equal, when so stated.
" $l, l_{\text{I}}, l_{\text{II}} =$ the length of respective spans, in inches, all being equal.
" $u, u_{\text{I}}, u_{\text{II}} =$ uniform loads on each span, in pounds, all being equal.
" $p, r, s, q, =$ the amount of respective reactions, in pounds.
" $m =$ the bending moment, in pounds-inch.

TABLE XVIII.

TRUSSED BEAMS.

Illustrations.	Description.	Compression in Struts.	Compression in Beam.	Tension in Rods.	Amount of Reactions.
	*Trussed Beam* with one centre load $= w$ $A\,B = B\,C$	*Compression in* $B\,D$ $= + w$	*Compression in* $A\,B$ $= + \frac{w}{2} \cdot \frac{A\,B}{B\,D}$ *Compression in* $B\,C$ same as in $A\,B$	*Tension in* $A\,D$ $= - \frac{w}{2} \cdot \frac{A\,D}{B\,D}$ *Tension in* $C\,D$ same as in $A\,D$	$p = \frac{w}{2}$ $q = \frac{w}{2}$
	*Trussed Beam* with one load $= w_1$ (not central,) at any point $A\,B \lesseqgtr B\,C$	*Compression in* $B\,D$ $= + w_1$	*Compression in* $A\,B$ $= + p \cdot \frac{A\,B}{B\,D}$ *Compression in* $B\,C$ same as in $A\,B$, or $= + q \cdot \frac{B\,C}{B\,D}$	*Tension in* $A\,D$ $= - p \cdot \frac{A\,D}{B\,D}$ *Tension in* $C\,D$ $= - q \cdot \frac{C\,D}{B\,D}$	$p = w_1 \cdot \frac{B\,C}{A\,C}$ $q = w_1 \cdot \frac{A\,B}{A\,C}$
	*Trussed Beam* with uniform load $= u$ and one central strut. $A\,B = B\,C$	*Compression in* $B\,D$ $= + \frac{5}{8} \cdot u$	*Compression in* $A\,B$ $= + \frac{5}{16} \cdot u \cdot \frac{A\,B}{B\,D}$ *Compression in* $B\,C$ same as in $A\,B$	*Tension in* $A\,D$ $= - \frac{5}{16} \cdot u \cdot \frac{A\,D}{B\,D}$ *Tension in* $C\,D$ same as in $A\,D$	$p = \frac{u}{2}$ $q = \frac{u}{2}$
	*Trussed Beam* with uniform load $= u$ and two struts, dividing beam into three equal parts. $A\,B = B\,C = C\,F$	*Compression in* $B\,D$ $= + \frac{11}{30} \cdot u$ *Compression in* $C\,E$ same as in $B\,D$	*Compression in* $A\,B$ $= + \frac{11}{30} \cdot u \cdot \frac{A\,B}{B\,D}$ *Compression in* $B\,C$ *and* *Compression in* $C\,F$ same as in $A\,B$	*Tension in* $A\,D$ $= - \frac{11}{30} \cdot u \cdot \frac{A\,D}{B\,D}$ *Tension in* $D\,E$ $= -$ compression in $A\,B$ *Tension in* $F\,E$ same as in $A\,D$	$p = \frac{u}{2}$ $q = \frac{u}{2}$

[Table XVIII continued.]

	*Trussed Beam* with two equal loads each $= w_{\mathrm{I}}$ and two struts at equal distances from ends. $AB = CF$	*Compression in* BD $= + w_{\mathrm{I}}$ *Compression in* CE same as in BD	*Compression in* AB $= + w_{\mathrm{I}} \cdot \frac{AB}{BD}$ *Compression in* BC *and* *Compression in* CF same as in AB	*Tension in* AD $= - w_{\mathrm{I}} \cdot \frac{AD}{BD}$ *Tension in* DE $=$ — compression in AB *Tension in* FE same as in AD	$p = w_{\mathrm{I}}$ $q = w_{\mathrm{I}}$
	*Trussed Beam* with two unequal loads w_{I} and w_{II} at any points. Providing p smaller than w_{I} and q larger than w_{II}	*Compression in* BD $= + p$ *Compression in* BE $= + (q - w_{\mathrm{II}}) \cdot \frac{BE}{BD}$ *Compression in* CE $= + w_{\mathrm{II}}$	*Compression in* AB $= + p \cdot \frac{AB}{BD}$ *Compression in* BC same as in CF *Compression in* CF $= + q \cdot \frac{CF}{BD}$	*Tension in* AD $= - p \cdot \frac{AD}{BD}$ *Tension in* DE $=$ — compression in AB *Tension in* FE $= - q \cdot \frac{FE}{BD}$	$p = \frac{w_{\mathrm{I}} \cdot BF + w_{\mathrm{II}} \cdot CF}{AF}$ $q = \frac{w_{\mathrm{I}} \cdot AB + w_{\mathrm{II}} \cdot AC}{AF}$
	*Trussed Beam* with two unequal loads w_{I} and w_{II} at any points. Providing p larger than w_{I} and q smaller than w_{II}	*Compression in* BD $= + w_{\mathrm{I}}$ *Compression in* CD $= + (p - w_{\mathrm{I}}) \cdot \frac{CD}{BD}$ *Compression in* CE $= + q$	*Compression in* AB $= + p \cdot \frac{AB}{BD}$ *Compression in* BC same as in AB *Compression in* CF $= + q \cdot \frac{CF}{BD}$	*Tension in* AD $= - p \cdot \frac{AD}{BD}$ *Tension in* DE $=$ — compression in CF *Tension in* FE $= - q \cdot \frac{FE}{BD}$	$p = \frac{w_{\mathrm{I}} \cdot BF + w_{\mathrm{II}} \cdot CF}{AF}$ $q = \frac{w_{\mathrm{I}} \cdot AB + w_{\mathrm{II}} \cdot AC}{AF}$

Where $p =$ the amount of the left reaction, in pounds.
" $q =$ the amount of right reaction, in pounds.
" $w, w_{\mathrm{I}}, w_{\mathrm{II}} =$ concentrated loads, in pounds.
" $u =$ uniform load, in pounds, over whole beam.
" $AB, BC, CF, BD, BE, CD, CE, AD, DE, FE =$ the length of ***longitudinal central*** axes of these pieces, and must *all* be expressed *uniformly*, that is, *all* expressed either in feet or inches.

The amounts of compression in either struts or beam-parts will be the total compression in each, expressed in pounds; to obtain the compression *per square inch*, divide the amount by the area of cross-section of the strut or part.

The amounts of tension in rods will be the total tension in each part, expressed in pounds; to obtain the tension *per square inch*, divide the amount by the area of cross-section of rod.

Where δ = the greatest deflection at centre, in inches, of a wrought-iron beam or plate girder of uniform cross-section throughout, and carrying its total safe uniform load, calculated for rupture only.

Where L = the length of span, in feet.

Where d = the total depth of beam or girder in inches.

If beam or plate girder is of steel, use $64\frac{1}{2}$ instead of 75.

If the load is not uniform, change the result, as provided in cases (1) to (8), Table VII.

For a centre load we should use $93\frac{3}{4}$ in place of 75 or

Uniform Cross-section, Centre Load.

$$\delta = \frac{L^2}{93\frac{3}{4} \cdot d} \tag{80}$$

Where values are the same, as for Formula (79) except that beam or girder carries its total safe centre load, calculated for rupture only.

If beam or girder is of steel use $80\frac{2}{3}$ instead of $93\frac{3}{4}$.

Therefore not to crack plastering and yet to carry their full safe loads, wrought-iron beams or plate girders should never exceed in length (measured in feet) twice and a quarter times the depth (measured in inches), if the load is uniform, or

Safe length, uniform Cross-section and Load.

$$2\tfrac{1}{4} \cdot d = L \tag{81}$$

Where L = the ultimate length of span (not to crack plastering), in feet, of a wrought-iron beam or plate girder, of uniform cross-section throughout and uniformly loaded with its total safe load.

Where d = the total depth of beam or girder in inches.

If beam or girder is of steel, use 2 instead of $2\frac{1}{4}$.

If the load is central the length in feet should not exceed $2\frac{4}{5}$ times the depth in inches, or

Safe length, uniform Cross-section, Centre Load.

$$2\tfrac{4}{5} \cdot d = L. \tag{82}$$

Where L = the ultimate length of span in feet (not to crack plastering), of a wrought-iron beam or plate girder, of uniform cross-section throughout, and loaded at its centre with its total safe load.

Where d = the total depth of beam or girder in inches.

If beam or girder is of steel use $2\frac{2}{5}$ instead of $2\frac{4}{5}$.

Deepest beam most economical. One thing should always be remembered, when using iron beams, and that is, that the deepest beam is *always* not only the stiffest, but the most economical. For instance, if we find it necessary to use a $10\frac{1}{2}''$ beam — 105 pounds per yard,

it will be cheaper to use instead the 12″ beam — 96 pounds per yard. The latter beam not only weighs 9 pounds per yard less, but it will carry more, and deflect less, owing to its extra two inches of depth. This same rule holds good for nearly all sections.

Deflection of Trusses. To obtain the deflections of trussed beams or girders by the rules already given would be very complicated. For these cases, however, Box gives an approximate rule, which answers every purpose. He calculates the amount of extension in the tension (usually the lower) chord, and the amount of contraction in the compression (usually the upper) chord, due to the strains in each, and from these, obtains the deflections. Of course the *average* strain in each chord must be taken and not the greatest strain at any one point in either. In a truss, where each part is proportioned in size to resist exactly the compressive or tensional strain on the part, every part will, of course, be strained alike; the strain in the compressive member being $= \left(\frac{c}{f}\right)$ per square inch, throughout the whole length, and in the tension member $= \left(\frac{t}{f}\right)$ per square inch, throughout the whole length.

The same holds good practically for plate girders, where the top and bottom flanges are diminished towards the ends, in proportion to the bending moment. But where, as in wrought-iron beams (and in many trusses), the flanges are made, for the sake of convenience, of uniform cross-section throughout their entire length, the "average" strain will, of course, be much less, and consequently the beam or girder stiffer.

Average Strain in Chords. If we construct the graphical representation of the bending moments at each point of beam (as will be explained in the next Chapter) and divide the area of this figure in inch-pounds by the length of span in inches, we will obtain the average strain in either flange, provided the flange is of uniform cross-section throughout, or

Uniform Cross-section.

$$v = \frac{a}{l} \qquad (83)$$

Where $v =$ the *average* strain, in pounds, on top or bottom flange or chord, where beam or girder is of uniform cross-section throughout.

Where $l =$ the length of span, in inches.

Where $a =$ the area in pounds-inch of the graphical figure giving the bending moment at all points of beam.

To obtain the dimensions of this figure measure its base line (or horizontal measurement) in inches, and its height (or vertical measurement) in pounds, assuming the greatest vertical measurement as $=\left(\frac{c}{f}\right)$ or $=\left(\frac{t}{f}\right)$, in pounds, according to which flange we are examining.

Thus, in the case of a uniform load, this figure would be a parabola, with a base of length equal to the span measured in inches, and a height equal to the greatest fibre strains in pounds; the average strain therefore in the compression member of a beam, girder or truss, of uniform cross-section throughout would be, — (remembering that the area of a parabola is equal to two-thirds of the product of its height into its base),

Uniform load and Cross-section.

$$v=\frac{\frac{2}{3}.l.\left(\frac{c}{f}\right)}{l} \text{ or}$$

$$v=\frac{2}{3}.\left(\frac{c}{f}\right) \qquad (84)$$

Where $v=$ the *average* strain, in pounds, in compression flange or chord of a beam, girder or truss of uniform cross-section throughout and carrying its total safe uniform load.

Where $\left(\frac{c}{f}\right)=$ the safe resistance to compression per square inch of the material.

It is supposed, of course, that at the point of greatest bending moment — or where the greatest compression strain exists — that the part is designed to resist or exert a stress $=\left(\frac{c}{f}\right)$ per square inch. If the greatest compression stress is less, insert its value in place of $\left(\frac{c}{f}\right)$. Of course, it must never be greater than $\left(\frac{c}{f}\right)$.

Similarly we should have

Uniform Load and Cross-section.

$$v=\frac{2}{3}.\ \left(\frac{t}{f}\right) \qquad (85)$$

Where $v=$ the *average* strain, in pounds, in tension flange or chord of a beam, girder or truss of uniform cross-section throughout, and carrying its total safe uniform load.

Where $\left(\frac{t}{f}\right)=$ the safe resistance to tension, per square inch, of the material.

It being understood that at the point of greatest bending moment — or where the greatest tension strain exists — that the part is de-

signed to resist or exert a stress $=\left(\frac{t}{f}\right)$ per square inch. If this greatest tensional stress is less than $\left(\frac{t}{f}\right)$ insert its value in its place in Formula (85). Of course, it must never be greater than $\left(\frac{t}{f}\right)$.

For a beam, girder or truss with a load concentrated at the centre, but with flanges or chords of uniform cross-section throughout, the average strain would be just one-half that at the centre; for, the bending-moment graphical-figure will be a triangle, and inserting the values in Formula (83) would give for the compression member:

Centre Load Uniform Cross-Section.

$$v=\tfrac{1}{2}\cdot\left(\frac{c}{f}\right) \tag{86}$$

and for the tension member:

$$v=\tfrac{1}{2}\cdot\left(\frac{t}{f}\right) \tag{87}$$

The meaning of letters being the same as in Formulæ (84) and (85), but the total safe load being concentrated at the centre instead of uniformly distributed.

To obtain the amount of contraction or expansion due to this average strain, use the following Formula:

Expansion or Contraction from Strain.

$$x=\frac{v.\,l}{e} \tag{88}$$

Where $v=$ the *average* strain, in pounds per square inch, in either chord or flange.

Where $l=$ the length of span, in inches.

Where $e=$ the modulus of elasticity of the material, in pounds-inch.

Where $x=$ the total amount of extension or contraction, in inches, of the chord or flange.

Now let us apply the above rules to beams, plate girders, and trussed beams. Taking the case of a beam or plate girder or truss with parallel flanges or chords.

Figure 145 shows the same, after the deflection has taken place. We can now assume approximately, that $C\,A$ is equal to one-half the difference between the contraction of $G\,C$ and the elongation of $H\,B$, or, what amounts to the same thing, that $C\,A$

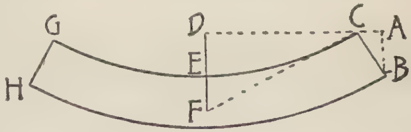


Fig. 145.

is equal to one-half the sum of the contraction of the one and the elongation of the other.

Further, we can assume that approximately, $A\ B = d$ or the depth of beam, and $C\ D = \frac{l}{2}$ or one half the span.

The curve $C\ E\ C$ will approximate a parabola, so that if we draw a tangent $C\ F$ to the same at C, we know that $D\ E = E\ F = \frac{D\ F}{2}$ or $D\ F = 2.\ D\ E$. But as $D\ E$ represents the deflection (δ) of the beam, we have

$$D\ F = 2.\ \delta$$

Now as $C\ F$ is normal to $C\ B$, and $C\ D$ normal to $A\ B$, we know that angles $D\ C\ F = A\ B\ C$; further, as both triangles are right angle triangles, we know that they are similar, therefore:

$$D\ F : C\ A \therefore D\ C : A\ B, \text{ or}$$

$$2.\ \delta : C\ A \therefore \frac{l}{2} : d \text{ or}$$

$$\delta = \frac{C\ A.\frac{l}{2}}{2\ .\ d} = \frac{C\ A.\ l}{4.\ d.}$$

If now we assume the sum of the extension and contraction of the two flanges or chords to be $= x$.

We have $C\ A = \frac{x}{2}$ or

Deflection of Parallel Flanges or Chords, any Cross-section.

$$\delta = \frac{x.\ l}{8.\ d} \qquad (89)$$

Where δ = the deflection, in inches, of a beam, plate girder or truss, with parallel flanges or chords.

Where x = the sum of the amount of extension in tension chord, plus the amount of contraction in compression chord.

Where l = the length of span, in inches.

Where d = the total depth of beam, girder or truss in inches.

Take the case of a wrought-iron plate girder or beam of uniform cross-section throughout carrying its full uniform load, we should have the strain at the centre on the extreme fibres = 12000 pounds per square inch. Now the average strain on both upper and lower flanges would be, Formulæ (84) and (85).

$$v = \tfrac{2}{3}.\ \ 12000 = 8000 \text{ pounds}$$

per square inch. Therefore amount of contraction in upper flange

Formula (88), (and remembering that, from Table IV, $e = 27000000$)

$$x = \frac{8000. l}{27000000} = \frac{l}{3375}$$

The elongation of the bottom flange would be an equal amount, therefore the sum of the two

$$x_1 = 2. x = \frac{2. l}{3375}$$

$$= \frac{l}{1687,5}$$

Inserting these values in Formula (89) we have the deflection

$$\delta = \frac{l^2}{8.1687,5. d} = \frac{l^2}{13500. d}$$

and inserting for $l^2 = 144. L^2$, we have

$$\delta = \frac{144. L^2}{13500. d.}$$

$$= \frac{L^2}{93\frac{3}{4}. d.}$$

Had we assumed that the area of flanges or chords diminished towards the supports in proportion to the bending moment or actual stresses required, the average strain would, of course, be 12000 pounds per square inch throughout the entire length, no matter how the load might be applied.

Inserting this value in Formula (88) we should have had, for the amount of contraction of top flange

$$x = \frac{12000. l}{27000000} = \frac{l}{2250}$$

The same for the extension of bottom chord, or

$$x_1 = 2. \frac{l}{2250} = \frac{l}{1125}$$

Inserting this in Formula (89) we have for the deflection:

$$\delta = \frac{l^2}{8.1125. d} = \frac{l^2}{9000. d}$$

Inserting 144 $L^2 = l^2$ we have

$$\delta = \frac{144. L^2}{9000. d} \text{ or}$$

Parallel flanges or Chords, Diminished Cross-section, any loads.

$$\delta = \frac{L^2}{62\frac{1}{2}. d} \qquad (90)$$

Where δ = the greatest deflection, in inches, of a wrought-iron plate girder, or wrought-iron truss, with parallel flanges or chords, and where the areas of flanges or chords are gradually diminished

towards supports, and no matter how the load is applied; in no part however must the stresses, per square inch exceed respectively either $\left(\frac{c}{f}\right)$ or $\left(\frac{t}{f}\right)$.

Where $L =$ the length of span, in feet.

Where $d =$ the total depth (height) in inches, from top of top flange or chord to bottom of bottom flange or chord.

If girder or truss is of steel, use $53\frac{3}{8}$ instead of $62\frac{1}{2}$.

From Formula (90) and Formula (28) we get the rule that (no matter how the load is applied) if we want to carry the full safe load and not have deflection enough to crack plastering the length in feet must not exceed $1\frac{7}{8}$ times the total depth in inches.

For:

$$L.\ 0{,}03 = \frac{L^2}{62\frac{1}{2}.\,d} \text{ or}$$

$$L = 62\tfrac{1}{2}.\ 0{,}03.\ d$$

$$= 1{,}875.\ d \text{ or say}$$

Safe Length, Diminished Cross-section, any Load, Parallel Flanges or Chords.

$$L = 1\tfrac{7}{8}.\ d \qquad (91)$$

Where $L =$ the length, in feet, of a wrought-iron plate girder or wrought-iron truss, with parallel flanges or chords and with area of flanges or chords diminishing gradually towards supports and no matter how the load is applied; in no part however must the stresses, per square inch, exceed respectively either $\left(\frac{c}{f}\right)$ or $\left(\frac{t}{f}\right)$.

Where $d =$ the total depth (height), in inches, from top of top flange or chord to bottom of bottom flange or chord.

If girder or truss is of steel, use $1\frac{3}{8}$ instead of $1\frac{7}{8}$.

We see therefore that a beam of diminishing cross-section throughout is only about $\frac{2}{3}$ as stiff, as one with uniform cross-section, as its amount of deflection will be one-half more than that of the latter. Both deflections are approximate only, however, as we see by comparing the amount for the uniform cross-section to that obtained from Formula (79). The deflection for varying cross-sections how-

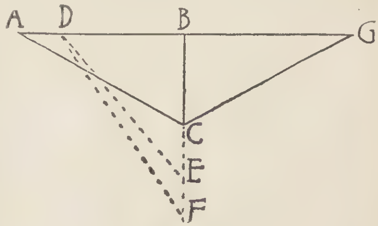


Fig. 146.

ever can be assumed as nearly enough correct, as these are never diminished so much practically as we have assumed in theory. Now taking the case of a trussed beam.

Deflection Trussed Beam. In Figure 146, let $A\ B$ be one half of a trussed beam, let $B\ C$ be the strut and $A\ C$ the tie. We will consider the load concentrated at B. Now the first effect is to shorten $A\ B$ by compression, let us say to $D\ B$.

Then, of course, $A\ D$ will represent one half of the contraction in the whole beam $A\ G$. Now the end of rod A moving to D will, of course, let the point C down to E, if we make $D\ E = A\ C$.

But there will be an elongation in $D\ E$ besides, due to the tension in it, which will let it down still further, say to F, if $D\ F = A\ C +$ elongation in $A\ C$, of course the point B will move down too, but we can overlook this to avoid complication. We now have $C\ F$ representing the amount of the deflection. To this should be added the amount of contraction of $B\ C$ due to the compression in it. We can readily find $C\ F$.

We know that

$$B\ F = \sqrt[2]{D\ F^2 - D\ B^2}$$

Now $D\ F$ we know is $= A\ C$ plus the elongation of $A\ C$ due to the tension in it, which we can find from Formula (88). From same formula we find the amount of contraction in $A\ G$ of which $A\ D$ is one-half, subtracting this from $A\ B$ or $\frac{l}{2}$ leaves, of course, $D\ B$.

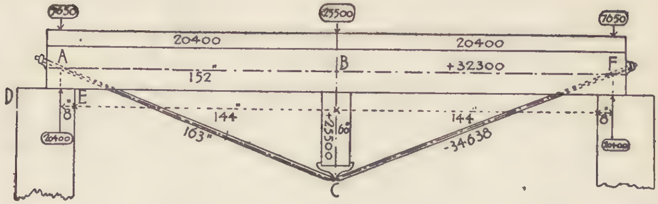
Now having found $B\ F$ we substract from it $B\ C$, the length of which is known, and the balance is of course the deflection $C\ F$; to this we add the contraction of $B\ C$ and obtain the total deflection of the whole trussed beam.

If the load had been a uniform load, instead of a concentrated one over the strut, there would be a deflection in that part of $A\ G$ which would be acting as a continuous girder. But this deflection would take place between B and G and between B and A and would not affect the deflection of the whole trussed beam.

An example will make much of the foregoing more clear.

Example.

Trussed Beam. *A trussed Georgia Pine beam is 16″ deep and of 24 feet clear span; it bears 16″ on each support and is trussed as*



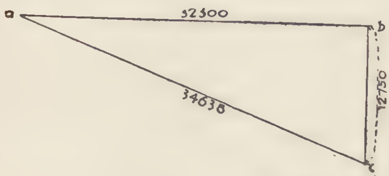


Fig. 147.

shown in Figure 147. *The beam carries a uniformly distributed load of* 40800 *pounds on the whole span including weight of beam and trussing. Of what size should the parts be?*

We draw the longitudinal neutral axes of each part, namely $A\ B$, $B\ C$ and $A\ C$. The latter is so drawn that the neutral axis of the reaction, which is of course half way between end of girder and E (or 8″ from E) will also pass through A.

In designing trusses this should always be borne in mind, that so far as possible all the neutral axes at each joint should go through the same point.

Cross-strains in Beams. The beam $A\ F$ virtually becomes a continuous girder, of two equal spans of 12 feet or 144″ each, uniformly loaded with 20400 pounds each, and supported at three points A, B and F. From table XVII we know that the greatest bending moment is at B and

$$=\frac{u.\,l}{8}=\frac{20400.144}{8}=367200 \text{ pounds-inch.}$$

The modulus of rupture for Georgia pine (Table IV) is $\left(\frac{k}{f}\right)=1200$, therefore moment of resistance (r) from Formula (18) and Table I, section No. 2,

$$r=\frac{b.\,d^2}{6}=\frac{367200}{1200} \text{ or}$$

$$b.\,d^2=1836$$

Now we know that $d=16$, or $d^2=256$, therefore

$b=\frac{1836}{256}=7{,}2$ or say we need a beam $7\frac{1}{4}''$ x 16″ for the

transverse strain. We must add to this however for the additional compression due to the trussing.

Compression in Strut. The amount of the load carried by strut $C\,B$, see Table XVII, is

$= \frac{5}{8}.\,u$ from each side, or

$= 25500$ on the strut $B\,C$, of which

$= 12750$ from each side.

If now we make at any scale a vertical line $b\,c =$ *half* the load carried at point B or $= 12750$ in our case, and draw $b\,a$ horizontally and $a\,c$ parallel to $A\ C$, we find the strain in $B\,A$ by measuring $b\,a = (32300$ pounds) or in $A\,C$ by measuring $a\ c = (34638$ pounds) both measured at same scale as $b\,c$. We find, further, in passing around the triangle $c\,b\,a\,c$ — ($c\,b$ being the direction of the reaction at A), that $b\,a$ is pushing towards A, therefore compression; and that $a\,c$ is pulling away from A, therefore tension. Using the usual signs of $+$ for compression, and — for tension, we have then:

Compression in Beam.

$A\,B = +\ 32300$ pounds.

$A\ C = -\ 34638$ pounds.

$B\,C = +\ 25500$ pounds.

Had we used Table XVIII we should have had the same result for:

Compression in $A\,B = \frac{25500}{2}.\frac{A\,B}{B\,C} = +\ 32300$ pounds and

Tension in $A\ C = \frac{25500}{2}.\frac{A\ C}{B\ C} = -\ 34638$ pounds.

Now the safe resistance of Georgia pine to compression along fibres (Table IV) is

$$\left(\frac{c}{f}\right) = 750 \text{ pounds.}$$

If $A\ B$ were very long, or the beam very shallow or very thin, we should still further reduce $\left(\frac{c}{f}\right)$ by using Formulae (3), or (5). But we can readily see that the beam will not bend much by vertical flexure due to compression, nor will it deflect laterally very much, so we can safely allow the maximum safe stress per square inch, or 750 pounds, that is, consider $A\ B$ a short column.

The necessary area to resist the compression, Formula (2) is:

$32300 = a.\ 750$ or

$$a = \frac{32300}{750} = 43 \text{ square inches.}$$

As the beam is 16″ deep, this would mean an additional thickness $= \frac{43}{16} = 2\frac{11}{16}$

Adding this to the $7\frac{1}{4}''$ already found to be necessary, we have

$$7\tfrac{1}{4} + 2\tfrac{11}{16} = 9\tfrac{15}{16}$$

or the beam would need to be, say 10″ x 16″.

Size of Strut. Now the size of $B\ C$ must be made sufficient not to crush in the soft underside of the beam at B. The bearing here would be across the fibres of the beam, and we find (Table IV) that the safe compressive stress of Georgia pine across the fibres is $\left(\frac{c}{f}\right) = 200$ pounds. We need therefore an area

$$a = \frac{25500}{200} = 128 \text{ inches.}$$

As the beam is only 10″ wide the strut $B\ C$ will have to measure, $\frac{128}{10} = 12\frac{4}{5}$ inches the other way, or we will say it could be 10″ x 12″. This strut itself might be made of softer wood than Georgia pine, say of spruce; the average compression on it is

$$\frac{25500}{10.12} = 212 \text{ pounds per square inch.}$$

Now spruce will stand a compression on end (Table IV) of $\left(\frac{c}{f}\right) = 650$ or, even if spruce is used, the actual strain would be less than one-third of the safe stress. At the foot of the strut $B\ C$ we put an iron plate, to prevent the rod from crushing in the wood. The rod itself must bear on the plate at least

Iron Shoe to Strut. $\frac{25500}{12000} = 2{,}1$ square inches, or it would crush the iron — (12000 pounds being the safe resistance of wrought-iron to crushing).

Size of Tie-rod. The safe tensional stress of wrought-iron being 12000 pounds per square inch (Table IV), we have the necessary area for tie-rod $A\ C$ from Formula (6)

$$34638 = a \,.\, 12000 \text{ or}$$

$$a = \frac{34638}{12000} = 2{,}886 \text{ square inches.}$$

From a table of areas we find that we should require a rod of $1\frac{15}{16}''$ diameter, or say a 2″ rod.

The area of a 2″ rod being $= 3{,}14$ square inches the actual tensional stress, per square inch on the rod, will be only

$$\frac{34638}{3{,}14} = 11312 \text{ pounds per square inch.}$$

Size of Washer. We must now proportion the bearing of the washer at "A" end of tie-rod. The amount of the crushing coming on washer will be whichever of the two strains at A, (viz. $B\ A$ and $A\ C$) is the lesser, or $B\ A$ in our case, which is 32300 pounds. We must therefore have area enough to the washer not to crush the end of beam (or along its fibres), the safe resistance of which we already found to be: $\left(\frac{c}{f}\right) = 750$ pounds per square inch; we need therefore

$$\frac{32300}{750} = 43 \text{ square inches.}$$

The washer therefore should be about

$6\frac{1}{2}''$ by $6\frac{1}{2}''$

Upset Screw-end. The end of the rod *must* have an "upset" screw-end; that is, the threads are raised above the end of rod all around, so that the area at the bottom of sinkage, between two adjoining threads, is still equal to the full area of rod. If the end is not "upset" the whole rod will have to be made enough larger to allow for the cutting of the screw at the end, which would be a wilful extravagance.

It is unnecessary to calculate the size of nuts, heads, threads, etc., as, if these are made the regulation sizes, they are more than amply strong. **Central Swivel.** It should be remarked here that in all trussed beams, if there is not a central swivel, for tightening the rod, that there should be a nut at *each* end of the rod; and not a head at one end and a nut at the other. Otherwise in tightening the rod from one side only it is apt to tip the strut or crush it into the beam on side being tightened. We must still however calculate the vertical shearing across the beam at the supports, which we know equals the reaction, or 20400 pounds at each end. To resist this we have $10'' \times 16'' = 160$ square inches, less $3'' \times 16''$, cut out to allow rod end to pass, or say 112 square inches net, of Georgia pine, across the grain; and as $\left(\frac{g}{f}\right) = 570$ pounds per square inch (see Table IV); the safe vertical shearing stress at each support would be (Formula 7)

$112.570 = 63840$ pounds or more than three times the actual strain. **Bearing of Beam.** Then, too, we should see that the bearing of beam is not crushed. It bears on each reaction 16 inches, or has a bearing area $= 16.10 = 160$ square inches.

$\left(\frac{c}{f}\right)$ for Georgia pine, across the fibres, Table IV, is

$\left(\frac{c}{f}\right) = 200$, therefore the beam will bear safely at each end

$160.200 = 32000$ pounds or about one-half more than the reaction. There will be no horizontal shearing, of course, except in that part of beam under transverse strain, and this certainly cannot amount to much. The beam is therefore amply safe.

Deflection of Beam. Now let us calculate the deflection. The modulus of elasticity for Georgia pine, Table IV is: $e = 1200000$ pounds-inch. The average compression strain in $A\ F$ was 750 pounds per square inch, therefore the amount of contraction (Formula 88)[1]

$$x = \frac{750.304}{1200000} = 0{,}19 \text{ inches.}$$

Now $A\ D$ (in Fig. 146) will be one-half of this, or 0,095 inches.

The amount of elongation in $A\ C$ will be, remembering that we found the average stress to be only 11312 pounds per square inch, and that for wrought-iron $e = 27000000$ (Formula 88)

$$x = \frac{11312.163}{27000000} = 0{,}0682$$

The exact length of $A\ C$ (Fig. 147 should be 163,41 not 163″). Therefore $D\ F$ (Fig. 146) will be

$$D\,F = 163{,}41 + 0{,}0682 = 163{,}4782''$$
$$D\,B = 152 - 0{,}19$$
$$= 151'', 81$$

Therefore (Fig. 146)

$$B\,F = \sqrt[2]{163{,}4782^2 - 151{,}81^2}$$
$$= 60'', 655$$

Now $B\ C$ (Fig. 147) would be $= 60''$, deducting this from the above we should have a deflection $= 0'', 655$.

To this we must add the contraction of $B\ C$. The strut will be less than 60″ long, say about 50″. The average compressive stress per square inch we found $= 212$ pounds. The modulus of elasticity

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[1] In reality the contraction of $A\ F$ would be much less, as the part figured for transverse strain only would very materially help to resist the compress'on, one half of it being in tension.

for spruce, Table IV, is $e = 850000$, therefore contraction in strut (Formula 88)

$$x = \frac{212.50}{850000} = 0{,}0125$$

Adding this to the above we should have the total deflection

$$\delta = 0{,}655 + 0{,}0125$$
$$= 0{,}6675$$

This would be the amount we should have to "camber" up the beam, or say $\frac{3}{4}''$.

The safe deflection not to crack plastering, would be (Formula 28)

$$\delta = L.\ 0{,}03$$
$$= 24.0{,}03$$
$$= 0{,}72$$

So that our trussed beam is amply stiff.

Explanation of Tables XIX to XXV. Table XIX gives all the necessary data in regard to the use of Tables XX, XXI, XXII, XXIII, XXIV and XXV. (See foot note p. 205). These Tables give all the necessary information in regard to all architectural sections, which are used in this work. Where, after the name of the Company formerly rolling the section, there are several letters, it means that practically the same sections were rolled by several Companies. It should be re-

I Sections not Economical. marked that except in the case of the simplest kind of beam work, it is cheaper to frame up plate girders, or trusses, of angles, flat-irons, tees, etc.

Steel beams and sections are sold cheaper than iron, if the latter can be had at all, as they are very much cheaper to manufacture, and where their uniformity can be relied on, should be used in preference, as they are much stronger and also a trifle stiffer. As a rule, however, the uniformity of steel in beams or other rolled sections cannot be implicitly relied upon, but—as already stated—the architect has no option now, he *must* use steel, and hence should be extra vigilant.

One example of an iron beam will make the application of the Tables to transverse strains clear, and help to review the subject, before taking up the graphical method of calculating transverse strains.

Example.

Use of Tables XIX to XXV. *A wrought-iron I-beam of* 25-*foot clear span, carries a uniform load of* 500 *pounds per foot including weight of beam; also a concentrated load of* 1000 *pounds* 10 *feet from*

the right hand support. The beam is not supported sideways. What size beam should be used?

The total uniform load $u = 500.25 = 12500$ pounds of which one-half or 6250 pounds will go to each reaction; of the 1000 pounds load $\frac{180}{300}$ or $\frac{3}{5}$ will go to the nearer support q (Formula 15), therefore

$$q = 6250 + \tfrac{3}{5}.1000 = 6850$$

Similarly we should have (Formula 14)

$$p = 6250 + \tfrac{2}{5}.1000 = 6650$$

As a check the sum of the two loads should $= 13500$, and we have, in effect:

$$6850 + 6650 = 13500$$

To find the point of greatest bending moment begin at q pass to load 1000, and we will have passed over ten feet of uniform load or 5000 pounds, add to this the 1000 pounds making 6000 pounds, and we still are 850 pounds short of the reaction, we pass on therefore towards p one foot, which leaves 350 pounds more, and pass on another $\frac{7}{10}$ of

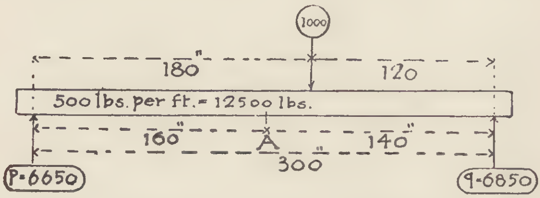


Fig. 148.

a foot (to A) which very closely makes the amount. The point of greatest bending moment therefore is at A, say 1′ 8″ to the left of the weight, or 140″ from q: As a check begin at p and we must pass along 160″ or 13′ 4″ of uniform load before reaching the point A, at 500 pounds a foot this would make $13\tfrac{1}{3}.500. = 6666$ or close enough to amount of reaction p for all practical purposes.

The uniform load per inch will be $\frac{500}{12} = 41\tfrac{2}{3}$ pounds.

Now the bending moment at A will be, taking the right-hand side (Formula 24)

$$m_A = 6850.140 - 41\tfrac{2}{3}.140.70 - 1000.20$$
$$= 530\,667 \text{ pounds-inch.}$$

As a check take the left-hand side (Formula 23)

$$m_A = 6650.160 - 41\tfrac{2}{3}.160.80$$

$=530614$ pounds-inch, or near enough alike for all practical purposes.

Now the safe modulus of rupture for wrought-iron (Table IV) is $\left(\frac{k}{f}\right)=12000$ pounds, therefore the required moment of resistance r from Formula (18)

$$r=\frac{530667}{12000}=44,2$$

Looking at the Table XX we find the nearest moment of resistance to be 46,8 or we should use the 12″ — 120 pounds per yard I-beam. But the beam is unsupported sideways. The width of top flange is $b=5\frac{1}{2}''$. We now use Formula (78) to find out how much extra strength we require.

Reduction for Lateral Flexure. In inserting value for y, we use the second column of Table XVI, as the beam is, of course, of uniform cross-section throughout, and have

$$y=0,0192.$$

In place of w we can insert the **actual value r** of the beam, and see what proportion of it is left to resist the transverse strength, after the lateral flexure is attended to,

$$\text{or } r_1=\frac{r}{1+\frac{0,0192.25^2}{5\frac{1}{2}^2}}=\frac{r}{1+0,3966}\text{ or}$$

$r_1=\frac{46,8}{1,3966}=33,6$ or the beam would not be strong enough. The next size would be the $12\frac{1}{4}''$ — 125 pounds per yard beam, but as the 15″ — 125 pounds per yard beam would cost no more and be much stronger we will try that. Its width of flange is $b=5''$ and moment of resistance $r=57,93$. Inserting these values in (Formula 78) and using r in place of w we have

$$r_1=\frac{57,93}{1+\frac{0,0192.25^2}{5^2}}=\frac{57,93}{1,48}$$

$$=39,14$$

The required moment of resistance was

$r=44,2$ so that this is still short of the mark, and we should have to use the next section or the 15″ — 150 pounds per yard beam. The moment of resistance of this beam is $r=69,8$ its width of flange the same as before, therefore:

$$r_1=\frac{69,8}{1,48}=47,1$$

Or this beam would be a trifle too strong even if unsupported sideways. We need not bother with deflection, for the length of beam is only 1⅔ times the span, and besides not even ⅔ of the actual transverse strength of the beam is required to resist the vertical strains, and, of course, the deflection would be diminished accordingly.

Safe Uniform Load. The column in Table XX headed "Transverse Value," gives the safe uniform load, in pounds, if divided by the span in feet, for beams supported sideways. Of course the result should correspond with Table XV, except that the uniform load will be expressed in pounds here, while it is expressed in tons of 2000 pounds each in that table. For Tables XXI, XXII, XXIII, XXIV and XXV the use of the "Transverse Value" is similar, and as more fully explained in Table XIX.

CHAPTER VII.

Graphical Analysis of Transverse Strains.

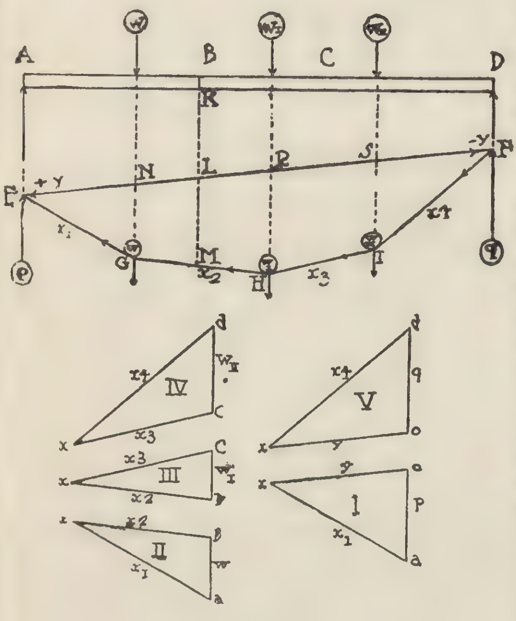


Fig. 149.

ALL the different calculations to ascertain the amounts of bending-moments, the required moments of resistance and inertia, the amounts of reactions, vertical shearing on beam, deflections, etc., can be done graphically, as well as arithmetically. In cases of complicated loads, or where it is desired to economize by reducing size of flanges, the graphical method is to be preferred, but in cases of uniform loads, or where there are but one or two concentrated loads,

the arithmetical method will probably save time. As a check, however, in important calculations, both methods might be used to advantage.

Basis of Graphical Method. If we have three concentrated loads w, w_{I}, and w_{II} on a beam $A\ D$ (Fig. 149), as represented by the arrows, we can also represent the reactions p and q by arrows in opposite directions, and we know that the loads and reactions all counterbalance each other. The equilibrium of these forces will not be disturbed if we add at E a force $= + y$, providing that at F we add an equal force, in the same line, but in opposite direction or $= - y$.

We have now at E two forces, $+ y$ and p. If we draw at any scale a triangle $a\,o\,x$ (or I) where $a\,o$ parallel and $= p$, and where $o\,x$ parallel and $= + y$, we get a force $x\,a$, which would just counterbalance them, or $a\,x$, which would be their *resultant.* That is, a force $G\,E$ thrusting against E with an amount $a\,x$ (or x_{I}) and parallel $a\,x$ would have the same effect on E as the two forces $+ y$ and p. Continuing x_{I} till it intersects the vertical neutral axis through load w at G, we obtain the resultant x_2 of the two forces acting at G, namely x_{I} and w (see triangle $b\,a\,x$ or II). Similarly we get resultant x_3 at H, of load w_{I} and x_2, (see triangle $c\,b\,x$ or III); also resultant x_4 at I of load w_{II} and x_3 (see triangle $d\,c\,x$ or IV); and finally resultant $+ y$, at F of reaction of q and x_4 (see triangle $o\,d\,x$ or V). As this resultant is $+ y$ it must, of course, be resisted by a force $- y$ that the whole may remain in equilibrium. By comparing the triangles I, II, III, IV and V, we see that they might all have been drawn in one figure (Fig. 150) for $q + p = w_{II} + w_{I} + w$, therefore:

$$d\,o + o\,a = d\,c + c\,b + b\,a,$$

further	both	V	and	IV	contain	$d\,x = x_4$
"	"	V	"	I	"	$o\,x = y$
"	"	II	"	I	"	$a\,x = x_{I}$
"	"	II	"	III	"	$b\,x = x_2$
"	"	III	"	IV	"	$c\,x = x_3$

We know further that the respective lines are parallel with each other.

In Fig. 150 then, we have $d\,c = w_{II}$

$c\,b = w_{I}$

$b\,a = w$

$a\,o = p$ and

$o\,d = q$

The distance $x\,y$ of *pole* x from *load line* $d\,a$ being arbitrary, and the position of pole x the same. The figure $E\,G\,H\,I\,F\,E$ (Fig 149) has many valuable qualities. If at any point K of beam we draw a vertical line $K\,L\,M$, then $L\,M$ will represent (as compared with the other vertical lines) the proportionate amount of bending moment at K. If we measure $L\,M$ in parts of the length of $A\,D$ and measure $x\,y$ (the distance of pole, Fig. 150) in units of the load line $d\,a$, then will the product of $L\,M$ and $x\,y$ represent the actual bending moment at K. That is, if we measure $L\,M$ in inches and — (having laid out $d\,c$, $c\,b$, etc., in pounds) — measure $x\,y$ in pounds, the bending moment at K will be $= x\;y.\;L\;M$ (in pounds-inch.) Similarly at w the bending moment would be

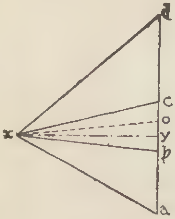


Fig. 150.

$= x\;y.\;\;N\;G$ (in pounds-inch.)
and at $w_{\prime}$ it would be $= x\;y.\;\;R\;H$ " " "
and at $w_{\prime\prime}$ it would be $= x\;y.\;\;S\;I$ " " "
measuring, in all cases, $x\,y$ in pounds and $N\,G$, $R\,H$ and $S\,I$ in inches.

Average Strain on Extreme Fibres. The area of $E\,G\,H\,I\,F\,E$, divided by the length of span in inches will give the average strain for the entire length on extreme top or bottom fibres of beam, providing the beam is of uniform cross-section throughout. The area should be figured by measuring all horizontal dimensions in inches, and all vertical dimensions in parts of the longest vertical ($R\;H$ in our case), this longest vertical being considered $=\left(\frac{c}{f}\right)$ for top, or $\left(\frac{t}{f}\right)$ for bottom fibres, or where these are practically equal $=\left(\frac{k}{f}\right)$.

The greatest bending moment on the beam will occur at the point where the longest vertical can be drawn through the figure. From this figure can also be found the shearing strains and deflection of beam, as we shall see later.

Distance of Pole. If now instead of selecting arbitrarily the distance $x\,y$ of the pole from load line $d\,a$ (Fig. 150) we had made this distance equal the safe modulus of rupture of the material, or $x\;y=\left(\frac{k}{f}\right)$ — measuring $x\,y$ in pounds at same scale as the load line $d\;a$ — it stands to reason that any vertical through the Figure

$EGHIFE$ (Fig. 149) measured in inches, will represent the required moment of resistance, for if $LM. x y = m$, we know from

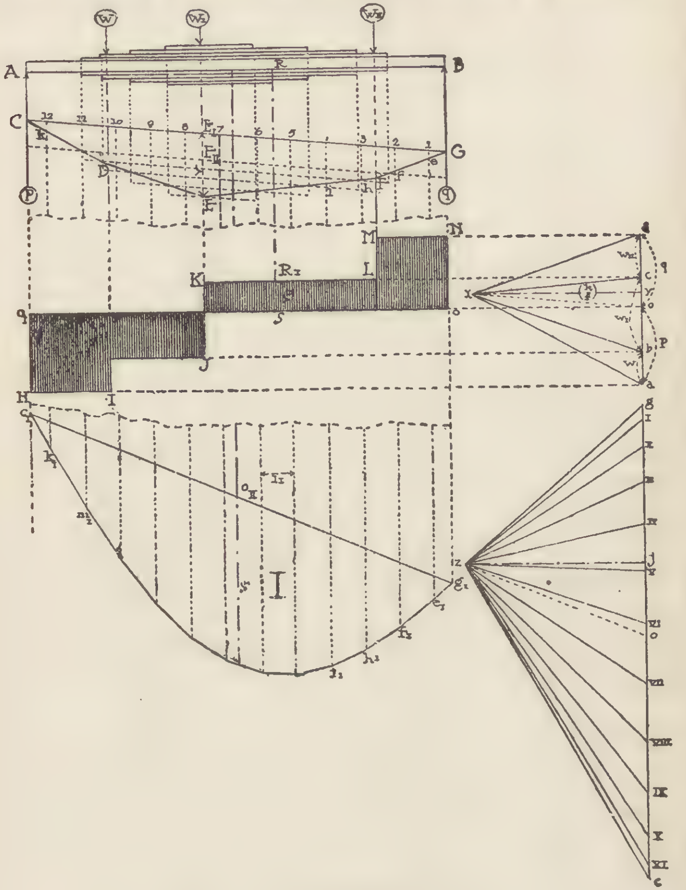


Fig. 151.

Formula (18), that $m = r. \left(\frac{k}{f}\right)$ and as we made $x y = \left(\frac{k}{f}\right)$, we have, inserting values in above:

$$LM. \left(\frac{k}{f}\right) = r. \left(\frac{k}{f}\right) \text{ or}$$
$$LM = r$$

Having thus shown the basis of the graphical method of analyzing transverse strains, we will now give the actual method without wasting further space on proofs.

Several Concentrated Loads. If there are three loads w, w_{I} and w_{II} on a beam $A\ B$ (Fig. 151) we proceed as follows: at any convenient scale — to be known as the pounds-scale — lay off in pounds, $d\,c = w_{\text{II}}$; also $c\,b = w_{\text{I}}$ and $b\,a = w$. Let $A\ B = l$ measured in inches — this scale being called the inch-scale. Now select pole x at random,

Strain Diagram. but at a distance (measured with pounds-scale) $x\,y = \left(\frac{k}{f}\right) =$ the safe modulus of rupture of the material. Draw $x\,d$, $x\,c$, $x\,b$ and $x\,a$. Now begin at any point G of reaction q, draw $G\ F$ parallel $d\,x$, till it intersects vertical w_{II} at F: then from F draw $F\ E$ parallel $c\,x$ to vertical w_{I}; then draw $E\ D$ parallel $b\,x$ to vertical w; and then $D\ C$ parallel $a\,x$ to reaction p. From C draw $C\ G$, and through x draw $x\,o$ parallel $C\ G$.

Reactions. We now have the following results:

$o\,d =$ reaction q (measured with pounds-scale.)
$a\,o =$ " p " " " "

any *vertical* through figure $C\,D\,E\,F\,G\,C$, (measured with inch-scale) gives the amount of $r =$ required moment of resistance in inches, at point of beam where vertical is measured. The longest vertical passes through the point of greatest bending-moment in beam. Multiply any vertical (in inches) with $x\,y$ (in pounds) to obtain amount of bending-moment at point of beam through which vertical passes.

Moment of Resistance. or we should have: $r = v$ (92)

Where $r =$ the required moment of resistance, in inches, at any point of beam, provided pole distance $x\,y = \left(\frac{k}{f}\right)$.

Where $v =$ the length (measured with inch-scale) of the vertical through upper figure $C\,D\,E\,F\,G\,C$ at point of beam for which r is sought.

And further:

Bending-moment. $m = v \, . \, x\,y$ (93)

Where $m =$ the bending moment at any point of beam in pounds-inch.

Where $v =$ the same value as in Formula (92)

Where $x\,y =$ the length, (measured with pound-scale) of distance of pole x from load line, in upper strain diagram $x\,a\,d$.

If now we draw horizontal lines through d, c, b and a; and through o the horizontal line for horizontal axis; and continue these lines until they intersect their respective load verticals w_{II}, w_I and w, the shaded figure $O_I\ H\ I\ J\ K\ L\ M\ N\ O\ O_I$ will give the vertical shearing strain along beam. Any vertical (as $R_I\ S$) drawn through this figure to horizontal axis and measured with pounds-scale, gives the amount of vertical shearing at the point of beam (R) through which vertical is drawn. O1,

Vertical Cross-shearing.

$$s = v_{II} \tag{94}$$

Where s = the amount of vertical shearing strain in pounds, at any point of beam.

Where v_{II} = the length (measured with pounds-scale) of vertical through figure $O_I\ H\ I\ J\ K\ L\ M\ N\ O\ O_I$ dropped from point of beam for which strain s is sought.

We now divide $G\ C$ into any number of *equal* parts — say twelve in our case — and begin with a half part, or

$$G \text{ to } 1 = 12 \text{ to } C = \tfrac{1}{24}.\ G\ C; \text{ also}$$

$$1 \text{ to } 2 = 2 \text{ to } 3 = 3 \text{ to } 4 = 4 \text{ to } 5, \text{etc.} = \tfrac{1}{12}.\ G\ C$$

and make the new lower load line $g\ c$ with inch-scale so that

Deflection Diagram. g to I = length of vertical 1 e

further I to II = length of vertical $2\ f$
" II " III = " " " $3\ h$
" III " IV = " " " $4\ i$, etc. until
" XI " c = " " " $12\ k$

Now select arbitrarily a pole z at any distance $z\ j$ from load line $g\ c$.

Now draw below the beam where convenient (say I. Fig. 151) beginning at g_I, the line $g_I\ e_I$ parallel $g\ z$ till it intersects the prolongation of 1 e (from above) at e_I; then draw $e_I\ f_I$ parallel I z till it intersects vertical $2\ f$ at f_I; and similarly draw $f_I\ h_I$ parallel II z; also $h_I\ i_I$ parallel III z, etc., to $m_I\ k_I$ parallel XI z and finally $k_I\ c_I$ parallel $c\ z$. The more parts (l_I) we divide the beam into, the nearer will this line $g_I\ e_I\ f_I\ m_I\ k_I\ c_I$ approach a curve. The real line to measure deflections would be a curve with the above lines as tangents to it; we need not, however, bother to draw this curve for practical work. Now draw $c_I\ g_I$ and parallel thereto $z\ o$. Divide $g_I\ c_I$ at o_{II} so that[1]: $g_I\ o_{II} : c_I\ o_{II} = c\ o : g\ o$, then will o_{II} be the point of greatest deflection along beam. This will be further proven by

[1] Note that the division of the line $g_I\ o_{II}\ c_I$ is the reverse of the division of the line $g\ o\ c$.

the fact that the greatest vertical (in lower figure I) will pass through

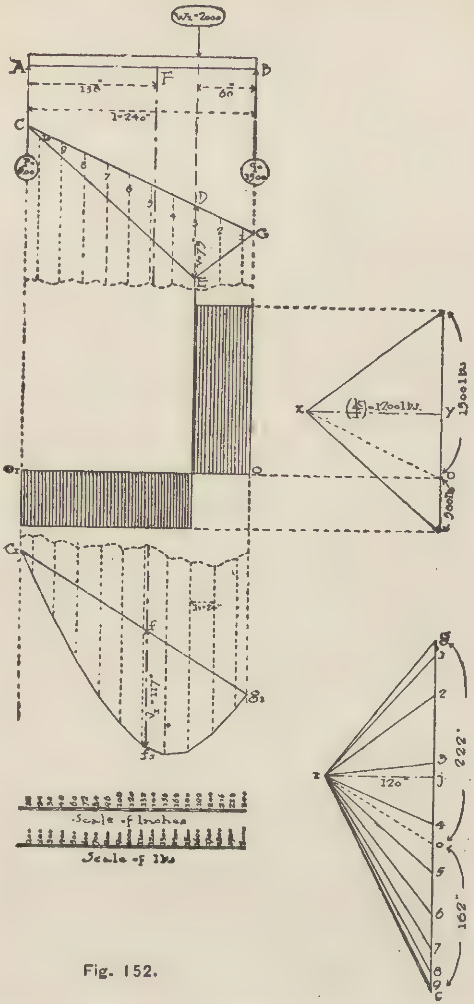


Fig. 152.

o_{11}, if the real curve were drawn. The figure $g_1\, e_1\, f_1\, h_1\, i_1\, m_1\, k_1\, c_1\, g_1$ will measure the amount of deflection of beam at all points of

beam. The deflection at any point of beam being proportionate to length of its vertical through lower figure I. The *amount* of this deflection will be

Amount of Deflection, Definite Pole Distance.

$$\delta = \frac{v_1 . l_1 . zj . \left(\frac{k}{f}\right)}{e . i .} \tag{95}$$

Where δ = the deflection, in inches, at any point of beam, if pole distance of upper strain diagram $(x\, y) = \left(\frac{k}{f}\right)$.

Where v_1 = the length of vertical, in inches, dropped from said point through lower figure I (see Fig. 151)

Where l_1 = the length, in inches, of each equal part 1 to 2, 2 to 3, 3 to 4, etc., into which beam was divided, [in our case $l_1 = \frac{1}{12}\, l$.]

Where i = the moment of inertia, of cross-section at said point, in inches.

Where zj = the distance (measured with inch-scale) of pole z from load line in lower strain diagram.

Where $\left(\frac{k}{f}\right)$ = the safe modulus of rupture, per square-inch, of the material.

Where e = the modulus of elasticity, in pounds-inch, of the material.

If we were to so proportion the beam that the moment of resistance at each point would exactly equal the required moment of resistance as found above, we should have:[1]

Deflection varying Cross-section.

$$\delta = \frac{v_1 . l_1 . zj . \left(\frac{k}{f}\right)}{v . \frac{d}{2} . e} \tag{96}$$

Where $\delta, v_1, zj, \left(\frac{k}{f}\right)$, e and l same value as in Formula (95).

Where v = length of corresponding vertical in upper figure $C\ D\ G\ E\ C$, (to vertical v_1 of lower Fig. I) to be measured in inches.

Where $\frac{d}{2}$ = one-half the total depth of beam, in inches. Had we not made $x\, y = \left(\frac{k}{f}\right)$, we should have

Deflection Pole Distance arbitrary.

$$\delta = \frac{v_1 . l_1 . zj . x\, y .}{e . i .} \tag{97}$$

[1] This would be the greatest possible deflection. If the beam were not so proportioned, but of uniform cross-section throughout, the deflection would be less.

Where δ, v_1, l_1, e, z j, and i same value as in Formula (95).

Where $x\,y =$ the length of pole distance from load line in upper strain diagram, measured in pounds.

The same formulæ and methods could be applied to cantilevers, but for these the arithmetical calculations are so very simple that it would be taking unnecessary trouble.

A few practical examples will make all of the foregoing more clear.

Example I.

Single concentrated Load. *A Georgia pine girder A B of 20-foot span carries a load w_1 of 2000 pounds 5′ 0″ from right reaction B. What size should the girder be?*

We draw (Figure 152) $A\,B = 240''$ at inch-scale, and locate w_1 at $60''$ to the left of B. Now draw a vertical line $b\,a = 2000$ pounds at pounds-scale. Select point x anywhere, but distant $x\,y = 1200$ pounds. (1200 pounds being $= \left(\frac{k}{f}\right)$ or the safe modulus of rupture, per square-inch, of Georgia pine). Draw $x\,b$ and $x\,a$. Draw verticals through A, w_1 and B. On vertical A begin at any point C, draw $C\,E$ parallel $x\,a$, till it intersects verticals w_1 at E; then draw $E\,G$ till it intersects vertical B at G. Draw $G\,C$ and $o\,x$ parallel to $G\,C$. We scale $o\,b$, it scales 1500 pounds, so this, is the reaction at B. We scale $a\,o$, it scales 500 pounds and this is the reaction at A. The longest vertical through $C\,E\,G$ is vertical w_1, therefore greatest bending-moment is at w_1 which we know is the case. We scale $E\,D$ at inch-scale, it scales 75 inches, therefore the (greatest) *required* moment of resistance will be at w_1 and will be Formula (92).

$$r = 75.$$

From Table I, section No. 2, we know for rectangular beams,

$$r = \frac{b.\,d^2}{6}, \text{ therefore:}$$

$$\frac{b.\,d^2}{6} = 75, \text{ or}$$

$$b.\,d^2 = 450.$$

We will suppose the girder is not braced sideways, and needs to be pretty broad; let us try $b = 5''$, we have then:

$$5.\,d^2 = 450 \text{ or}$$

$$d^2 = \frac{450}{5} = 90 \text{ and}$$

$$d = 9,5'' \text{ or the girder}$$

would have to be $5'' \times 9\frac{1}{2}''$ or say $5'' \times 10''$. The bending-moment at

w_1 is, of course, Formula (93) $= E\ D.\ x\ y = 75.1200 = 90000$ (pounds-inch).

Had we calculated arithmetically, we should have had, Formulæ (14) and (15):

$$\text{reaction } A = \frac{60}{240}.\ 2000 = 500 \text{ pounds.}$$

$$\text{“} \quad B = \frac{180}{240}.\ 2000 = 1500 \text{ pounds.}$$

Bending moment at w_1 would be (right side) Formulæ (23) and (24). $m_{w_1} = 1500.60 - 0.2000 = 90000$ (pounds-inch) or check (left) side $m_{w_1} = 500.180 - 0.2000 = 90000$ (pounds-inch.) Therefore required moment of resistance, Formula (18)

$$r = \frac{90000}{1200} = 75.$$

or same result as graphically.

By drawing the horizontals from b between verticals B and w_1; from a between verticals A and w_1; and from o between verticals A and B we get the etched figure for measuring vertical shearing strains. We see at a glance that the shearing to the right of load is equal to the right reaction, and is constant at all points of right side of beam; while on the left side of load it is equal to the left reaction, and is constant at all points of the left side of beam. And this we know is the case. We need not bother with shearing, however, for we can readily see there is no danger. For even immediately to the right of the load, the weakest point in our case, we know that one-half of the fibres of cross-section are not strained at all, or we should have one-half of area or $\frac{5.10}{2} = 25$ square-inches to resist 1500 pounds of shearing, or $\frac{1500}{25} = 60$ pounds per square-inch, while the safe resistance, per square-inch, of Georgia pine to shearing across the grain is (Table IV) $\left(\frac{g}{f}\right) = 570$ pounds.

There is, however, some danger of excessive deflection; we draw, therefore, the figure $c_1 f_1 g_1$ by dividing the beam into ten equal parts, beginning and ending with half parts at the reaction, (each whole part being 24″ long, or $l_1 = \frac{240}{10} = 24''$)

We draw the verticals through these parts and get their lengths through figure $C\ E\ G$. These lengths we carry down in their proper succession on the load line $g\ j\ c$ of the lower strain diagram, begin-

ning at the top with the right vertical 1, putting immediately under this the length of second vertical 2, then 3 and so on till $g\,c =$ sum of lengths of all ten verticals through $C\,E\,G$. We now select z at random (in our case 120 inches from load line or $z\,j = 120''$). We now draw lines from z to g I, II, III, etc., to c. Construct figure $g_1 f_1 c_1$ by beginning at g_1 drawing line parallel to $z\,g$ until it intersects

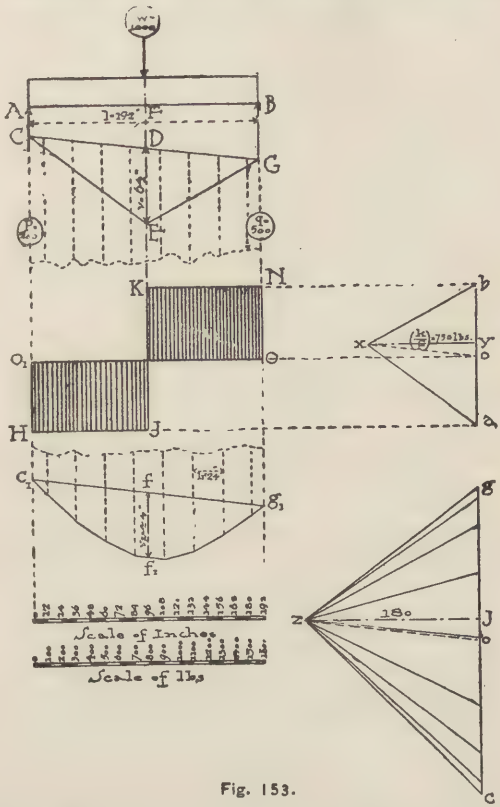


Fig. 153.

prolongation of first vertical 1; then line parallel to z I till it intersects prolongation of second vertical 2, etc. We now draw $z\,o$ parallel $c_1\,g_1$. We scale $g\,o$ and find it scales $222''$, also $c\,o$ which scales $162''$; we divide $c_1\,g_1$ at f, so that

$$c_1 f : f g_1 = 222 : 162.$$

Carrying vertical ff_1 through figure we find it scales $(v_1) = 117''$ continuing ff_1 up to beam it gives us point F as the point of greatest deflection, we find $A\ F$ scales $138''$. Had we used Formula (43) we should have located F at a distance from A or $A\ F = \sqrt{\frac{240^2 - 60^2}{3}} = 134,17''$. So that we have a sufficiently accurate result.

For the amount of deflection at F we use Formula (95); we know that (Table I, Section No. 2) $i = \frac{b.\,d^3}{12} = \frac{5.\,10^3}{12} = 417$, further for Georgia pine $\left(\frac{k}{f}\right) = 1200$ pounds.

$$e = 1200000 \text{ (inch-pounds.)}$$
$$l_1 = 24''$$
$$v_1 = ff_1 = 117''$$
$$zj = 120'', \text{ therefore:}$$
$$\delta = \frac{117.\,24.\,120.\,1200}{1200000.\ \ 417}$$
$$= 0,808''$$

Had we calculated the deflection by Formula (41) we should have had:
remembering that $m = 180''$ and $n = 60''$ and $l + n = 240 + 60 = 300''$

$$\delta = \frac{2000.\,180.\,60.\,300}{9.\,240.\,1200000.\,417} \cdot \sqrt{\frac{180.\,300}{3}}$$
$$= 0,803''$$

Which proves the accuracy of the graphical method.

For a beam of 20 feet span the deflection not to crack plastering should not exceed, Formula (28).

$$\delta = 20.\,0,03 = 0,6''$$

Therefore, if our beam supports a plastered ceiling, it must be redesigned to be *stiffer*. Either made deeper, in which case it can be thinner, if braced sideways, or it can be thickened sufficiently to reduce the deflection, see Formula (31).

Example II.

Single centre load. *A hemlock girder $A\ B$ (Fig. 153) of 16-foot span, carries a centre load w of 1000 pounds. What size should the girder be?*

We make $A\ B = 192''$ at inch scale; locate w at its centre F; make $b\,a$ at any scale — (pounds-scale) — equal 1000 pounds. Select pole x distant, $x\,y = 750$ pounds, from load line $b\,a$, (as 750

pounds $=\left(\frac{k}{f}\right)$ the safe modulus of rupture per square inch of hemlock). Draw $x\,b$ and $x\,a$. Begin at G, draw $G\,E$ parallel $b\,x$ to vertical through load, and then draw $E\,C$ parallel $a\,x$. Draw $C\,G$ and then $x\,o$ parallel $C\,G$, we find that o bisects $b\,a$ or $a\,o = o\,b = 500$ pounds. Each reaction is therefore one-half of the load; this we know is the case. Greatest line through $C\,G\,E$ we find is at $D\,E$, so that greatest bending-moment is at load; this we know is the case. $D\,E$ scales 64″ at inch-scale, therefore the required moment of resistance for the beam is, Formula (92):

$$r = 64.$$

and the greatest bending-moment at load, Formula (93):

$$m_w = 64.\,x\,y = 64.750$$
$$= 48000$$

Had we calculated arithmetically we should have obtained the same results, for Formula (22)

$$m_w = \frac{1000.\,192}{4} = 48000$$

and Formula (18):

$$r = \frac{48000}{750} = 64$$

Now from Table I, Section No. 2, we know that for rectangular sections:

$$r = \frac{b.d^2}{6} \text{ or}$$
$$b.d^2 = 64.6 = 384.$$

If we assume the beam as 4″ thick, we have then:

$$4.\,d^2 = 384.$$
$$d^2 = \frac{384}{4} = 96 \text{ or}$$
$$d = \sqrt{96} = 9{,}8'' \text{ or we will make the beam } 4'' \times 10''.$$

We draw the figure $O_1\,H\,J\,K\,N\,O$ for shearing and find it is constant throughout the whole length of beam and equal to length $O_1\,H$ or $N\,O$ measured at pounds scale, or 500 pounds. This is so small we need not bother with it.

To obtain the deflection diagram we divide $G\,C$ into eight equal parts, each part $l_1 = \frac{192}{8} = 24''$ and begin at each end with half parts, drawing the eight verticals through $C\,E\,G$.

We lay off their exact lengths in proper succession on the lower load line $g\,c$, beginning at the top with the right vertical. Select

pole z at random, in this case distant from load line $z j = 180''$. We now draw the figure $c_1\, g_1\, f_1$ and find greatest deflection is at its centre $f f_1$; for $z\, o$ parallel $c_1\, g_1$ bisects $g\, c$. We scale $f f_1$ at inch scale

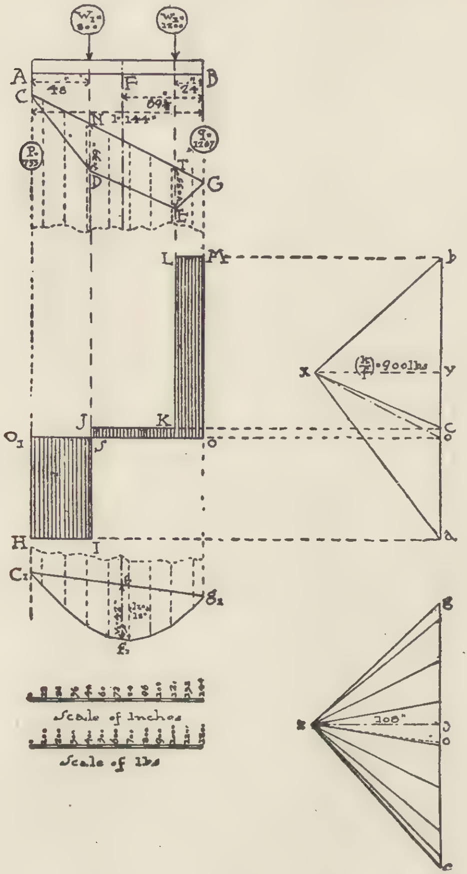


Fig. 154.

$= 44''$. therefore greatest deflection of beam at centre, Formula (95), remembering that (Table I, Section No. 2) $i = \frac{4.10^3}{12} = 333$ and

(Table IV) $e = 800000$

$$\delta = \frac{44.\ 24.\ 180.\ 750}{800000.\ 333} = 0{,}535''$$

Had we calculated the deflection arithmetically from Formula (40) we should have had:

$$\delta = \frac{1}{48} \cdot \frac{1000.\ 192^3}{800000.\ 333} = 0{,}548''$$

or practically the same result.

If the beam supported plastered work the deflection should not exceed, Formula (28)

$$\delta = 16.\ 0{,}03 = 0{,}48''$$

Still, unless we were very particular, the beam could be passed as practically stiff enough.

Example III.

Two concentrated loads. *A white pine beam* $A\ B$ *Fig.* 154, *of* 12 *foot span carries two loads, one* $w_{\text{I}} = 800$ *pounds, four feet from left support, the other* $w_{\text{II}} = 1200$ *pounds, two feet from right support. What size should the beam be?*

Make $A\ B$ at inch scale $= 144$ inches, locate w_{I} so that $A\ w_{\text{I}} = 48''$, and w_{II} so that $B\ w_{\text{II}} = 24''$. At any (pounds scale) make $b\ c = 1200$ pounds and $c\ a = 800$ pounds. Select pole x distant from $b\ a$; $x\ y = 900$ pounds, the safe modulus of rupture per square inch of white pine; draw $x\ b$, $x\ c$ and $x\ a$. Construct $C\ D\ E\ G$ parallel to these lines. Draw $C\ G$, and parallel to same $x\ o$, then will $a\ o = 733$ pounds be reaction at A, and $o\ b = 1267$ pounds be reaction at B. We scale vertical $D\ N$ at $w_{\text{I}} = 39''$ and $T\ E$ at $w_{\text{II}} = 35''$, therefore greatest bending-moment is at w_{I} and Formula (93)

$$m_{w_{\text{I}}} = 39.\ 900 = 35100$$

Further, the required moment of resistance at w_{I} Formula (92) will be:

$$r = D\ N = 39.$$

Now from Table I, Section No. 2,

$$r = \frac{b.d^2}{6}, \text{ or}$$

$$b.d^2 = 6.39 = 234$$

Now if $b = 3''$ we should have

$$d^2 = \frac{234}{3} = 78 \text{ and}$$

$d = \sqrt{78} =$ say $9''$, or the beam would need to be $3'' \text{ x } 9''$.

We should have obtained practically the same results arithmetically, for: Formulæ (16) and (17):

$$\text{Reaction at } A = \frac{800.96}{144} + \frac{1200.24}{144} = 733$$

$$\text{Reaction at } B = \frac{800.48}{144} + \frac{1200.120}{144} = 1267$$

check: $A + B = w_{\text{I}} + w_{\text{II}} = 800 + 1200 = 2000$ pounds and $733 + 1267 = 2000$ pounds.

Beginning at B we have to pass over load w_{II} (1200 pounds) and on to w_{I}, before passing amount of reaction B (1267 pounds) therefore greatest bending-moment at w_{I}. We know from Formula (24) it would be:

$$m_{w_{\text{I}}} = 1267.96 - 72.1200 = 35232$$

and check from Formula (23)

$$m_{w_{\text{I}}} = 733.48 - 0.800 = 35184$$

being near enough for practical purposes. From Formula (18) we should have had:

$$r = \frac{35184}{900} = 39{,}09$$

We now draw the shearing diagram $O_{\text{I}}\ H\ I\ J\ K\ L\ M\ O$, as shown in Figure 154, and find the amount of shearing

from A to $w_{\text{I}} = O_{\text{I}}\ H =$ 733 pounds,
from w_{I} to $w_{\text{II}} = J\ S =$ 67 pounds,
from w_{II} to $B = M\ O = 1267$ pounds.

We can overlook it, for even at the weakest point of beam for resisting cross-shearing we have half the area, or $\frac{3.9}{2} = 13\frac{1}{2}$ square inches.

White pine will safely resist 250 pounds per square inch in cross-shearing (Table IV) or the beam would resist.

$13\frac{1}{2}.\ 250 = 3375$ pounds at its weakest point for cross-shearing, (viz.: at w_{I}) and twice as much at the reactions.

To find the deflection we divide $G\ C$ into eight equal parts, beginning with half parts (or $l_{\text{I}} = \frac{144}{8} = 18''$) and draw the verticals hrough $C\ D\ E\ G$. We now make the lower load line $g\ c$ equal the ım of these verticals, beginning at the top with the right vertical.

Select z distant from $g\ c$ (the load line) $z\ j = 108''$. Draw $z\ g$, $z\ c$, etc., and construct $g_{\text{I}}\ c_{\text{I}}\ f_{\text{I}}$ as before.

We draw $z\ o$ parallel $c_{\text{I}}\ g_{\text{I}}$. Now $g\ o$ measures 116 inches and $o\ c$

108 inches, therefore divide $c_1\, g_1$ at f so that:

$c_1 f : f\, g_1 = 116 : 108$

Carrying the vertical $f\, f_1$ up to point F of beam, we find the point of greatest deflection F, where

$B\ F = 69\frac{1}{2}''$ and $A\ F = 74\frac{1}{2}''$

We find $f\, f_1$ scales 42″, remembering that (Table I, Section No. 2) $i = \frac{3.9^3}{12} = 182$, and that for white pine Table IV $e = 850000$ pounds we have Formula (95):

$$\delta = \frac{42.\ 18.\ 108.\ 900}{850000\ .182} = 0{,}475''$$

Had we attempted to get this result arithmetically by inserting the values in Formula (41) (and remembering that n is always the nearer support, or in our case respectively 48″ and 24″, while m respectively 96″ and 120″) we should realize the advantage of the graphical method, for:

$$\delta = \frac{800.\ 96.\ 48.\ (144+48).\ \sqrt{\frac{96.\ (144+48)}{3}} + 1200.\ 96.\ 48.\ (144+24).\ \sqrt{\frac{120.\ (144+24)}{3}}}{9.\ 144.\ 850000.\ 182}$$

If we figure out the above tedious formula we should have

$$\delta = 0{,}422''$$

or practically the same result as we obtained graphically.

The safe deflection, were the beam to carry plastering, should not exceed Formula (28)

$$\delta = 12.\ 0{,}03 = 0{,}36''$$

Our beam is therefore not nearly stiff enough, and we must make it thicker; or else if we wish to save material, we will make it thinner, but deeper; and then brace it sideways, see Formula (31).

Example IV.

Five Concentrated Loads. *A spruce girder A B of 18-foot span carries five loads, as shown in Figure 155. What size should the girder be?*

We draw $A\ B = 216''$ (inch scale); further

$b\, a = 2700$ pounds (sum of loads at pounds scale); make

$b\, h = w_{\text{v}} = 540$ pounds,

$h\, e = w_{\text{iv}} = 180$ pounds,

$e\, d = w_{\text{iii}} = 360$ pounds,

$d\, c = w_{\text{ii}} = 720$ pounds, and

$c\, a = w_{\text{i}} = 900$ pounds.

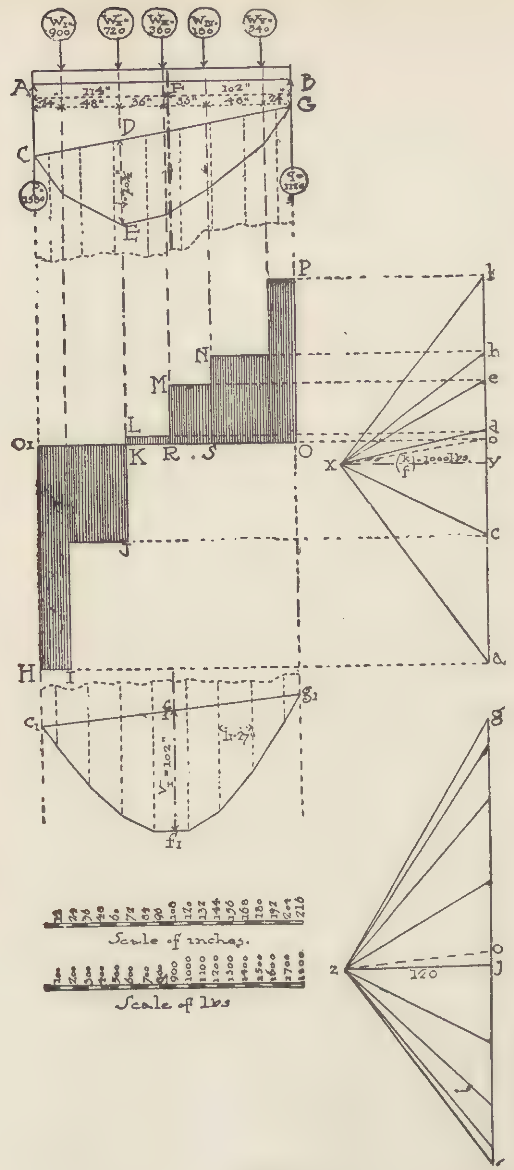

W I 900
W II 720
W III 360
W IV 180
W V 540
A
B
F
114"
102"
24"
48"
36"
30"
40"
24"
G
D
C
E
P
k
N
h
M
e
L
d
O1
K
R
S
O
o
x
y
c
J
H
I
a
g1
c1
f
V1 = 102"
f1
Scale of inches.
Scale of lbs
g
o
z
J
120

Fig. 155.

Select x distant $x\,y = 1000$ pounds from $b\,a$, (as $1000 = \left(\frac{k}{f}\right)$ for spruce, see Table IV). Draw $x\,b$, $x\,h$, $x\,e$, etc., and figure $C\,D\,G$. Draw $x\,o$ parallel $C\,G$; it divides load line as follows:

$a\,o = 1580$ pounds or reaction at A.

$o\,b = 1120$ pounds or reaction at B.

We find longest vertical through $C\,D\,G$, is at load w_{II}, therefore greatest bending-moment on beam at w_{II}; now $D\,E$ scales $70\frac{1}{2}''$, therefore Formula (93):

$$m_{w_{\text{II}}} = 70\tfrac{1}{2}.\ 1000 = 70500$$

and Formula (92)

$$r = 70,\ 5$$

From Table I, Section No. 2

$$r = \frac{b.d^2}{6} = 70,\ 5 \text{ and if } b = 5, \text{ we have}$$

$$5.\ d^2 = 6.\ 70,\ 5 \text{ or}$$

$$d^2 = 84,\ 6 \text{ and}$$

$$d = \sqrt{84,\ 6} = 9,2'' \text{ or say } 10'' \text{ which is the nearest size}$$

larger than 9,2″, and of course wooden beams are never ordered to fractions of inches.

Had we worked arithmetically we should have had practically the same results.

From Formulæ (16) and (17) we should have had:

reaction at $A = 1580$ pounds,

reaction at $B = 1120$ pounds.

From rule for finding greatest bending-moment we should have located it at w_{II} and then had Formula (23)

$$m_{w_{\text{II}}} = 1580.\ 72 - 48.900 = 70560$$

and from Formula (18)

$$r = \frac{70560}{1000} = 70,\ 56$$

We now draw the shearing diagram $O_{\text{I}}\ H\ I\ J\ K\ L\ M\ N\ P\ O$ and find as follows:

Cross-shearing A to $w_{\text{I}} = H\ O = 1580$ pounds.
Cross-shearing w_{I} to $w_{\text{II}} = J\ K = 680$ pounds.
Cross-shearing w_{II} to $w_{\text{III}} = K\ L = 40$ pounds.
Cross-shearing w_{III} to $w_{\text{IV}} = M\ R = 400$ pounds.
Cross-shearing w_{IV} to $w_{\text{V}} = N\ S = 580$ pounds.
Cross-shearing w_{V} to $B = P\ O = 1120$ pounds.

We need not bother with it, therefore. For deflection we now

divide $C\,G$ again into eight equal parts, (or $l_1 = \frac{216}{8} = 27''$) beginning with half parts at C and G. We now make lower load line $g\,c =$ the sum of the eight verticals, putting the right vertical at the top from g down. We select pole z at a distance $z\,j = 120''$ from $g\,c$ and draw $z\,g$, $z\,c$, etc. We construct figure $g_1 f_1 c_1$ and draw $z\,o$ parallel to $c_1\,g_1$. We now divide $c_1\,g_1$ at f, so that

$g_1 f : f c_1 = c\,o : o\,g$, carrying $f f_1$ up to beam, we have the point F, distant $102''$ from B, and $114''$ from A, which is the point of greatest deflection. We find that $f f_1$ scales $102''$, remembering that $e = 850000$ for spruce (Table IV), and that $i = \frac{5.10^3}{12} = 417$ (See Table I, Section No. 2) we have, Formula (95).

$$\delta = \frac{102.\,27.\,120.\,1000}{850000.\,417} = 0{,}93''$$

This would be too much for plastering, for if the girder supported plastering, the deflection should not exceed Formula (28)

$$\delta = 18.\,0,\,03 = 0{,}54''$$

We must therefore deepen the beam very materially.

We use Formula (31),

$$x = \frac{1}{b.d^3}$$

In our case it would be

$$x = \frac{1}{5.10^3} = \frac{1}{5000} = 0{,}0002$$

Supposing we were to make the beam $4'' \times 12''$, then we should have

$$x = \frac{1}{4.12^3} = 0,\,000\,144$$

The deflection of the latter, then, would be

$$\delta : 0,\,93 = 0,\,000144 : 0,\,0002 \text{ or}$$

$$\delta = \frac{0,\,93.\,0{,}000144}{0{,}0002} = 0{,}67'' \text{ still too much deflection.}$$

Were we to make the beam $3'' \times 14''$, we should have:

$$x = \frac{1}{3.14^3} = 0{,}0001215$$

The corresponding deflection for this beam would be:

$$\delta : 0,\,93 = 0{,}0001215 : 0{,}0002 \text{ or}$$

$$\delta = \frac{0,\,93.\,0{,}0001215}{0{,}0002} = 0{,}565''$$

or just about what would be required in the way of stiffness.

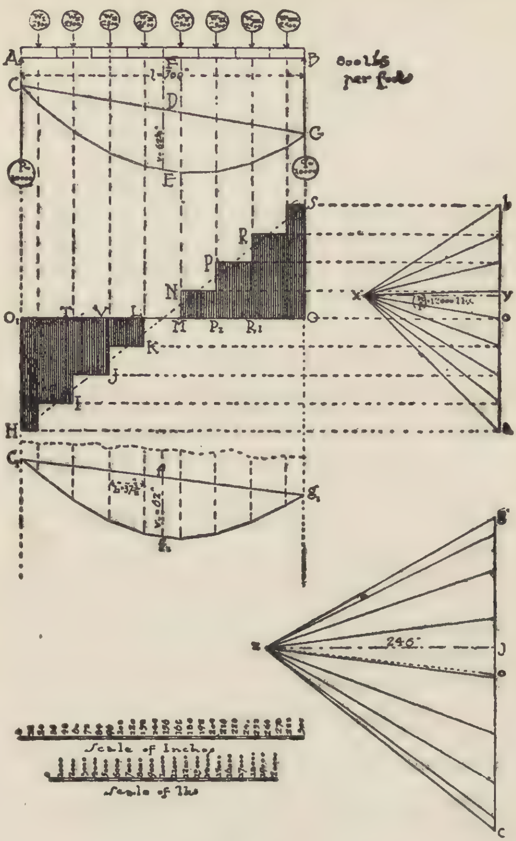


Fig. 156.

Had we used Formula (95) we should have had, remembering that now

$$i = \frac{3.14^8}{12} = 686$$

$$\delta = \frac{102.\,27.\,120.\,1000}{850000.\,686} = 0{,}568''$$

showing that we have made no mistake in applying Formula (31).

If we have any doubts as to whether a 3″ x 14″ stick is as strong as a 5″ x 10″ we use Formula (30) and have for the former

$$x = 3.14^2 = 588$$

while for the latter

$x = 5.10^2 = 500$, so that the 3″ x 14″ stick is actually much stronger, as well as much stiffer than the 5″ x 10″. It is, however, a very thin beam, and would be apt to warp or twist, unless braced sideways about every five feet of its length.

To attempt to get the deflection of the girder arithmetically would be a very tedious operation. It could be done, however, by inserting in Formula (41) the different values for n and m, remembering every time to make n the distance from each weight to the *nearer* support to respective weight, and m the distance from same weight to the further support.

Example V.

Uniform Load. *A wrought-iron beam of 25-foot span (Figure 156) carries a uniform load of 800 pounds per running foot of beam, including weight of beam. The beam is thoroughly braced sideways What beam should be used?*

We draw $A B = 300''$ at inch scale, and then divide our uniform load into a number of equal sections, say eight, each

$$l_{\text{I}} = \frac{300}{8} = 37\tfrac{1}{2}'' \text{ long.}$$

The total load on beam is

$$u = 25.800 = 20000 \text{ pounds.}$$

Each section therefore carries:

$$\frac{u}{8} = \frac{20000}{8} = 2500 \text{ pounds.}$$

We place our arrows $w_{\text{I}}, w_{\text{II}}$, etc., at the centre of each section, which will bring the end ones at $\frac{l_{\text{I}}}{2}$ distant from each support, so that these same verticals will answer when obtaining deflection figure.

We now make $b\,a = 20000$ pounds at pound scale, and divide it into eight equal parts, each equal $w_{\text{I}} = w_{\text{II}} = w_{\text{III}}$ etc., $= 2500$ pounds. We make $x\,y = 12000$ pounds, which is the $\left(\frac{k}{f}\right)$ for wrought-iron, see Table IV. We draw $x\,b$, $x\,a$, etc., and construct figure $C\,E\,G$, which will approach a parabola in outline. The more parts we take the nearer will it be to a parabola.

We draw $x\,o$ parallel $C\,G$ and find it bisects $b\,a$, or each reaction

is one-half the load or $= 10000$ pounds. This we know is the case. The longest vertical will, of course, be at the centre D of $C\ G$, or greatest bending-moment will be at the centre, this we know is the case. $D\ E$ scales (inch scale) $62\frac{1}{2}''$ which will be the *required* r or moment of resistance (Formula 92). The bending-moment at the centre will be, Formula (93).

$$m = 62\tfrac{1}{2}.\ 12000 = 750000$$

Had we used Formula (21) we should have had

$m = \dfrac{20000.\ 300}{8} = 750000$ or same result, and from Formula (18) for

$r = \dfrac{750000}{12000} = 62\frac{1}{2}$ also the same as before. From Table XX we find the nearest r to our required r (62,5) is 69,8 which calls for a $15''$—150 pounds beam; as the beam is braced sideways this will do, if sufficiently stiff.

In regard to shearing, we draw the figure $O_I\ H\ I\ J\ K\ L\ M\ N\ P\ R\ S\ O$ and find shearing on both sides of beam similar, increasing gradually from the centre to ends.[1]

It would be:

Cross shearing from A to $w_I = O_I\ H = 10000$ pounds.
Cross shearing from w_I to $w_{II} = T\ I = 7500$ pounds.
Cross shearing from w_{II} to $w_{III} = V\ J = 5000$ pounds.
Cross shearing from w_{III} to $w_{IV} = L\ K = 2500$ pounds.
Cross shearing from w_{IV} to $w_V = O = 0$ pounds.
Cross shearing from w_V to $w_{VI} = M\ N = 2500$ pounds.
Cross sheering from w_{VI} to $w_{VII} = P\ P_I = 5000$ pounds.
Cross shearing from w_{VII} to $w_{VIII} = R\ R_I = 7500$ pounds.
Cross shearing from w_{VIII} to $B = S\ O = 10000$ pounds.

The area of web of a $15''$—150 pounds beam (Table XX) is 7,59 square inches; the safe resistance of wrought-iron to cross-shearing per square inch being $\left(\dfrac{g}{f}\right) = 8000$ pounds, we need not worry any further on that score.

To find the deflection we now make the lower load line $g\ c$ equal to the sum of the lengths of verticals w_{VIII}, w_{VII}, w_{VI}, etc., through parabola $C\ E\ G$, beginning at top g with length of right vertical w_{VIII}. We select z at random, scale $z\ j = 246''$ (inch scale), draw $z\ g$, $z\ c$, etc., and figure

[1] Had we taken more parts, the steps in shearing figure would become smaller and smaller till they would finally assume the straight line $H\ S$, which is the real outline of shearing figure.

$c_1 f_1 g_1$. We now draw $z\,o$ parallel $c_1\,g_1$ and find it bisects $g\,c$, or greatest deflection will be at centre of beam, which we know is the case. We scale $f\,f_1 = 62''$ (inch scale); find from Table XX for our $15'' - 150$ pounds beam $i = 523,5$ and from Table IV for wrought-iron $e = 27000000$, therefore, Formula (95):

$$\delta = \frac{62.37, 5.246.12000}{27000000.523,5} = 0,486''$$

Had we figured arithmetically, Formula (39), we should have had

$$\delta = \frac{5.20000.300^3}{384.27000000.523,5} = 0,497'$$

or practically the same result.

The safe deflection for plastering should not exceed (28)

$$\delta = 25.0,03 = 0,75''$$

so that we are perfectly safe, providing our beam is well braced sideways.

Example VI.

Uniform and Concentrated Load. *A wrought-iron beam, braced sideways, of 30-foot span, Figure 157, carries a uniform load of 200 pounds per foot, including weight of beam. It carries also a concentrated load $w_1 = 10000$ pounds ten feet from the right-hand support. What beam should be used?*

We draw beam $A\,B = 360''$ at inch scale, we divide uniform load into, say, six equal parts, each 5 feet long, or $l_1 = 60''$. The total uniform load will be $u = 30.200 = 6000$ pounds, therefore each part $\frac{u}{6} = \frac{6000}{6} = 1000$ pounds. We draw arrows at the centre of each uniform part, so that the end arrows will be one-half part from supports. These arrows will therefore answer for our verticals, when drawing deflection figure.

At $120''$ from right hand support we locate the load $w_1 = 10000$ pounds.

We now make load line $b\,a = 16000$ pounds the total load and divide it, so that

$$b\,l = w_{VII} = 1000 \text{ pounds.}$$
$$l\,h = w_{VI} = 1000 \text{ pounds.}$$
$$h\,f = w_{I} = 10000 \text{ pounds.}$$
$$f\,e = w_{V} = 1000 \text{ pounds.}$$
$$e\,d = w_{IV} = 1000 \text{ pounds.}$$
$$d\,c = w_{III} = 1000 \text{ pounds.}$$
$$c\,a = w_{II} = 1000 \text{ pounds.}$$

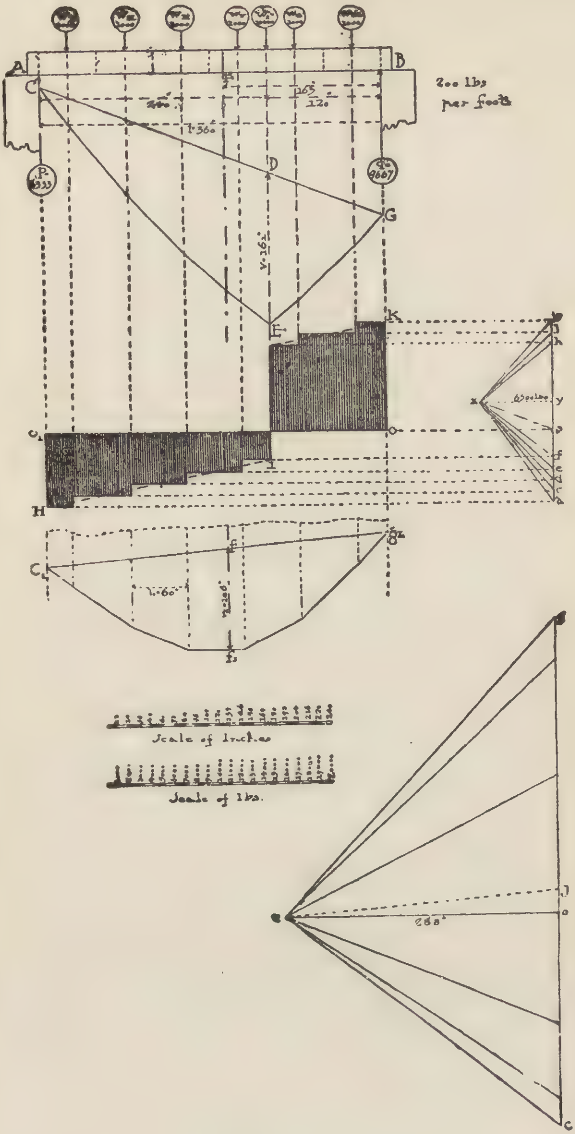


Fig. 157.

Select pole x distant from load line at random (for the sake of illustration, though it would be better to make $x\,y = \left(\frac{k}{f}\right) = 12000$ pounds.) We find $x\,y$ scales 6500 pounds. We now draw $x\,b, x\,l, x\,h, x\,f$, etc. And construct figure $C\,E\,G$. Draw $x\,o$ parallel $C\,G$ and we find $a\,o$ (or reaction A) scales $= 6333$ pounds, and $o\,b$ (or reaction B) scales $= 9667$ pounds.

The longest vertical is $D\,E = 161''$ (inch scale) therefore greatest bending-moment is at w_{I} and from Formula (93)

$$m_{w_{\text{I}}} = 161.6500 = 1046500$$

For the *required* moment of resistance we have from Formula (18)

$$r = \frac{1046500}{12000} = 87{,}2$$

The *cheapest* or most economical nearest section we find — to this required r (87,2) is the $20''$ — 200 pounds beam of which the moment of resistance is $r = 123{,}8$.

Had we combined the formulæ for uniform and concentrated loads and worked out the problem arithmetically it would have been tedious, but we should have had similar results.

We can safely overlook shearing, but note that the real shearing figure would not be the shaded figure, but dotted figure $O_{\text{I}}\,H\,I\,J\,K\,O$

For finding the deflection we now draw lower load line $g\,c =$ the sum of the verticals through $C\,E\,G$, beginning at top with length of w_{VII}, then w_{VI}, w_{V}, w_{IV}, w_{III}, and w_{II} in their order. We take no notice of vertical w_{I} as it does not fall in one of the even divisions of $C\,G$ or $A\,B$ into lengths l_{I}. We select pole z distant $z\,j = 288''$ from load line, draw $z\,g$, $z\,c$, etc., and then figure $c_{\text{I}} f_{\text{I}} g_{\text{I}}$. We now draw $z\,o$ parallel $c_{\text{I}}\,g_{\text{I}}$ it divides $g\,c$, so that $g\,o = 295''$ and $o\,c = 245''$, we divide $c_{\text{I}}\,g_{\text{I}}$ in same proportion at f, and carry this up to F at beam, which is the point of greatest deflection of beam, and is distant $163''$ from B, and $197''$ from A. We scale $f\,f_{\text{I}} = 106''$ (inch scale) and have from Formula (97)

$$\delta = \frac{106.\,60.\,288.\,6500}{27000000.\,1238} = 0{,}357''$$

1238 being $= i$, the moment of inertia of beam as found in Table XX. The beam is therefore amply stiff even to carry plastering.

Irregular Cross-sections. The graphical method lends itself very readily to finding centres of gravity and neutral axes, as explained in the chapter on arches, and also for finding the moments of inertia of difficult cross-sections.

If we have an irregular figure $A\ B\ C\ D\ E$ (Figure 158) we divide it into simple parts I, II, III and IV. We find the centres of gravity g_{I}, g_{II}, g_{III} and g_{IV} of each part and draw their respective horizontal neutral axes through these. Anywhere's make a line $a\,e$ = area of whole figure and divide it, so that:

To find Neutral Axis.

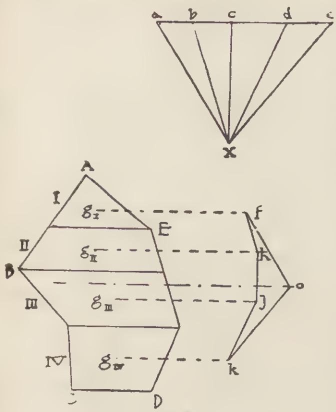


Fig. 158.

$a\,b$ = area of I
$b\,c$ = area of II
$c\,d$ = area of III and
$d\,e$ = area of IV.

Select pole x at random, draw $x\,a$, $x\,b$, $x\,c$, $x\,d$, and $x\,e$.

From any point of horizontal g_{I} draw $f\,h$ parallel $b\,x$ till it insects horizontal g_{II}; then draw $h\,j$ parallel $c\,x$ to horizontal g_{III}; then $j\,k$ parallel $d\,x$ to last horizontal, and finally $k\,o$ parallel $x\,e$; and $f\,o$ parall°l $a\,x$ till they intersect at o. A horizontal through o is the main neutral axis of the whole.[1] If we multiply the area of the figure $f\,o\,k\,j\,h$ by the area of the figure $A\,B\,C\,D\,E$ (both in square inches) we have the value of moment of inertia i of $A\,B\,C\,D\,E$ in inches, around its horizontal neutral axis o.

To find Moment of Inertia.

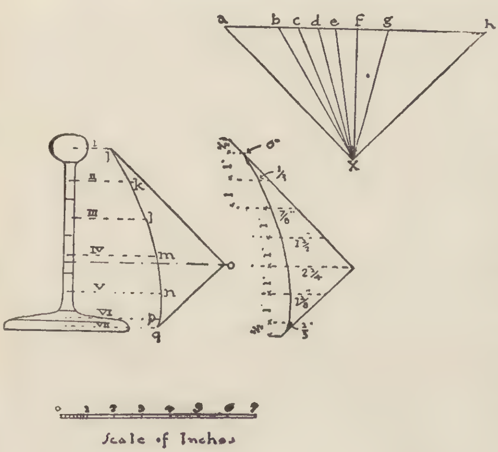


Fig. 159.

[1] The point of intersection of this line with a main neutral axis, found similarly, in any other direction, would be the centre of gravity of the whole figure.

A simple way of obtaining the area of the figure $f\,o\,k$ would be to draw horizontal lines through it at *equal* distances beginning with half distances at *top* and *bottom*, and to multiply the sum of these horizontals in length by the distance apart of any two horizontals, all measurements in inches. This will approximate quite closely both the area and moment of inertia. Of course the more parts we take in all of the processes, the closer will be our result.

To find area.

A practical example will more fully illustrate the above.

Example VII.

Rolled Deck-beam.

Find horizontal neutral axis and the corresponding moment of inertia of a 7″ — 55 pounds per yard deck beam, resting on its flat flange (Figure 159).

We will take the roll as one part, divide the web into four equal parts, the flange into two parts, one the base which will be practically rectangular, and its upper part which will be practically triangular. The whole area we know is for wrought-iron:

$$a = \frac{55}{10} = 5,5 \text{ square inches.}$$

The bottom rectangular part of flange will be

$$a_{vii} = 4\tfrac{1}{2} \cdot \tfrac{3}{8} = 1,7 \text{ square inches}$$

next triangular part

$$a_{vi} = \frac{4\frac{1}{2} \cdot \frac{3}{8}}{2} = 0,9$$

The web parts

$$a_{ii} = a_{iii} = a_{iv} = a_v = \frac{\frac{5}{16} \cdot 5\frac{1}{2}}{4} = 0,4 \text{ square inches each.}$$

Leaving for the roll at top $a_i = 1,3$

We now make the horizontal line $a\,h = 5,5''$ and divide it, so that

$a\,b = 1,3$ inches

$b\,c = c\,d = d\,e = e\,f = 0,4$ inches

$f\,g = 0,9$ inches and

$g\,h = 1,7$ inches.

Select x at random and draw $x\,a$, $x\,b$, $x\,c$, etc.

Draw the horizontal neutral axes I, II, III, etc., through their respective parts. Begin anywhere on I and draw $j\,k$ parallel $b\,x$ to line II; then $k\,l$ parallel $c\,x$ to III; then $l\,m$ parallel $d\,x$ to IV; then $m\,n$ parallel $e\,x$ to V; then $n\,p$ parallel $f\,x$ to VI; then $p\,g$ parallel

$g\,x$ to VII; Now draw from q the line $q\,o$ parallel $x\,h$, and from j the line $j\,o$ parallel $a\,x$ till they intersect at o. A horizontal through o is the neutral axis of whole beam. We will now make a new drawing of figure $j\,o\,q$ for the sake of clearness. Draw horizontals through it every inch in height beginning at both top and bottom with one-half inch. The top one scales nothing, the next $\frac{1}{3}''$, then $\frac{7}{8}''$, then $1\frac{1}{2}''$, then $2\frac{1}{4}''$, then $1\frac{1}{8}''$, and the bottom one $\frac{1}{8}''$, the sum of all being $6\frac{5}{12}''$ or 6,416″. This multiplied by the height of the parts, which is one inch, would give us, of course, 6,416 square inches area. Multiplying this area by the area of the cross-section of deck beam 5,5 square inches, we should have

Horizontal Neutral Axis.

Area of Diagram.

$$i = 5{,}5.\,6{,}416 = 35{,}288.$$

Moment of Inertia of Beam.

In Table XX it is given as 35, 1 so that we are not very far out.

If we had taken more parts, of course the result would have been more exact.

Reducing Flanges, Plate Girders.

When constructing plate girders of large size, much material can be saved by making the flanges heaviest at the point of greatest bending-moment, and gradually reducing the flanges towards the supports.

This is accomplished by making each flange at the point of greatest bending-moment of several thicknesses or layers of iron, the outer layer being the shortest, the next a little longer, etc. Of course the angles, which form part of the flange are kept of uniform size the whole length, as it would be awkward to attempt to use different sized angles. Generally (though not necessarily) the inner or first layer of the flange plates, is also run the entire length. Of course, where the flanges are gradually reduced in this way, it becomes necessary to figure the bending-moment and moment of resistance at many points along the plate girder to find where the plates can be reduced. This would be a wearisome job. By using the graphical method, however, it can be easily accomplished. Referring back to Figure 151, we take the point of greatest bending-moment (at w_1) of the beam $A\;B$. The required moment of resistance at this point, it will be remembered was the length (inch scale) of vertical E through $C\,D\,E\,F\,G$. We now decide what size angles we propose using and settle the necessary thickness of the flanges by Formula (36), inserting for the value of r, the length (inch scale) of v or vertical at E. Further a_1 will, of course, be the sum of the area of two angles, d the

total depth of girder in inches and b the breadth of flange, in inches, less rivet holes. The above is on the assumption that the distance $x\,y$ of pole x from load line $d\,a$ was equal to the safe modulus of rupture $\left(\frac{k}{f}\right)$ of steel or wrought-iron according to whichever material we were using, or we should have:

Thickness of Flanges.

$$x = \frac{\frac{v}{d} - a_{\text{I}}}{b} \qquad (98)$$

Where x = the thickness, in inches, of each flange of a plate girder at any point of its length.

Where v = the length of vertical, inch scale, through upper or resistance figure, providing we have assumed the distance of pole from load line (pound scale) $= \left(\frac{k}{f}\right)$ of the material.

Where d = the total depth, in inches, of the plate girder.

Where b = the width, less rivet holes, in inches, of the flange.

Where a_{I} = the sum of the areas of cross-section, in square inches, of *two* of the angles used.

We now calculate as above, the thickness x of flange at point of greatest bending-moment and then decide into how many layers or thicknesses we will divide the flanges. Say, in our case we decide to make the flange of four layers of plates, each $\frac{x}{4}$ or one quarter x in thickness. Then make

$$E_{\text{I}}\,E_{\text{II}} = a_{\text{I}}.\,d \qquad (99)$$

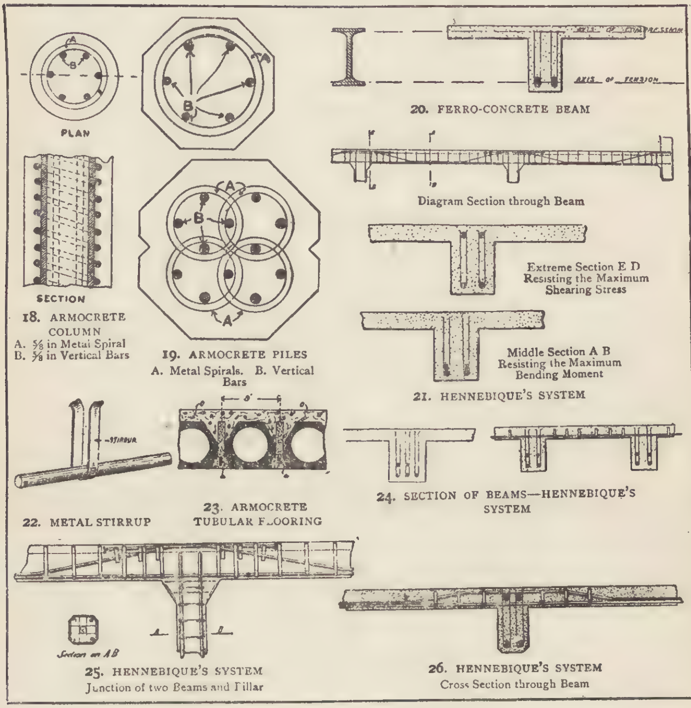
Where $E_{\text{I}}\,E_{\text{II}}$ = the amount to be substracted (inch scale) from moment of resistance or vertical v and representing the work of two angles.

Where a_{I} = the sum of the area of cross-section, in square inches, of the two angles.

Where d = the total depth, in inches, of the girder.

Where to drop off Plates. Now draw through E_{II} a parallel to base of figure $C\,G$, divide $E_{\text{II}}\,E$ into as many parts as we decide to use thicknesses of plates (four in our case) and draw parallel lines to base $C\,G$ through these parts. Vertically over the points where these lines intersect the curve or outline of figure $C\,D\,E\,F\,G$ will be the points at which to break off plates, as illustrated in drawing. This method, of course, is approximate, but it will be found sufficiently accurate for all practical purposes. It is not necessary that x

or $E\ E_{\text{II}}$ be divided into equal parts. Had we decided to use plates of varying thicknesses we should simply divide $E\ E_{\text{II}}$ in proportions to correspond to thicknesses of plates *in their proper order*, beginning at E_{II} with plate immediately next to angles and ending at E with extreme central outside plate. An example, more fully illustrating the above, will be given in the chapter on plate girders.



18. ARMOCRETE COLUMN
A. ⅝ in Metal Spiral
B. ⅝ in Vertical Bars

19. ARMOCRETE PILES
A. Metal Spirals. B. Vertical Bars

20. FERRO-CONCRETE BEAM

Diagram Section through Beam

Extreme Section E D
Resisting the Maximum
Shearing Stress

Middle Section A B
Resisting the Maximum
Bending Moment

21. HENNEBIQUE'S SYSTEM

22. METAL STIRRUP

23. ARMOCRETE TUBULAR FLOORING

24. SECTION OF BEAMS—HENNEBIQUE'S SYSTEM

25. HENNEBIQUE'S SYSTEM
Junction of two Beams and Pillar

26. HENNEBIQUE'S SYSTEM
Cross Section through Beam

CHAPTER VIII.

REINFORCED CONCRETE CONSTRUCTION.

Modern useful concrete constructions began in France, in the beginning of the last century; but owing to the nature of the material, the constructions were necessarily confined to mostly heavy work, such as foundations, retaining walls, dams and similar structures.

Historical Facts.

As a matter of fact concrete was well-known to the Ancients. The dome of the Pantheon is of concrete; while during the various Gothic periods, vaultings, floors, etc., were made of concrete, and frequently walls and other parts of cities, forts and castles as well. But even further back the Aztecs of Mexico, the old Greeks and other very ancient nations used concrete. They however used lime or ground shells or similar substances.

It was with the discovery of the method for making Portland cements, that concrete became an active factor in modern above-ground constructions.

The French used it largely in floors, forcing between the iron beam flanges, corrugated sheet iron, in segmental arches, and filling above this with concrete. By the time the concrete had set thoroughly, it mattered little whether the sheets lasted or not.

The subject of concrete itself, and of concrete underground constructions has already been treated in Chapter II.

In over-ground constructions the use of concrete began, (after dams, retaining walls, etc.) by the use of moulded, generally hollow, interlocking blocks, formed in moulds, of fine concrete. This system is still in vogue, and is so simple and well-known it needs no further comment here.

But with the beginning of reinforced concrete, invented by the French about the time of the ending of our Civil War, a new era dawned for construction in this country, and to-day *re-inforced* concrete bids fair, before very long to re-place many—if not most—other systems of masonry or timber construction.

What Reinforced Means.

As is well known concrete properly made of Portland cement continues to set until it becomes as hard as (almost) the hardest stones used for construction purposes. It resists compression strains admirably,

and is therefore used in piers, walls and similar constructions, to carry super-imposed loads, providing it cannot give way laterally.

But, where it is not braced laterally, the bending tendencies in walls or columns of great height, compared to their bases, would quickly crack and let the walls or columns down, as concrete is very poor indeed in resisting tension.

Similarly in floors, the concrete would be admirable for the upper layers of floors, beams or girders, which layers are in compression, but the bottom layers being in tension, the bottom would tear apart, being unable to resist much tension strain, and the whole give way in consequence.

Metal Bars, Etc. To off-set this weakness steel square, round, rectangular, or other shaped bars are introduced into the concrete and so designed as to act *together* with the concrete, the latter attending to the compression, the steel taking up and resisting the tension.

The methods of doing this are innumerable. Almost every large city in the United States has several companies doing concrete construction, each in their own way.

The essential thing is to roughen the steel so that the concrete will take hold of the metal, and not allow the steel to slip past the concrete, without bearing on each other, and every part of both working together.

Many companies do this by means of specially rolled shapes, with projections, indentations, ribs, etc., to form roughened surfaces for the concrete to clinch to. Other companies use wire, others still, punched metals, patent laths, expanded metals, etc. The best thing for the architect, lacking experience, is to select the method that appeals to him and study it carefully; it would be impossible to cover the subject in a general work of this kind.

These metal forms are imbedded into the wooden moulds and frequently wired together, before the concrete is poured in.

It should be explained that for all floor constructions wooden moulds on centres (with proper support), are formed underneath, into which the concrete is poured. **Removing Moulds.** Moulds should never be removed until the concrete is known to be thoroughly set. At least two weeks, or longer, is a good rule for floors; three weeks or longer for girders, columns and walls.

Continuous Floors. Floors that are built *over* girders and beams, that is on top of them, instead of in between are much stronger, in the same ratio as the continuous girder is stronger than the girder butting over supports. But, it must be

borne in mind that this very strength is obtained by a reversal of the strains, (see Figs. 143 and 144, Chap. VI) the top layers over and near the supports being in tension, the bottom in compression. It is necessary therefore where bars are used, to rivet them together over supports, to get the benefit of the additional strength, and where rods and similar shapes are used to bring them up from the lower layers of concrete to near the top.

Plate L gives a good idea of what is meant, and shows the steel skeleton and wire construction for columns and girders, walls, etc. as well.

Where floors are supported on all four sides, that is built in squares over interlacing or crossing girders, thus forming square panels, they are still stronger.

Concrete roofs are built similarly to floors, but generally sloped to form pitch and to save extra weight. Domes are similarly moulded up and reinforced.

In all floor constructions, care should be taken to avoid a steel that is very soft and will easily stretch; as, while it might hold, it will badly crack the whole construction.

One system of construction avoids this difficulty, by fixing each rod of steel in vises at each end, so that the length between secured ends cannot change, and then twisting the steel, thus taking out the stretch to close to the elastic limit, and at the same time the twisted, cork-screw shaped rods give the concrete a good chance to take hold.

No formulae, that are sufficiently scientific can be given as yet, as most of them are empirical, many even rule-of-thumb only; it is remarkable however, how much a thin floor of flat concrete slabs will hold, when properly reinforced, and what wide spans may be used.

As a rule, where the floor slabs are supported only at the two ends the following will be sufficient for the architect to approximate his construction.

Allowable Spans. Not including the load of the construction, for a live load of 150 pounds per square foot, the safe thickness of slabs should not be less than:

3 " thick for spans of 7 feet or less.
3½" thick for spans of 8 feet.
4 " thick for spans of 9 feet.
4½" thick for spans of 10 feet.
5 " thick for spans of 11 feet.
6 " thick for spans of 13 feet.

But in each case the slabs must be well supported and thoroughly reinforced.

Where the slabs are square and supported on all four sides, very much greater spans may be used. The deeper the slab, the stronger of course, but the greater the expense and the weight on other parts.

Girders and Beams. Girders and beams are formed similarly to floor slabs. The necessary moulds are built, in forms of boxes, open on the top, and thoroughly supported from below. The steel re-inforcement is then put in, and is usually in the shape of trussed bar work, with additional U bars or stirrups. These latter are spaced, similarly to stiffeners on riveted girders, closer together at supports, and further apart at centre or near point of greatest bending moment of the concrete girder. Their object is to take care of the shearing strains at supports, and prevent cracking of concrete there. Girders thus trussed up, with U straps, etc. are frequently called armored girders. The concrete is poured into the moulds from the top, and the frames and supports of the moulds should be left in place as long as possible.

For purposes of laying out his work, the architect should make the depth of the girders, in inches, not less than the span in feet.

Columns and Piers. Columns and piers are built similarly as girders, but the boxes are whole boxes with only the ends open. The vertical rods are placed in position and then wired together when the concrete is poured in from the top and allowed to set before removing the jacketings from mould.

Walls. Walls are similarly built. Moulds are made, placed in position and thoroughly secured in place. The vertical rods and horizontal truss rods are placed inside, wired together, concrete poured in and the whole allowed to set thoroughly. A building, that is its floors, columns, girders, walls, can be made a practical monolith by attaching all ends together, both of rods, wires and concrete.

The diameter or sizes of columns, piers, etc. and the thicknesses of walls in most cities is regulated by Law, or Building Department regulations.

But after all is said, the greatest thing for the architect to do, is to watch, eternally watch, the construction, to see that the concrete is properly mixed, and allowed to set sufficiently long before removing moulds. Great care must be taken too, by the architect to see that no frost gets into any part of the work.

The question of exterior finish of concrete hardly enters into

a work of this kind, but the writer has discussed this quite fully in a letter, one of a series asked for by the Editors of the American Architect on the subject: "The Aesthetic Treatment of Concrete." This letter was published in the edition of May 4, 1907. (American Architect.)

Colored Treatment of Facades.

The main and most original suggestion made by the writer was that where a colored treatment of the exterior or main facade was desired, the constructional concrete be kept back some two or more inches, with sufficient anchoring wires left to anchor outside finish. That the outside finish then be cast into moulds, specially made for that purpose, with a concrete made of very small parts of either broken stone, brick or terra-cotta (or a mixture thereof), Portland cement, water and finely ground similar stone, brick or terra-cotta grains, (or a mixture thereof) the latter to take the place of the sand. By experimenting and allowing for the effect of the cement coloring in connection with the mixtures of small broken parts and finely ground grains, as above, almost any scheme of coloring for the facade can be thus obtained, and it has the benefit of being *natural* to the construction, a part of it, and very durable.

The reader is referred to pages 95 to 99 (incl.) for further information on reinforced concrete in sub-soil constructions.

CHAPTER IX.

RIVETS, RIVETING AND PINS.

WHEN it is necessary to secure two or more pieces of iron or steel together in such a manner that they can be readily separated, bolts are used. These are iron or steel pins with solid heads at one end and threads cut on the other end, onto which the nut is screwed, thus holding the two pieces together. How closely the two pieces may be held together depends, of course, entirely upon the man who handles the wrench. Then too, bolts or pins do not completely fill the holes through which they pass, which frequently is a cause of great weakness, besides the danger of water getting into the spaces and rusting them. Where, therefore, it is not necessary to ever separate the pieces — and the latter are of either wrought-iron or steel — rivets should be used, which, for all practical purposes, might be considered as permanent bolts or pins. Cast-iron, of course, always has to be bolted, as it would break if riveting were attempted on it. A rivet is a piece of metal (wrought-iron or steel) with a solid head at one end and a long circular shank. In its shortest description the process of riveting consists in heating the rivet, passing its shank through the two (or more) holes, while hot, and then forging another solid head out of the projecting end of the shank. The hammering forces the heated shank to fill all parts of the holes, and the shank contracting in its length, as it cools, draws the heads nearer together, thus firmly forcing and holding the pieces together.

Description of Rivet.

Rivets are made of mild steel or the very best wrought-iron, the latter being the most reliable. According to some writers, the shank is made tapering in length and circular in shape, being larger at the head and smaller at the end. In this country, however, the shank is always of uniform diameter. The length of shank from head to end varies with the thickness of pieces to be riveted together, but is long enough not only to allow for passing through the pieces, but has also enough additional length to

Length of Tail.

provide for filling of holes and forming head, the additional length being about $2\frac{1}{2}$ times the diameter of hole. The rivets are manufactured by heating rods of the diameter of rivets, which are pushed while hot into a machine, which at one operation forms the head and cuts off the shank to the desired length.

The shank before heating is about one-sixteenth smaller in diameter than the hole, to allow for its expansion when heated, *i. e.*, for $\frac{3}{4}''$ rivets, $\frac{13}{16}''$ holes are punched and for $\frac{7}{8}''$, $\frac{15}{16}''$ holes. There are four kinds of rivets, all answering the same uses, and only distinguishable by the shapes of their heads. These are the button or round headed rivet; the conical headed rivet; the pan or flat headed rivet; and the countersunk rivet. The latter is only used when a smooth surface is desirable. The first is the most used shape. Figure 163 shows the different shapes, the dotted lines indicating how the second head is finally shaped. Sometimes a rivet is countersunk on one end only.

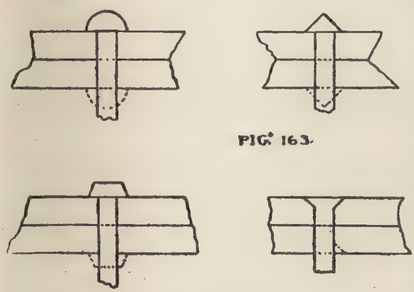


FIG. 163.

The exact sizes of heads, shapes, etc., vary in different mills.

Size of Head. The diameter of head should be from $1\frac{1}{2}$ to 2 times the diameter of shank,[1] according to shape adopted and the height of head should be about $\frac{2}{3}$ the diameter of hole. In countersunk work, the head may extend entirely through the plate or not, its diameter being accordingly smaller or larger. Where it extends through the sharp edges will shear the rivet, countersinking therefore, should be avoided in the plates. In showing riveted work the diameter of the shank is always drawn and figured where the hole is to be left open and the size of rivets is spoken of accordingly, the hole is always made $\frac{1}{16}''$ larger. Where the riveting is to be done at the shop or mills, the size of head is shown.

Pitch of Rivets. The spacing of rivets will be considered later, the direct distance from *centre of hole to centre of hole* is known as the

[1] The Franklin Institute standard for button-heads (which are usually used in the United States) is to make the head $\frac{1}{8}''$ larger in diameter than $1\frac{1}{2}$ times the diameter of shank.

"*pitch.*" The pitch should never be less than $2\frac{1}{2}$ diameters; nor should the centre of any hole (if possible) be nearer to any edge than $1\frac{1}{2}$ diameters. By diameters is understood the diameter of shank. In riveted angle work the distance is frequently necessarily less. In thick plates it should be more. In drilled work the pitch might be reduced to 2 diameters. If rivet-heads are countersunk the pitch should be increased, according to the amount of metal cut away to make room for the rivet-head.

Punching of Holes.

In punching rivet-holes the position of holes are usually marked off on a wooden template and then through this marked or indented by a hand-punch on the iron plate; the plate is then passed under a punch which is usually worked by steam-power, the die and the punch being adjusted to the size of the rivet-hole wanted, the punch is usually $\frac{1}{16}''$ larger than the rivet, and the die about $\frac{1}{8}''$ larger. The thickness of plates to be punched should not, as a rule, be greater than $\frac{3}{4}$ of an inch; nor in any case should the thickness of plate be as large as, nor larger than, the rivet-hole, as, unless the diameter of the hole is larger than the thickness of the plate the punch is apt to break. Where holes are punched at the building, small screw or hydraulic (alcohol) punches are used, which can be readily carried around by one or two men, the power being obtained by screwing or pumping; or sometimes, where mechanics are not quite up to the times, a rather more clumsy lever-punch is used, the power being obtained by increasing the direct pressure by leverage. Punching makes a ragged and irregular hole, and as it gives the plate a sudden blow or shock it injures the metal considerably, unless the rivet-holes are so close, that the entire plate is practically cold-hammered. The loss

Loss by Punching.

in strength to the remaining fibres in a punched wrought-iron plate is about 15 per cent, this loss being, of course, in addition to the loss of area, and it is a loss that cannot be restored. In steel plates the remaining fibres are damaged about 33 per cent, but in them the loss can be restored by annealing the plate which, however, adds considerably to the expense.

In drilled-work there is no loss, and the holes are not only accurately located but are accurately cut. But drilled-work is very expensive, as it has to be done by hand or by machine-drills, the process being slow at best and consequently meaning a very large

addition to the charge for labor. In riveted girders it would probably double the expense of the girder.

Advantages of Drilling.

The advantages of drilling are, that the holes can be cut after the plates have been partially secured together, thus assuring a perfect fit of the holes over each other; and that the holes being perfectly smooth and even bear more evenly on the rivet, and the work is less apt to fail by compression, than where the bearing of plate against rivet is ragged and uneven, as in punched work. On the other hand, the edges of drilled holes are so sharp that they promote shearing, and for this reason the edges of drilled holes in plates should be filed or reamed off. As a rule, however, the architect will find the bearing and cross-breaking strengths of rivets less than their shearing, excepting where rivets are small in comparison to thickness of plates being riveted, which is not often the case.

To settle, then, whether work should be drilled or punched, is mainly a matter of expense. Drilled-work, of course, is far preferable as regards strength and it costs accordingly.

The rule of the mills is to punch all holes, excepting for countersunk rivets, which, after punching, of course, have to be drilled, to obtain the slanting sides of the hole.

A medium course between drilling and punching would be to punch the holes smaller than desired and then drill or ream them to accurate size when partially secured together.[1] Steel should always be drilled unless annealed after punching.

In most work, however, the architect will have to be satisfied with punching, and must therefore allow sufficient material to make good the damage done and to allow for inaccuracies.

Machine-Riveting.

In riveting proper, that is, filling the holes, there are also the two methods of doing it, by hand (hand-riveting), or by machinery (machine-riveting), but unlike the making of the hole, in this case, the machine-work is both better and cheaper.

A machine-driven rivet is driven and completed by one powerful squeeze of the steam (or compressed-air) riveting-machine; this squeeze not only forces the plates more closely together, but more completely fills the hole with the rivet metal, besides the great ad-

[1] If this is not done the "drift pin" will be used to force all the holes into line, and this means crushing and possibly buckling of the plates.

vantage of doing the entire work while the rivet is hottest, and while it is, of course, at one temperature.

In hand-riveting these advantages are lost, the power being only equal to that of the mechanic's blow, and as in hand-work the process consumes some time, the rivet changes its temperature and cools considerably.

In riveting, the entire rivet, including the head, should be heated to at least a red heat. It should not be heated beyond this for fear of "burning," particularly with steel rivets. Rivets that have been heated once and allowed to cool without working should be discarded. If rivets are driven at a lower heat than a red one, they will be greatly damaged, unless riveted cold.

Hand-Riveting.

In hand-riveting at least two men are required, one to hold the head, the "holder-up," the other to do the riveting; but generally there is a boy to heat and bring the rivets, one holder-up, and two riveters, whose strokes alternate and thus accelerate the process.

The riveter puts a punch or drift-pin through the holes to clear them and force them into line; the holder-up seizes the hot rivet with his pincers and puts its shank through the hole, he then covers the head with a holding-up-iron shaped to fit it, and the riveters at the other end begin hammering down the projecting end of shank. When this is roughly shaped the use an iron (called "button set," for round-headed rivets), which is properly shaped to make the head. Before beginning to hammer down the end of shank, the riveters should always thoroughly hammer the plates around the hole, to bring the plates closely together.

How to Distinguish Riveting.

Hand-riveted work can sometimes be distinguished from machine-riveted work by the many marks at the made head. In machine-work there is but one mark, and this may be a little out of the centre with the shank and so show the squeezed material around its edge. But usually the work cannot be distinguished in this way. If hand-riveting has been conscientiously done and by careful mechanics, it is difficult to distinguish it from machine-riveting. But the shanks of hand-riveted work, as a rule, do not fill the hole as well as those in machine-riveted work, and they can more easily be "backed out" after the head has been cut off with a chisel and hammer. The only reliable test in both methods is to hold the hand one side of the head and

strike the other side with a light hammer — the hammer test — when the sound will quickly disclose loose shanks. If in machine-riveting the plate has been sprung, and the pressure is quickly removed while the rivet is still hot, the plate may settle back, lift the hot head, and so form a loose shank.

In designing riveted work, whether to be hand or machine riveted, the architect should bear in mind the necessity of placing the rivets so that they can be inserted in the holes from one side and hammered from the other, and for machine-work, that the machine can reach them.[1]

Steel rivets damaged.

Steel rivets are very seriously damaged during the process of riveting. Box gives as the average of a number of tests of bar steel a tensional-strength of 47,84 tons (gross), which, after riveting, was equal in shearing-strength to only 23,77 tons (gross), a loss of 50 per cent as between the tensional-strength of the steel and shearing-strength of the riveted work, whereas in ordinary steel work this loss never exceeds $33\frac{1}{3}$ per cent, as between tension and shearing.

In wrought-iron the loss is about 15 per cent. In steel the strength of the rivets will depend greatly upon the composition and nature of the steel itself, but in order to be able to rivet, the steel will necessarily have to be of a mild character. The safe values given in Table IV, for compression and shearing of wrought-iron and steel, can therefore be used with perfect safety.

Use larger modulus of rupture.

When calculating bending-moments on rivets, a modulus of rupture 25 per cent greater than given in Table IV may be used, as the rivet-heads answer the same purpose as nuts and heads on pins in holding together the plates which are pulling in opposite directions, and thus reducing the bending-moment by friction. In figuring the number of rivets required an architect should err on the side of liberality, rather than to stint them, as there will necessarily be more or less of them poorly driven. He should particularly do this where strains are small and the number of rivets required are few, as one weak rivet in a small lot would quickly diminish the factor-of-safety, where in a large lot it would scarcely vary it appreciably.

---

[1] The minimum distance, from inside face of one leg of an angle-iron to centre of *nearest* rivet-hole, in other leg, should be at least $1\frac{1}{4}''$ for $1''$ diameter rivet; $1\frac{1}{8}''$ for $\frac{7}{8}''$ diameter rivet; $1''$ for $\frac{3}{4}''$ diameter rivet; $\frac{7}{8}''$ for $\frac{5}{8}''$ diameter rivet; 13-16″ for $\frac{1}{2}''$ diameter rivet: and if possible these distances should be increased.

Of course the more rivets there are, the more will the plates be injured and cut away; this loss, however, can be largely overcome by what is called "chain-riveting," or "zig-zag" riveting, and by making the laps or cover-plates pointed.

Chain, zig-zag or staggered.

Chain riveting consists of placing the alternate rivets on different lines instead of all on one line, see Figure 164. This again is called

CHAIN RIVETED ZIG-ZAG

FIG. 164.

zig-zag if they alternate as shown.[1] A cross-cut through this plate at any point can only pass through two rivets if chain-riveted or through one rivet-hole if zig-zag riveted; so that the plate is only weakened by two or one rivet-hole, respectively, while it may have a large number of rivets.

Where plates are lapped or joined by cover-plates, the rivets have to transfer the full strain from one plate to the other (in case of a lap); or from one plate to the cover-plate and from that to the other plate (in case of cover-plates). Of course, it can be readily seen that this means a large number of rivet-holes and a very great weakening of the plates. If it were practical to suddenly enlarge the plate at the riveting point there would be no loss, but this would be clumsy and besides it would not be practicable to roll plates with certain points enlarged or thickened. As the whole plate, however, will be equal only to its strength at its least cross-section the rivets should be so disposed as to weaken the plate as little as possible. This is done by pointing the plate ends in the case of lapping, or the ends of cover-plates where these are used, and are not covered in the construction. Where the plates are to be ultimately hidden out of sight, this expense is saved, the plate ends are left square, but the rivets are placed pointedly or pyramidically.

Plates weakened by holes.

Thus, in Figure 165 is shown a lapped joint with staggered rivets; we will suppose that calculation has shown the necessity of nine rivets to equal the tensional strain

Lapped joints.

[1] Most mills use the term "staggered" in place of "zig-zag," but for convenience in writing the writer prefers to use the term staggered to mean zig-zag rivets placed pyramidically.

on each plate. The under plate A is dotted, the upper plate B drawn with full lines. By arranging the rivets as shown in Figure 165, each plate is weakened only by one rivet-hole. (As already explained the plates themselves need not necessarily be pointed, but can be left square, if expense must be considered and looks are no object.) For, while a section at C shows plate A weakened by two rivet-holes (Nos. 2 and 3) it must be remembered that the strain on A is no longer the full strain, but has been diminished by an amount equal to the work that would have come on the one rivet-hole; for rivet No. 1 has already transferred this amount to plate B. Similarly while a cut at D shows three rivet-holes (Nos. 4, 5 and 6) the plate is really not weakened at all here, for an amount of strain equal to what would have come on the metal taken from these rivet-holes has already been transferred to plate B by rivets Nos. 1, 2 and 3.

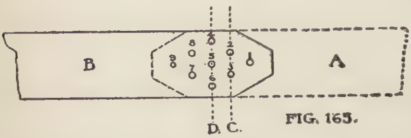


FIG. 165.

Similarly as the strain on B increases the rivet-holes in it diminish till at No. 9, plate B has got the full strain and is therefore only weakened by this one rivet-hole.

The disadvantage of lapping plate ends is obvious, as the plate would not continue in the same plane. For this reason joints are generally made by butting the ends of the plates and covering one or both sides with cover-plates. The principle of riveting is the same as for lapped joints, but it will require a different disposition of the rivets, and twice the number of rivets, as plate A (Figure 166) has to transfer its strain to the cover-plate by one series of nine rivets, and the cover-plate transfers it to plate B by another series of nine rivets. This can be readily seen in Figure 166. In this case the disposition of the rivets requires an extra rivet each side, or 20 in all.

Butt joint with cover-plates.

Where it is not necessary to keep plates A and B in the same plane it would be cheaper and better of course to lap them, rather than to use *one* cover-plate. If, however, two cover-plates can be used, one on top of the plates and the other under them, the advantage is very great, as the strain will be transmitted in a direct line or plane from plate A

Two cover-plates best.

to plate B, and besides this the joint is greatly strengthened by the friction between the cover-plates and plates A and B. In an experiment made by Clark with three $\frac{5}{8}$ inch thick plates riveted with one $\frac{7}{8}$ inch rivet through an oblong hole, it was found that friction added about 5 tons resistance against pulling out the centre plate. This would seem to add a safe-strength to each $\frac{7}{8}$ inch rivet of $\frac{5}{f}$ and if $f=5$ (or a factor-of-safety of 5 were used) it would add just one ton to the calculated strength of a $\frac{7}{8}$ inch rivet. This increase, however, is a doubtful one, and would probably be lost in time by a gradual wearing of the plates due to this very friction, or possibly from slight rusting or other causes; no

Gain by Friction.

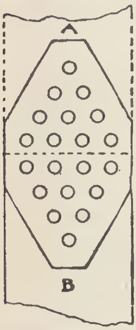


FIG. 166.

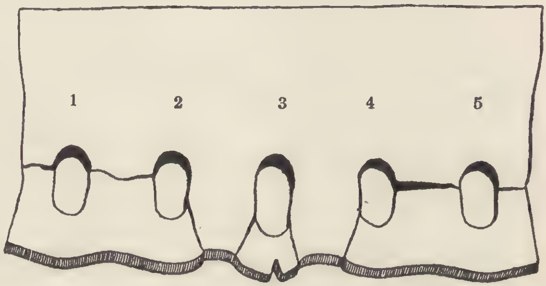


FIG. 167.

allowance should therefore be made for extra strength due to friction, but it certainly is a great advantage in making joints; and the above facts may account largely for the discrepancies in experiments on riveted joints, where no allowance is made for friction.

The pitch of rivets is, of course, governed by circumstances. The rule is to try to arrange the rivets, so that the strength of the plate between them shall equal the actual strength of the rivet.

Boiler Riveting.

In boiler-work, however, they must be located not only for strength, but must be placed close enough to make the joint steam-tight. For this reason, too, boiler-plates are always lapped, the joint being more easily caulked and made tight.

In constructional work, however, there will be a great loss and waste of material, if the rivets are placed too closely. In plate

girders and riveted joints in trusses the rule is not to make the pitch less than $2\frac{1}{2}$ diameters for punched-work, nor more than sixteen times the thickness of the least plate at the joint, or:

$$p = 16.t \qquad (107)$$

Greatest Pitch. Where p = the greatest pitch, in inches, for rivets of plate-girders or riveted trusses.

Where t = the thickness of the thinnest plate, in inches.

The pitch is measured from centre of hole to centre of hole on a direct (straight) line.

$$p_1 = 2\frac{1}{2}.d \qquad (108)$$

Least Pitch. Where p_1 = the least pitch, in inches, for rivets of plate-girders or riveted trusses.

Where d = the diameter of rivet-holes, in inches.

The exact pitch must be between these two limits; and is, of course, calculated.

Diameter of rivets. Different writers have attempted to lay down exact rules for the size of rivets to be used, using for a basis for the formulæ the thickness of thinnest plate to be riveted. Such rules, however, generally do not agree with good practice, as they either make the rivets too small for thin plates, or too large for thick ones, or *vice versa.*

As a rule the local circumstances must control the selection of the size of rivet; the following, however, may serve as a general guide:
For plates from $\frac{1}{4}''$ to $\frac{7}{16}''$ thick, use rivet-holes $\frac{5}{8}''$ diameter.
For plates from $\frac{1}{2}''$ to $\frac{5}{8}''$ thick, use rivet-holes $\frac{3}{4}''$ diameter.
For plates from $\frac{11}{16}''$ to $\frac{13}{16}''$ thick, use rivet-holes $\frac{7}{8}''$ diameter.
For plates from $\frac{7}{8}''$ to $1''$ thick, use rivet-holes $1''$ diameter.

Of course, larger or smaller rivets can be used, but as a rule $\frac{5}{8}$ inch, $\frac{3}{4}$ inch, and $\frac{7}{8}$ inch are most desirable.

Figure 167 shows the different ways in which riveted-work will yield. This figure is made from a photograph of an actual specimen, tested and torn apart at the Watertown arsenal.

How plate yields. It is evident that the plate began yielding by all of the rivets compressing or crushing the plates, and finally yielded completely by tearing apart from 2 to 1 and to left edge and the same from 4 to 5 and the right edge, while rivet 3 tore its way completely out, shearing off a piece of the plate, and rivets 2 and 4 partially so.

The iron plates tested were 15 inches wide, $\frac{1}{4}$ inch thick, with two

iron cover-plates 15 inches x $\frac{3}{16}$ inch each. The rivets were $\frac{15}{16}$ inch of iron and filled 1 inch drilled holes, pitch 3 inches.

The gross area of plate was 3,765 square inches, the net area 2,510. The total bearing-surfaces of rivets on the plates aggregated 1,255 square inches, and their aggregate shearing areas, (being in double shear) was 7,854 square inches. At 116715 pounds strain the edges contracted and scale on the specimen began to start and the plate yielded as shown at 167200 pounds strain. This was equal to 44410 pounds tension per square inch on the uncut plate, or 66610 pounds per square inch on a line at the rivet-holes (or net area).

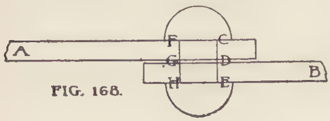


FIG. 168.

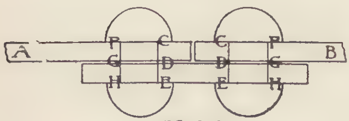


FIG. 169.

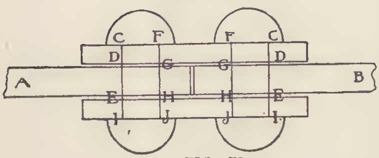


FIG. 170.

The compression from the rivets was 133230 pounds per square inch, while the shearing was only 21290 pounds per square inch.

This example shows clearly how the plate yields. Besides this the joint might yield by breaking or shearing off the rivets. We have then the following six manners in which a riveted-joint might yield.

How Riveted Joint yields.

1. By crushing either the rivet or the rivet crushing the plate.
2. By shearing off the rivet — in single shear.
3. By shearing off the rivet — in double shear.
4. By bending or cross-breaking of the rivet.
5. By tearing the plate apart or crushing it between rivet-holes.
6. By the rivets shearing out the part of the plate between them and the edge.

In Figures 168, 169 and 170 are shown three kinds of joints, each with a single rivet transferring the whole strain; in Figure 168 directly from plate *A* to plate *B;* in Figure 169 transferring strain from plate *A* to cover-plate and thence to plate *B;* and in Figure

170 transferring one-half of the strain A to *each* cover-plate and thence each half is transferred back again to plate B.

It should be remarked here that cover-plates (as in Figure 169) should be at least the full width and thickness of the original plates. In practice they are made a trifle thicker (about $\frac{1}{16}$ inch or more).

Cover-plates, as in Figure 170, should *each* be the full width of original plates and at least one-half the thickness of same; in practice they too are each made about $\frac{1}{16}$ inch (or more) thicker.

The plates A and B are themselves, of course, of the same thickness.

Failure by Compression.

Now to prevent failure by the first method, compression, there must be area enough at both $G\ H$ and $C\ D$, (if plates are in tension), not to crush the rivet or the plates at these points, ($C\ D + E\ I$ and at $G\ H$ in Figure 170). This area is considered equal to the thickness of either plate A or B (or of cover-plate or their sums) multiplied by the diameter of rivet-hole.

Failure by Shearing.

To resist failure by the second method, single-shearing of rivet, the area of cross-section of each rivet must be sufficient not to shear off under the total strain on either plate A or B. It will be readily seen that only the rivets in Figures 168 and 169 are subjected to single shearing, *viz:* at their sections $G\ D$. The rivets in Figure 170 have two areas resisting shearing, $G\ D$ and $H\ E$, hence are subjected to double shearing; therefore their area of cross-section need only be sufficient to resist a shearing strain equal to only one-half of the *total* strain on either plate A or B, in order to avoid failure by the third method.

Failure by bending.

To avoid failure by the fourth method, the rivet must be sufficiently strong to resist the load as a lever in Figures 168 and 169, and as a beam in Figure 170.

In Figures 168 and 169 we can consider the part $D\ C\ F\ G$ as the built-in part of a lever, with a free end $D\ E\ H\ G$ which carries a uniform load equal to the whole strain on either plate A or B.

In Figure 170 we have a beam supported at $C\ D\ G\ F$ and $E\ I\ J\ H$, with its span or central part $G\ H\ E\ D$ loaded with a uniform load equal to the whole strain on either plate A or B.

To prevent failure by the fifth method the area of cross-section

of either plate taken at right angles across same through the rivet-hole — (that is, deducting the rivet-hole from the area of cross-section) — should be sufficient to resist the tension or compression.

Failure of plate.

To prevent failure by the sixth method the rivets must be far enough from the edges of plates (cover and original plates) not to shear out the metal ahead of them. The rule is shown in Figure 171. Make angle $A\ O\ C = 90°$ that is a right angle (0 being the centre of rivet-hole and $C\ A$ part of its circumference), and so that the directions of $O\ A$ and $O\ C$ are at 45° with edge of plate $D\ B$. Then the sums of the areas $A\ B + C\ D$ — (that is, $A\ B + C\ D$ multiplied by thickness of plate) — must be sufficient to resist the longitudinal shearing strain, which in this case would be the strain on either plate A or B (Figures 168 to 170).

Failure by rivets tearing out.

To put the above in formulæ we should have:

Bearing.

$$x = \frac{s}{d.\,h.\left(\frac{c}{f}\right)} \qquad (109)$$

Use x for lap joints only.

Use $2\,x$ in place of x for butt joints with single or double cover-plates.

Single Shearing.

$$x = \frac{s}{0{,}7857.\,d^2.\left(\frac{g}{f}\right)} \qquad (110)$$

Use x for lap joints only.

Use $2\,x$ in place of x for butt joints with single cover-plate.

Double Shearing.

$$x = \frac{s}{1{,}5714.\,d^2.\left(\frac{g}{f}\right)} \qquad (111)$$

Use $2\,x$ or butt joints with double cover-plates.

Bending-moment Lever.

$$x = \frac{s.\,h.}{0{,}1964.\,d^3.\left(\frac{k}{f}\right)} \qquad (112)$$

Use x for lap joints only.

Use $2\,x$ for butt joints with single cover-plates.

Bending-moment Beam.

$$x = \frac{s.\,h}{0{,}7857.\,d^3.\left(\frac{k}{f}\right)} \qquad (113)^{1}$$

[1] The fourth decimal given in formulæ is not quite right, but is made to correspond with fractions used in Table I.

Use $2\,x$ for butt joints with double cover-plates.

Tension on Plate.

$$h = \frac{s}{b.\left(\frac{t}{f}\right)} \tag{114}$$

Use $\left(\frac{c}{f}\right)$ instead of $\left(\frac{t}{f}\right)$ if plate is in compression.

Shearing end of plate.

$$y = \frac{s}{2.\,h.\left(\frac{g}{f}\right)} \tag{115}$$

(If more than one rivet use $\frac{s}{x}$ instead of s for distance of each rivet from end as shown in Figure 171.)

Where $s =$ the whole load or strain, in pounds, to be transferred from one side of the joint to the other.

Where $d =$ the diameter of rivet-hole, in inches.

Where $h =$ the thickness of plate, in inches. Where more than one plate is used, take for h the *least* aggregate sum of thicknesses of all plates acting in one direction. (The sum of cover-plates should at least equal this aggregate in thickness and should be larger, where the net b of cover-plates is smaller than the net b of connected plates.)

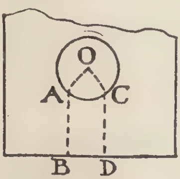


FIG. 171.

Where $b =$ the net breadth of plate, in inches, that is the breadth, less rivet-holes, at the weakest section; where more than one plate is used they should all of course be of same breadth. The net b of cover-plates will frequently be much less than that of original plates, as they lose the greatest number of rivet-holes at their centre, where they are carrying the full strain.

Where $x =$ the total number of rivets required at the joint for lap joints, and the number required *each side of joint* for butt joints with single or double cover-plates; that is in the latter two cases $2\,x$ will be the total number of rivets required.

Where $y =$ in inches, is the length $A\,B$ or $C\,D$ (Figure 171) from *any* rivet to free edge of plate; where more than one rivet is used, insert $\frac{s}{x}$ in formula, in place of s. It will only be necessary, of course, to calculate y for the line of rivets nearest free edge.

Where $\left(\frac{c}{f}\right) =$ safe compression stress, per square inch,

Where $\left(\frac{t}{f}\right) =$ safe tension stress, per square inch,

Where $\left(\frac{g}{f}\right) =$ safe shearing stress, per square inch,

Where $\left(\frac{k}{f}\right) =$ safe modulus of rupture, per square inch,

all in pounds, (see Table IV).

Safe Stresses on Rivets and Pins. The writer uses the following values, as a rule for *rivets and pins.*

For Wrought-Iron. $\left(\frac{c}{f}\right) = 12000$ pounds, per square inch.

$\left(\frac{t}{f}\right) = 12000$ pounds, per square inch.

$\left(\frac{g}{f}\right) = 8000$ pounds, per square inch.

$\left(\frac{k}{f}\right) = 15000$ pounds, per square inch.

For Steel. $\left(\frac{c}{f}\right) = 15000$ pounds, per square inch.

$\left(\frac{t}{f}\right) = 15000$ pounds, per square inch.

$\left(\frac{g}{f}\right) = 10000$ pounds, per square inch.

$\left(\frac{k}{f}\right) = 18000$ pounds, per square inch.

Example I.

Lap Joint. *A wrought-iron plate, which cannot be over* 12″ *wide, is required to be so long that it has to be made up from two lengths; the joint is to be a lap joint. The plate is in tension and is strained* 65000 *pounds. Design the joint.*

We will assume that we propose to design the joint as shown in Figure 165, with staggered rivets, in that case the plate will only be weakened by one rivet-hole. We can readily see that the plate will not need to be very thick and decide to use $\frac{3}{4}''$ rivets, (that is $\frac{3}{4}''$ rivet-holes)[1]; we then shall have a net breadth of plate

Size of plate. $b = 12'' - \frac{3}{4}'' = 11\frac{1}{4}''.$

Of course $s = 65000$ in this case, and we know that

$$\left(\frac{t}{f}\right) = 12000\,;$$

inserting these values in Formula (114) we have:

[1] Before heating, in this case, the rivets would be $\frac{11''}{16}$.

$$h = \frac{65000}{11\frac{1}{4}.\,12000} = 0{,}482''$$

Or we should use a plate one-half inch thick.

We next determine the number of rivets required.

Required number of Rivets.

In the first place there must be enough for bearing, that is not to crush the plate or get crushed by it. We use Formula (109) inserting the values, and have:

$$x = \frac{65000}{\frac{3}{4}.\,\frac{1}{2}.\,12000} = 14{,}44$$

Or we should need 15 rivets for bearing. Had we figured without using the formula, we should have said, the bearing area of each rivet is $\frac{3}{4}''$ by $\frac{1}{2}'' = \frac{3}{8}$ square inches, this at 12000 pounds, per square inch, would equal 4500 pounds safe-compression for each rivet, dividing this into 65000 pounds, the strain, would, of course, give the same result 14,44 or say 15 rivets.

We next see if there is any danger from shearing. The joint being a lap joint, the rivets will have, of course, only one sectional area to resist shearing, that is, will be in single shear, so that we use Formula (110) and inserting values have:

$$x = \frac{65000}{0{,}7857.\,(\frac{3}{4})^2.\,\left(\frac{g}{f}\right)} = 18{,}39$$

Or we must use 19 rivets to prevent the shearing.

Had we figured without the use of formula, we should have said, area of a $\frac{3}{4}''$ rivet is $= 0{,}4417$ square inches. This multiplied by 8000 pounds (the safe shearing stress per square inch) $= 3533{,}6$ pounds, or each rivet could safely assume this amount of the strain without shearing. This amount being *less* than the safe compression on each rivet, would, of course, require a larger number of rivets, and should therefore be used, rather than the latter. We have, in effect $\frac{65000}{3533{,}6} = 18{,}39$ or say 19 rivets, being four more than required for bearing.

We next take up the question of bending; the joint being lapped the rivets will practically become short levers. We use Formula (112); inserting values, we have:

$$x = \frac{65000.\frac{1}{2}}{0{,}1964.\,(\frac{3}{4})^3.\,15000} = 26{,}15$$

Or we should have to use at least 26 rivets to prevent bending;

which readily illustrates the great disadvantage of not transferring the strain in a direct plane, by using two cover-plates.

Had we not used the formula, we should have said, we have here a $\frac{3}{4}''$ circular lever, the free end projecting $\frac{1}{2}''$ and loaded uniformly with a load of 65000 pounds.

From Formula (25) or Table VII we have the bending-moment

$$m = \frac{65000 . \frac{1}{2}}{2} = 16250 \text{ pounds-inch.}$$

and from Table I, section No 7 the moment of resistance,

$$r = \tfrac{1}{14} . \left(\tfrac{3}{8}\right)^3 = 0{,}04143$$

From the formula on page 49, Volume I,

$$\frac{m}{r} = s$$

we have the total extreme fibre strains on all the rivets,

$$s = \frac{16250}{0{,}04143} = 392228 \text{ pounds.}$$

This divided by the safe strain, or safe modulus of rupture $\left(\frac{k}{f}\right) = 15000$ pounds, will give the number of rivets required, *viz:*

$$\frac{392228}{15000} = 26{,}15$$

or 26 rivets as before.

We still have to decide the distance y (or $A\ B$, Figure 171). We use Formula (115). As we have more than one rivet we use in place of s the strain on each rivet or $\frac{s}{26}$, which was the largest number required as above, therefore:

$$\frac{s}{26} = \frac{65000}{26} = 2500$$

Inserting this in Formula (115) we have:

$$y = \frac{2500}{2 . \frac{1}{2} . 8000} = 0{,}3125''$$

This, however, being less than our rule which requires $1\frac{1}{2}$ diameters from centre of hole to edge, we will stick to the rule.

We now design the joint.

We have a plate $12''$ wide, $\frac{1}{2}''$ thick, lapping, and require 26 rivets.

Designing the joint. They must be arranged not to weaken the plate by more than one rivet-hole.

If we arrange the rivet-holes as shown in Figure 172, we will find it the most economical arrangement. To be sure it allows for only

25 rivets, but that will probably be near enough, otherwise we should have to insert at least five more to keep them symmetrical. It will be readily seen that the weakest point is at section A where one rivet-hole is lost.

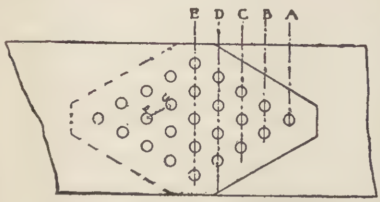


FIG. 172

Section B is of same strength, two rivet-holes being lost, but the strain has been reduced by an amount equal to the value of one rivet-hole.

At section C we lose three rivet-holes, but the strain has been reduced by the value of three rivet-holes, so that the plate practically has its full value here.

At sections D and E the plate is stronger to resist the remaining tension than required.

By figuring out E (12″ wide) it will be seen that the pitch on this line is more than required by the rule, Formula (108).

The pitch $F\,G$ between two adjacent lines of rivets, measured on the slant from centre to centre of rivets, should be at least $2\frac{1}{2}$ diameters, $2\frac{1}{2}\cdot\frac{3}{4} = 1\frac{7}{8}''$ or say $2''$.

It will be good practice for the student to carefully lay this joint out to scale.

Example II.

Butt joint single cover-plate.

A steel plate 10″ *wide has to be pieced, and for local reasons this can only be done by a cover-plate on one side. The plate is subjected to a tensional strain of* 135000 *pounds. Design the joint.*

Of course, the rivets must be of steel too.

We will again assume that we can stagger the rivets, so that we shall lose only one rivet-hole. The plate will evidently have to be thick and we will decide to use 1″ rivets; this would leave us a net breadth of plate

Size of plate. $b = 10'' - 1'' = 9''$

From Formula (114) we have:

$$h = \frac{135000}{9.\,15000} = 1''$$

Or the plate will have to be just one inch thick. The cover-plate

should be at least the same, if there were only one rivet, but as there will evidently be more than one and we propose staggering the rivets, the cover-plate will have to be considerably thicker; we can therefore leave the cover-plate out of consideration for the present as, being thicker, or in case of one rivet equal to the plates to be joined, it will certainly be as strong, and not crush.

Required number of rivets. Now, as for the number of rivets, from Formula (109) we have for bearing:

$$x = \frac{135000}{1.\,1.\,15000} = 9$$

Or nine rivets are required not to crush the plate or be crushed by it (each side of joint).

From Formula (110) we have for single shearing (as there is evidently only one area to each rivet to resist shearing):

$$x = \frac{135000}{0{,}7857.\,1^2.\,10000} = 17{,}2$$

Or seventeen rivets are required to resist the shearing (each side of joint). The rivets will evidently be levers in this case and we have from Formula (112)

$$x = \frac{135000\,.1}{0{,}1964.\,1^3.\,18000} = 38{,}1$$

Or it will require thirty-eight rivets to resist the bending-moment (each side of joint). This again shows the advantage of using both a top and bottom cover-plate and so avoiding the great leverage on the rivets.

Designing the joint. It must now be borne in mind that the joint is a butt joint, therefore, unlike the case of lap joint where the whole number of rivets bear on each plate, we must here use twice the number, as only one-half, or those each side of the joint bear on one plate, or we require in all 76 rivets.

The plate is so narrow that we cannot get more than three rivets on a line across the plate, without infringing on the rules for pitch. We can, therefore, stagger the end rivets and make the rest either chain riveted or zig-zag riveted. The chain riveting will require a little longer cover-plate, but in the case of a plate-girder flange would have the advantage of using the rivets that are needed for the angle-irons.

In that case we should not stagger the end rivets, for our plate would only be weakened by one additional hole, and, of course, the

gross breadth of plate would have to be 12 inches instead of 10 inches to give same strength.

Figure 173 shows this joint chain riveted, with end rivets, staggered, to correspond to our calculated example.

We see that it takes 39 rivets each side of joint for symmetry.

Figure 174 shows this joint zig-zag riveted. It has three advantages over the other, it is shorter, takes just the right number of

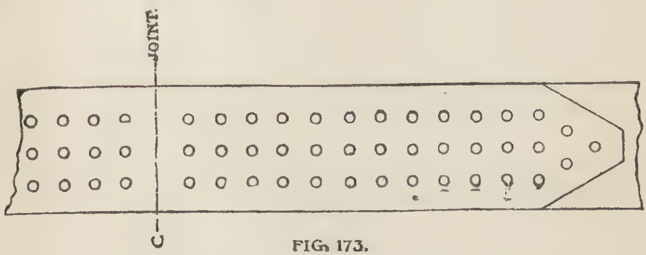


FIG. 173.

rivets and requires a thinner cover-plate. For in Figure 173 at the first line of rivets next to the joint (C), the cover-plate loses three rivet-holes and bears the full strain, or its clear breadth would be

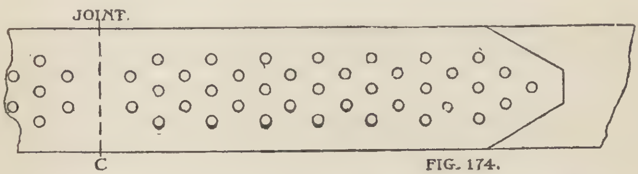


FIG. 174.

only 7 inches and from Formula (114) would require a cover-plate of thickness

$$h = \frac{135000}{7.15000} = 1{,}3$$

or say $1\frac{5}{16}''$ thick.

Whereas, in Figure 174, next to the line C we lose only two rivets and have consequently a clear breadth of 8 inches and require from Formula (114) a cover-plate of thickness equal to

$$h = \frac{135000}{8.15000} = 1{,}125$$

or only $1\frac{1}{8}$ inch cover-plate.

Had we not used the formula we should have figured out our rivets, etc, as follows:

Required net or clear area of plate

$\frac{135000}{15000} = 9$ square inches. Net breadth being 9″ gives, of course, 1″ thickness.

Bearing area of each rivet $= 1.1 = 1$ square inch, which will safely bear 15000 pounds or we should need

$\frac{135000}{15000} = 9$ rivets.

Single shearing area of each rivet (or area of a circle 1″ diameter) $= 0,7854$ square inches, which at 10000 pounds per square inch, would give a resistance to shearing per rivet $=$ 7854 pounds or we should need

$\frac{135000}{7854} = 17,2$ rivets. For bending we should have a one inch circular lever, projecting one inch and uniformly loaded with 135000 pounds.

The bending-moment would be Formula (25)

$$m = \frac{135000.1}{2} = 67500 \text{ pounds.}$$

The moment or resistance would be Table I, section No. 7

$$r = \tfrac{11}{14} \cdot (\tfrac{1}{2})^3 = 0,0982$$

The strain s, therefore, on all the rivets will be [page 49].

$$s = \frac{67500}{0,0982} = 687373$$

This divided by the safe modulus of rupture for steel rivets $\left(\frac{k}{f}\right) = 18000$ will give the required number, as before,

$\frac{687373}{18000} = 38,2$

Example III.

Butt-Joint Two Cover-plates. *Same problem as before, but two cover-plates to be used, one above and one below the joint.*

We will again decide to stagger the rivets and losing only one rivet-hole will again require a 1″ thick plate.

Now, for bearing we will have the same result as before, *viz.:* 9 rivets required, but in shearing it is evident that now each rivet has two resisting areas or is in double shear, so that we will need only one-half of the previous quantity or $\frac{17,2}{2} = 8,6$ or say, nine rivets. Had we used Formula (111) we should have obtained this result, for:

Required number of rivets.

$$x = \frac{135000}{1,5714.\ 1^2.\ 10000} = 8,6$$

For the bending-moment we use Formula (113) and have

$$x = \frac{135000.\ 1}{0,7857.\ 1^3.\ 18000} = 9,5$$

or say ten rivets required to resist cross-breaking. This shows how much better proportioned the different strains (shearing, bending and bearing) are to each other, where we use two cover-plates. Had we calculated directly without use of formula we should have obtained the same results. We have already calculated net area of plate and bearing value of rivets in *Example II*, also the single shearing value, which was 7854 pounds per rivet; as we now have two areas this would be doubled or the resistance of each rivet to shearing would be 15708 pounds and the number required $\frac{135000}{15708} = 8,6$ as before.

Advantage of Two Covers.

For bending-moment we should have a 1″ circular beam with a clear span of 1″, uniformily loaded with 135000 pounds.

From Formula (21) we have the bending-moment

$$m = \frac{135000.\ 1}{8} = 16875 \text{ pounds-inch}$$

Moment of resistance will be as before

$$r = 0,0982$$

Therefore the total strain on all the rivets

$$s = \frac{16875}{0,0982} = 171843$$

This divided by $\left(\frac{k}{f}\right) = 18000$ gives the required number of rivets as before

$$x = \frac{171843}{18000} = 9,5$$

Designing the Joint.

We now design the joint, as before, remembering to stagger the rivets and to place the required number each side of joint. The greatest number required was to resist cross-breaking, *viz.*, ten.

We can design as shown in Figure 175 or as shown in Figure 176; both require eleven rivets each side of joint, but cover-plates in Figure 176 need only aggregate $1\frac{1}{8}''$ in thickness, that is, be $\frac{9}{16}''$ thick each; while those in Figure 175 would have to aggregate $1\frac{5}{16}''$ in thickness, or be, say, $\frac{11}{16}''$ thick each.

Joint shown in Figure 175 looks a little better, but otherwise there is no preference.

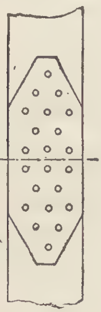


FIG. 175.

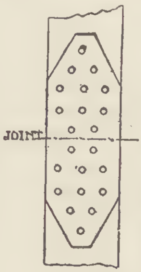


FIG. 176.

Covers of same thickness.

If cover-plates are not equal in thickness each side of plate, it would require very many more rivets. Each rivet would become a double lever, with its central part built-in and a projecting free arm each side, the length of arms being equal to their respective thicknesses of cover-plates. The load on each arm would be the proportion of whole strain, that the thickness of its respective cover-plate would be of the whole required (aggregate) thicknesses of cover-plates.

There would be no sense in such an arrangement however. It would produce all sorts of unequal stresses, in shearing, bearing, cross-breaking, etc., and should be avoided. Riveted work at best is very theoretical, as the calculations depend entirely upon the accuracy and fit of each rivet. If a single rivet fails to do its share, it will at once disarrange all the strains and produce unequal stresses in different parts. Still if the above rules are followed, riveted work can be used with perfect safety. Where the result gives a fraction, a whole rivet should as a rule be used in place of the fraction. If the necessary spacing requires still more rivets, they can either be used, or, all the rivets can be reduced in size enough to bring them nearer to the allowable stresses.

No account has been taken of the loss due to punching, for this will affect the plate in tension mainly, and the safe stresses allowed for tension are very low. In compression the metal would not be strained as heavily as in tension, for the rivets will not weaken the plate so much if they entirely fill the holes, thus giving full bearing on the entire plate. Then, too, the butt joint if planed and carefully

made and joined, will transfer directly more or less of the compression.

But in all good work it is customary to place no reliance whatever on the butt, and to calculate in compression the same as for tension, namely, sufficient *net* area in each plate, at its weakest section, to resist the whole compression strain.

No reliance on butt.

Tables XXXV to XL inclusive, have been calculated and laid out to save most of the wearisome figuring necessary in riveted work and in connection with pins. The first three give the bearing value of pins and rivets against eye-bars or plates, and the latter three the values in tension, single and double shearing and in cross-breaking of pins and rivets. All the tables are laid out for both steel and wrought-iron.

Explanation of Tables.

The full heavy straight lines in Tables XXXV, XXXVI and XXXVII represent the thicknesses of plates or eye-bars against which the different sizes of rivets or pins bear. The thicknesses given are from $\frac{1}{4}''$ to $1\frac{1}{2}''$ in Table XXXV and from $\frac{1}{4}''$ to $2''$ in Tables XXXVI and XXXVII; all by $\frac{1}{16}''$. For thicker plates or eye-bars it will only be necessary to increase the bearing value found, in proportion to extra thickness.

Tables xxxv, xxxvi, xxxvii.

The columns to the left give the diameters of pins and rivets, running in Table XXXV from $\frac{1}{4}$ inch diameter (by $\frac{1}{32}$ inch) to 1 inch diameter; in Table XXXVI from 1 inch diameter (by $\frac{1}{16}$ inch) to 3 inches diameter; and in Table XXXVII from 3 inches diameter (by $\frac{1}{8}$ inch) to 6 inches diameter. The figures at the *tops* of these tables give the bearing values in pounds for wrought-iron, and those along the *bottoms*, the bearing values in pounds for average steel.

The full heavy curved lines in Tables XXXVIII, XXXIX and XL give the single and double shearing values for the same sized pins and rivets as in the previous three tables.

Tables xxxviii, xxxix, xl.

The additional vertical columns to the left in these tables give the areas of cross sections in square inches, of the different sized pins and rivets, which multiplied by ten give their weights in pounds per yard of length for wrought-iron, (for steel add 2 per cent to the weight of wrought-iron). There are also full heavy curve lines giving the strength, in tension, of tie-rods of same diameter as pins or rivets. The values selected for these curves are those *always* used by the writer in calculating pins, rivets or tie-rods.

For different values.

It sometimes becomes desirable, in temporary work to use higher values, or in very important permanent structures with moving loads to use lower values. But even in such cases the tables can be used, for, as all of these curves are directly dependent on the area (or double area), of cross section of the rivet or pin, they can, of course, be used interchangeably. That is, any one who wishes to figure the safe shearing $\left(\frac{g}{f}\right)$ for steel as $= 15000$ pounds, instead of 10000 pounds, can take the curve marked "15000 pounds — tension steel," in any of the three tables.

Or, if he wishes to figure his iron at only 10000 pounds safe stress for tension, that is $\left(\frac{t}{f}\right) = 10000$ pounds, he will, instead of using the curve marked "12000 pounds — tension wrought-iron," take the curve marked "10000 pounds — single shear steel."

The writer, however, always sticks to the one set of values for tension, and for pins and rivets to those given in tables (also after Formula 115), as they are certainly *safe* values, and yet not low enough to make the work excessively costly. Iron contractors will frequently quote off-hand opinions of celebrated engineers saying these values are much too low and thus backing up their economical tendencies in trying to add lightness (and beauty) to roof trusses and plate girders, but as a rule the opinions are either not authentic, or else it is found that the celebrated engineer, when delivering the opinion, had a similar axe to grind, as had the contractor. Good engineers as well as good architects, will attempt to save their clients all they can, but hardly at the risk of taking chances in their most important constructive works. The figures along the *tops* of Tables XXXVIII to XL give the values of either iron or steel, according, of course, to which curve is used.

Curve of Bending-moment.

The dotted curved lines in the Tables XXXVIII to XL give the safe bending-moments for iron and steel pins and rivets, and for these the *lower* horizontal lines of figures are used.

How to use Tables.

The use of the tables is simple, similar to the other tables with curves. Thus if we have a $\frac{5}{8}''$ steel rivet bearing against a $\frac{7}{16}''$ steel plate, we use Table XXXV and pass horizontally to the right from the vertical figure (or

diameter marked) $\frac{5}{8}''$ till we strike the (fourth) slanting full bearing line, marked $\frac{7}{16}''$. This is one-third way between

Bearing Values.

the vertical lines marked (*below* for steel) 4000 and the next vertical unmarked line to the right; as each vertical space at the bottom (steel) evidently is one-fourth of a thousand or 250 pounds, a third space will, of course, be 83 pounds, or a $\frac{5}{8}''$ steel rivet bearing against a $\frac{7}{16}''$ steel plate will resist safely 4083 pounds. Had we calculated arithmetically we should have had

$\frac{5}{8} . \frac{7}{16} . 15000 = 4101$ pounds.

Had the rivet and plate been of wrought-iron we should have used the *upper* line of figures; here the intersection is one-third way between the second and third unmarked lines after 3000; as there are five spaces above between each thousand, each space evidently represents one-fifth of a 1000 or 200 pounds for iron, so that the bearing value for a $\frac{5}{8}''$ iron rivet against a $\frac{7}{16}''$ iron plate would be $3000 + 200 + \frac{1}{3} . 200 = 3267$ pounds. By calculation we should have had $\frac{5}{8} . \frac{7}{16} . 12000 = 3281$ pounds.

The single shearing value of a $\frac{5}{8}''$ steel rivet at $\left(\frac{g}{f}\right) = 10000$ pounds would be $= 3067$ pounds; for, the horizontal line $\frac{5}{8}''$ in Table XXXVIII strikes the single shearing curve for steel

Shearing Values.

about one-third way between vertical line marked at the *top* 3000 and the next unmarked line to the right. Each space evidently represents 200 pounds, so that we should have for the steel $\frac{5}{8}''$ rivet in single shear $3000 + \frac{1}{3} . 200 = 3067$ pounds. By arithmetical calculation we should have had the area of a $\frac{5}{8}''$ circle multiplied by 10000 pounds or

$0{,}3068 . 10000 = 3068$ pounds.

The double shearing value of a $\frac{5}{8}''$ rivet would be double this, or $= 6136$ pounds, and this is confirmed by the Table (XXXVIII), as the horizontal line $\frac{5}{8}''$ strikes the double shearing curve for steel 20000 pounds about two-thirds way between vertical line marked *above* 6000 and its next unmarked line to the right, or

$6000 + \frac{2}{3} . 200 = 6134$ pounds.

For the safe bending-moment we find the horizontal line $\frac{5}{8}''$ strikes dotted curved line marked "safe bending-moment on steel at 18000 pounds" on the second vertical unmarked line to the left of the one marked *at the bottom* 500; each space below is evidently 20 pounds, and as they increase to the left we must add the two spaces, or

$500 + 40 = 540$ pounds, which would be the safe bending-moment on a $\frac{5}{8}''$ steel rivet or pin.

Or to illustrate further take the last *Example* (*III*). We had a 1″ steel plate and 1″ steel rivet. The actual bending-moment we found was

Bending-moment Curve.

$$\frac{u.l}{8} = \frac{135000.1}{8} = 16875$$

From Table XXXVIII we see that the safe bending-moment on a 1″ steel rivet is the intersection of horizontal line 1″ being one-third way between third and fourth vertical unmarked lines to the left of $1700 = 1700 + 3\frac{1}{3}.\ 20 = 1767$ pounds-inch.

Dividing the actual bending-moment at the joint 16875 pounds-inch, by the safe bending-moment on each rivet, will, of course, give the number of rivets required, or

$$\frac{16875}{1767} = 9{,}55$$

being the same result as before, namely, 10 rivets. Take *Example II*, we had the same rivet, plate and strain, but there was but one cover-plate and the rivet was practically a cantilever.

The bending-moment was

$$\frac{u.l}{2} = \frac{135000.1}{2} = 62500 \text{ pounds-inch.}$$

This divided by the safe bending-moment on each rivet (1767 pounds-inch) as found in Table XXXVIII gives the number of rivets required, or

$$\frac{62500}{1767} = 35{,}4 \text{ or same as before.}$$

The use of Tables for pins or tie-rods, is, of course, exactly similar to use for rivets, as already described.

Bearing on pins. For instance, we have a 2″ pin bearing against a $1\frac{1}{8}$ inch thick eye-bar or head of a tie-rod. We use Table XXXVI, the 2 inch horizontal line, strikes the full slanting line marked $1\frac{1}{8}$ inch exactly on one of the vertical lines. If both pin and eye-bar are wrought-iron — (or if either were wrought-iron we should use the smaller value) — we use the *top* line of figures, or we find our vertical line the third to the right of 24000 and as each space evidently represents 1000 pounds, the safe bearing value of our 2 inch pin against a $1\frac{1}{8}$ inch thick eye-bar would be 27000 pounds, if one or both are of wrought-iron. Had both pin and eye-bar been of steel, we should have used the bottom line of figures, where each space

evidently represents 1250 pounds, and we should have had 33750 pounds.

Shearing of Pins.

The safe shearing value for a 2 inch wrought-iron pin, would be from Table XXXIX = 25000 pounds, and for steel 31670 pounds.

By calculation we should have had for iron = 25133 pounds and for steel = 31416 pounds.

Tension on Rods.

If we had a 2 inch circular tie-rod its strength in tension from Table XXXIX would be, if of wrought-iron = 37600 pounds and if of steel = 47000 pounds.

By calculation we should have had for iron = 37752 pounds and for steel = 47190 pounds.

Calculation of Pins.

Pins are calculated similarly to rivets. The same formulæ can be used for bearing area (109) h being the thickness of head of eye-bar, in inches, and for single shear (110) and double shear (111).

For bending where there are but two eye-bars or rods, each pulling in opposite directions, Formula (112) could be used; using for h the thickness, in inches, of either rod; where there are two eye-bars or rods pulling in one direction with only one between them pulling in the opposite direction Formula (113) might be used; h in this case being the thickness, in inches, of the large (central) single rod, and twice the thickness of either of the smaller rods. In all of the formulæ x should be = 1.

Designing Pin-Joint.

As a rule, however, there are several rods at each pin-connected joint, and the bending-moment has to be carefully calculated for each special case. In designing pin-joints this should be borne in mind, and the pieces with largest strains brought as closely together, as possible, to avoid excessive bending-moments, which will require very large pins, and necessitate large eyes and heads to the rods and bars, which means, of course, very great expense; pin-connections for small trusses are more expensive than riveted joints; for large trusses they are cheaper.

Pin-Joint best.

They are very much better in both cases, more easy to transport and erect, and agree much nearer with the theoretical calculation, which assumes that all members around a joint are free to move, and not rigid, as is the case in riveted trusses. Then, too, in case of any movement in the truss, it can be readily adjusted by means of nuts, swivel-links,

sleeve-nuts, etc. This cannot be done in a riveted truss, as the joints not being flexible, the tightening of any one part might throw strains on some joint not able to bear it.

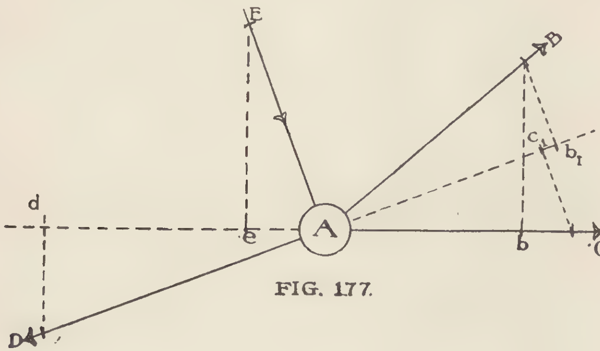


FIG. 177.

In calculating a pin-joint, the strength of pin, as a rule, should be calculated in the several lines or directions in which it is being strained, that is first along the line of one rod, then along the line of another rod (or compression piece) and so on. In doing this all the other strains must be projected (and reduced) to the same line. If this is correctly done, it will be found in every case, that the *sum* of all the strains acting in one direction along this line will be equal to the *sum* of all acting in the opposite direction along the line. Thus in Figure 177 we have the end elevation of a pin *A* strained by three forces in tension *B*, *C* and *D* and one force in compression *E*.

Strains reduced to same line.

We will first examine the pin along the line *A C*. Make, at any convenient scale, the lines *A C*, *A B*, *A D* and *A E*, each in length just equal to the amount of strain they are subjected to. Project them all on to *A C*, then will *A b* represent the resultant or equivalent of strain *A B* along the line and in the direction *A C*. Similarly *A d* will be the resultant of *A D* along the line of *A C*, but it will be in the opposite direction; *e A* will be the resultant of *E A* along the line *A C*, but it will be noticed that though on the opposite side of the pin it is in the same direction as *A C*.

Were we to examine the pin along the line $A\ D$, we should have the strain of $A\ D$ in one direction and in the opposite directions the resultant strains $A\ c_r$ and $A\ b_r$. It will be noticed that $E\ A$ has no resultant along the line $A\ D$, being normal to it; and it need not be considered, when calculating the pin along the line $A\ D$. We might still calculate the pin along the line $A\ E$; we should then have the strain $E\ A$ in one direction and in addition thereto the resultant of $A\ C$; in the opposite direction we should have the resultant of $A\ B$. The largest strain $A\ D$ has no resultant along the line $A\ E$. It is evident, however, that all of these resultants will be very small as compared to those along the other lines, and therefore need not be calculated.

Arrangement of Heads.

In Figure 178 we have a plan of the pin, showing in plan the strains and resultants acting along the line $A\ C$. As arranged in this figure the pin evidently becomes a double-ended cantilever, supported at the fulcrum d, and loaded on one free end with the load c and on the other free end with two loads b and e.

If the strains were small, this would probably be the most economical arrangement, as $A\ D$ could then be made in one piece. But if the strains were heavy it would require a tremendous pin.

In the latter case the arrangement shown in Figure 179 would be better. Here the pin becomes a beam, supported at both ends by $\frac{d}{2}$ (or one-half of the force $A\ D$); the beam carries three loads c, e and b. In this case $A\ D$ would be made up of two rods, separated. The forces c, e and b must be so distributed as to make the least

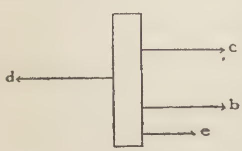


FIG. 178.

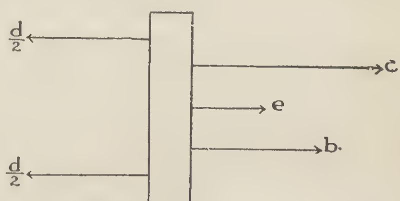


FIG. 179.

bending-moment on the beam or pin, that is, the larger ones placed nearer the supports or outer edges, and the smaller ones in the centre. Frequently it would be more economical to divide c the

larger force into two halves and place them immediately next to $\frac{d}{2}$. The small force e is frequently put on the outside, as it is not sufficient to affect the beam or pin seriously and only enlarges the span of beam, that is, length of pin or distance between supports $\frac{d}{2}$ thus greatly increasing the bending-moment. This arrangement is shown in Figure 180. Here we have a beam supported at two points $\frac{d}{2}$ with a free end carrying load e; the beam being loaded with three loads, two each $= \frac{c}{2}$ and one $= b$.

Pin with Unsymmetrical Strains.

To calculate the bending-moment at any point of this pin we have to consider the end of pin farthest from e as built in solidly (and after getting reactions) we should multiply all the forces to the right or left of the point into their distances from the point. If we select the forces on the right we deduct the sum of the moments of those (right hand forces) acting in one direction from the sum of the moments of those (right hand forces) acting in the opposite direction, the difference will be the bending-moment at the point. To check the calculation we take all the forces on the left side of the point. But the reactions will not be $\frac{d}{2}$ each as shown in the table unless e were divided and one-half put at each end of pin.

Forces act through Central Axes.

In every case of pin calculation, excepting where the forces form simple beams, as in Figure 179, the forces are supposed to act through their central axes, that is through a line drawn at half the thickness of their heads and at right angles to the pin. The heads are frequently thickened up, that is, made thicker than the rod or flat bar, in order to get the necessary bearing surface on the pin, but it will be readily seen that this lengthens the pin greatly, thus largely increasing the bending-moments. It is better, as a rule, to get the extra bearing surface by dividing the rod or flat bar into two or more parts and then distributing them along the

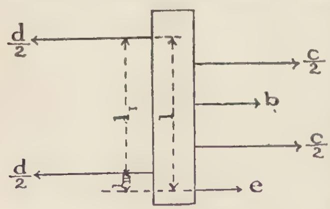


FIG. 180.

pin symmetrically and at the most favorable points to avoid bending-moment.

Forces arranged Unsymmetrically.

If the pin is arranged as shown in Figure 180 and we call the different lengths along the pin, as shown, l, l_{I} and l_{II} the reaction nearest e will be

$$= \frac{c+b}{2} + \frac{l}{l_{\text{I}}} . e$$

and the further reaction will be

$$= \frac{c+b}{2} - \frac{l_{\text{II}}}{l_{\text{I}}} . e$$

In practice, however, e would probably be so small and the convenience so great in making the two bars $\frac{d}{2}$ alike, that the unequal stresses caused by e in the two $\frac{d}{2}$ would probably be overlooked. But in calculating the bending-moment on pin, they will have to be considered as unequal, otherwise, the bending-moments calculated from the right hand or left hand of any point would not agree.

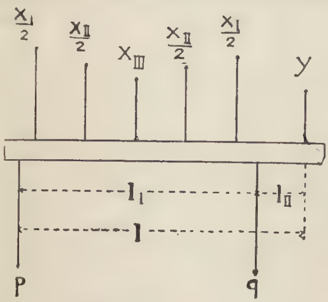


FIG. 181.

To put the above in formulæ we should have the following:

If a pin $p\,q$ is strained by a number of forces x_{I}, x_{II}, etc., in one direction, the forces p and q in the opposite direction will equal the reactions of a beam, see Formulæ (14, 15, 16 and 17).

If a single force y is placed beyond one of these resisting forces (say q Figure 181) the *additional reaction* on the nearer force (q) will be

Nearer Reaction Force.

$$q_{\text{I}} = + \frac{l}{l_{\text{I}}} . y \qquad (116)$$

and on the further force

Further Reaction Force.

$$p_{\text{I}} = - \frac{l_{\text{II}}}{l_{\text{I}}} . y \qquad (117)$$

or, we must add q_{I} to q and substract p_{I} from p to get the real strains or reactions in the tie-rods p and q.

Where q_{I} = the strain, in pounds to be added to nearer strain q owing to force y being placed on the (q) nearer free end of pin.

Where p_1 = the strain, in pounds, to be deducted from further strain p owing to force y being placed on the other further free end of pin.

Where q = the force (reaction) at q, in pounds, resisting the forces x_1, x_{11}, etc., (see Formulæ 14, 15, 16 and 17) and to which q_1 must be added.

Where p = the force (reaction) at p, in pounds, resisting the forces x_1, x_{11}, etc., (see Formulæ 14, 15, 16 and 17) and from which p_1 must be deducted.

Where x_1, x_{11} = the forces, in pounds, acting in opposite direction to p and q, and all projected to line p and q as shown in Figure 177.

Where l, l_1, l_{11}, = the respective lengths, or distances, in inches, measured along pin, from *centre lines to centre lines* of respective pins, as shown in Figure 181.

As the forces x_1, x_{11}, etc., should *always*, if possible, be located along pin to make the resisting forces p and q even, which is done by putting the smallest force (x_{111}) in the middle and dividing the others, we should have, *where this is the case:*

Nearer Reaction Force. $$q = \frac{\Sigma . x}{2} + \frac{l}{l_1} . y \qquad (118)$$

Further Reaction Force. $$p = \frac{\Sigma . x}{2} - \frac{l_{11}}{l_1} . y \qquad (119)$$

Where $\Sigma\, x$ = the *sum* of all the opposing forces (x_1, x_{11}, x_{111}, etc.,) to and between p and q, in pounds, provided that they are divided and the halves or fractions located symmetrically with respect to p and q.

Where p = the *total* resisting force (reaction), in pounds, or strain on p, the reaction furthest from free end.

Where q = the total resisting force (reaction), in pounds, or strain on q, the reaction next to free end.

Where l, l_1, l_{11}, and y = same as in Formulæ (116 and 117); of course, all the forces must be projected to one line as shown in Figure 177.

If one of the forces, x_1, x_{11}, etc., were = y and were placed to the left of p (Figure 181) the forces p and q would be equal and each = one-half of the sum of all the other opposing forces, provided always that the forces x_1, x_{11}, etc., are symmetrically located in respect to p and q. Where there are a number of forces on both sides of pin, the pin might be treated as a continuous girder (see Table XVII),

but the calculation would become very difficult and the parts of all rods (or compression piece) acting in the same lines and direction would become very irregular.

Locate Forces Symmetrically.

It is customary, therefore, to locate the forces along the pin symmetrically, regardless of their true resistances as they would be if treated as continuous girders, and to consider, that each takes its *proportionate load* (according to the thickness of its head) of the whole load along its respective line and direction. In each case it will require special study to obtain the most economical arrangement.

An example will more fully illustrate the method of calculating pins.

Example IV.

Calculation of Pin-Joint.

The joint A of a pin-connected wrought-iron truss is strained by the following members: The tie-rod $B = -28000$ *pounds: the tie-rod* $C = -70000$ *pounds; the strut* $D = +20000$ *pounds; and the tie-rod* $E = -88000$ *pounds. All as shown in Figure* 182; *design the joint.*

We will assume that for certain reasons we wish to use a $2\frac{3}{4}''$ diameter pin. Now the first thing to do is to settle the thickness of the (eye) heads. These, of course, must have sufficient bearing against pins not to crush the pin or be crushed by it. We use therefore the following formula:

Thickness of Heads.

$$t = \frac{s}{d.\left(\frac{c}{f}\right)} \qquad (120)$$

Where $t =$ the necessary thickness of eye-bar head, in inches.

Where $s =$ the strain on each eye-bar, in pounds.

Where $d =$ the diameter of pin, in inches.

Where $\left(\frac{c}{f}\right) =$ the safe-compression stress, per square inch, of the material of eye-bar or pin (whichever is the weaker in resisting crushing should be used.)

Accordingly we should have for thickness of the different eye-bar heads in our example the following:

$$A\,B = \frac{28000}{2\frac{3}{4}.12000} = 0{,}85 \text{ or say } \frac{3}{4}''$$

$$A\,C = \frac{70000}{2\frac{3}{4}.12000} = 2{,}12 \text{ or say } 2''.$$

$$D\,A = \frac{20000}{2\frac{3}{4}.12000} = 0{,}6 \text{ or say } \tfrac{5}{8}''$$

$$A\,E = \frac{88000}{2\frac{3}{4}.12000} = 2{,}67 \text{ or say } 2\tfrac{3}{4}''$$

The values for above thicknesses have been rather too broadly rounded off, but this is done to simplify the subsequent calculations.

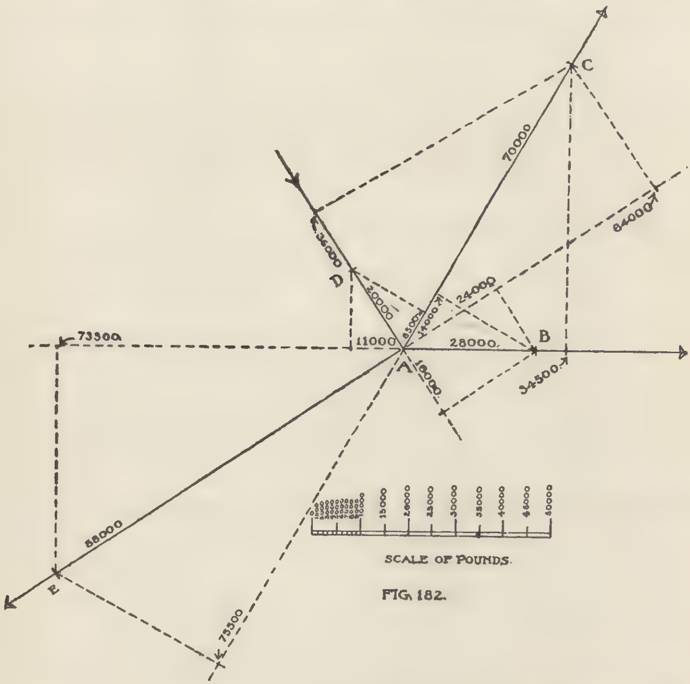


FIG. 182.

Thickness by Table XXXVI.

Had we used Table XXXVI we should have had the same results. For $A\ B$ (28000 pounds) the horizontal line $2\frac{3}{4}''$ and vertical line 28000 (from *above* for iron) meet between the heavy bearing lines $\frac{13}{16}''$ and $\frac{7}{8}''$, for convenience, however, we will call it $\frac{3}{4}''$ though this should not, of course, be done in a real calculation.

For $A\ C$ (70000 pounds) horizontal line $2\frac{3}{4}''$ and vertical line 70000 from above (the second to the right of 68000) meet just be-

yond the 2″ heavy bearing line. It should be, therefore, a little over 2″ thick, but we will call it even 2″.

For $D\ A$ (20000 pounds) the horizontal line $2\frac{3}{4}''$ and vertical line 20000 from above, meet between heavy bearing lines $\frac{9}{16}''$ and $\frac{5}{8}''$, we will call it $\frac{5}{8}''$.

To find $A\ E$ (88000 pounds) which is larger than $A\ C$, we shall evidently have to divide it in halves, and, of course, double the result. We find that horizontal line $2\frac{3}{4}''$ and vertical line 44000 from above, meet between the heavy bearing lines $1\frac{5}{16}''$ and $1\frac{3}{8}''$, or we will say $1\frac{3}{8}''$; this doubled or $2\frac{3}{4}''$ is the required thickness therefore, for head $A\ E$.

Of course, if we use more than one bar for either of the strains we will divide the required thickness of head accordingly. Thus, if we decide to use two bars along the line $A\ C$, each would be strained $= \frac{70000}{2} = 35000$ pounds, and the thickness of head required for each would be only $= \frac{2{,}12}{2} = 1{,}06$ or say 1″.

Arrangements of Heads along Pin.

Now to arrange the different heads along the pin, we first lay off along each line (Figure 182) the amount of strain (measuring all at the same scale) and project these strains on to the different lines. We measure these projections and have along the line $A\ B$ the strain $A\ B = 28000$ pounds pulling to the right; the projection of $A\ C = 34500$ pounds

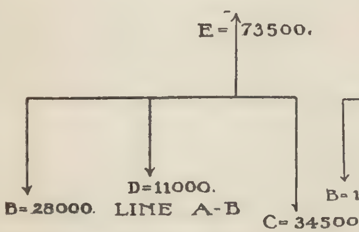


FIG. 183.

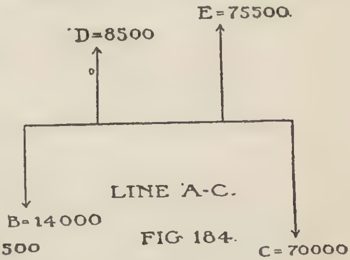


FIG 184.

also pulling to the right; also the projection of $D\ A = 11000$ pounds pushing to the right; the projection of $A\ E = 73500$ pounds pulling to the left, thus opposing the other three.[1]

If our measurements are right the sum of the forces acting in one

[1] These figures have been rounded off to simplify the calculations.

direction along line $A\ B$, must equal the sum of their opponents, or, $A\ B + A\ C + D\ A = A\ E$ and we have in effect,

$28000 + 34500 + 11000 = 73500$

The strains on the pin along line $A\ B$ are shown in Figure 183. Those along line $A\ C$ are shown in Figure 184 and those along $A\ E$

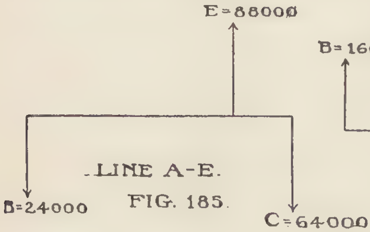


FIG. 185.

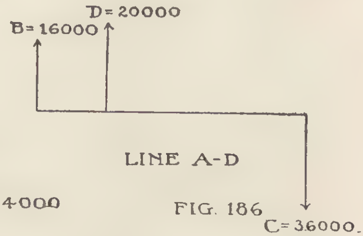


FIG. 186

are shown in Figure 185; it will be noticed that in the latter case $D\ A$ becomes $= O$. In Figure 186 are shown the strains along $D\ A$, in this case $A\ E$ becomes $= O$.

Design for largest Strain first.

As the largest strain in one direction (88000 pounds) is along line $A\ E$, we will select Figure 185 to design the joint and when we have settled the arrangement of heads along the pin to suit these strains, we will see how it affects the pin according to the strains along the other lines.

Shearing strain too great.

The simplest arrangement of the parts would be evidently that shown in Figure 185. We will first consider the shearing. The largest shearing strain will be between C and E and will be $= 64000$ pounds. From Table XL we find for wrought-iron, single shear (at 8000 pounds per square inch) that we should need a $3\frac{3}{16}''$ diameter pin to resist 64000 pounds single shear (as each of the vertical spaces at the top evidently represent 10000 pounds), we must pass down vertically not quite half-way between the second and third lines to the right of 50000, till we strike the single shear iron curve and then pass along the horizontal line to the left to find diameter of pin, which is between $3\frac{1}{8}''$ and $3\frac{1}{4}''$ or say $3\frac{3}{16}''$. Had we calculated arithmetically we should have had, area of cross-section required from Formula (7) transposed

$$a = \frac{64000}{8000} = 8 \text{ square inches.}$$

By referring to a table of areas of circles, or by calculation we find

that for an area of cross-section of 8 square inches we require a diameter of $3\frac{3}{16}''$. This is a larger pin than we want to use and besides seems a very large pin for the strains; it is evident, therefore, that our heads are badly arranged along the pin; we will decide, therefore, to divide the rods E and C each in two parts, making each head one-half the thickness as above found, and arrange them as shown in Figure 187.

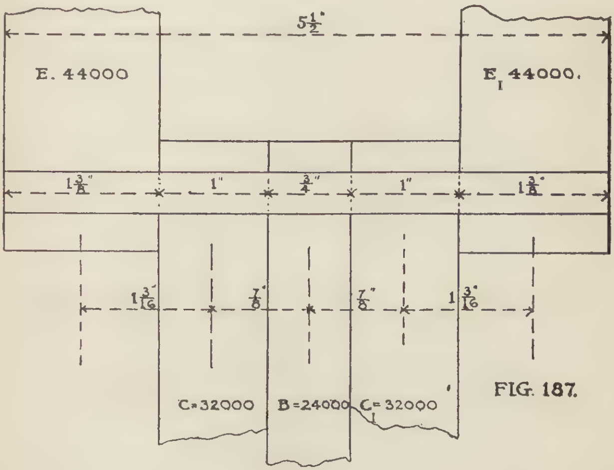


FIG. 187.

Now the safe cross-shearing on our pin ($2\frac{3}{4}''$) for single-shear would be from Table XXXIX $= 47600$ pounds, we pass along horizontal line $2\frac{3}{4}''$ till we strike iron single-shear curve and then pass upward about four-fifths way between the third and fourth vertical lines to the right of 40000; as each space is evidently 2000 pounds, we should have $40000 + 3\frac{4}{5}.2000 = 47600$ pounds. By calculation we should have had area of $2\frac{3}{4}''$ circle $= 5{,}939$ square inches, therefore safe (single) cross-shearing see Formula (7) $= 5{,}939.8000 = 47512$ pounds. The greatest cross-shearing strain on the pin with arrangement as shown in Figure 187 is 44000 pounds, and is between the heads E and C (or E_1 and C_1), so that we need not fear shearing.

Shearing from Table XXXIX.

The shearing area of pin being all right we now consider the bend-

ing-moment. We have marked along the pin, Figure 187, the thicknesses of heads, the length of pin required being $5\frac{1}{2}''$, to this must be added the head and nut and also sufficient for strut D ($\frac{5}{8}''$) which we remember did not come into the calculation along line $A\ E$ (Figure 182.)

Bending-moment on pin.

Immediately under the pin, Figure 187, we have marked the distances from centre to centre of heads, which are, of course, the distances we consider when calculating the bending-moment. Accordingly our pin becomes a circular wrought-iron beam of $2\frac{3}{4}''$ diameter, with a span or length of $4\frac{1}{8}''$ and supported at both ends by forces E and E_1. The beam is loaded with the forces C, C_1 and B as shown in Figure 187.

The greatest bending-moment will be at the centre (See p. 51, Vol. I), and will be

$$m_B = 44000.\ (1\tfrac{8}{16} + \tfrac{7}{8}) - 32000.\tfrac{7}{8} - 24000.0$$
$$= 90750 - 28000 - 0$$
$$= 62750 \text{ pounds-inch.}$$

This will be much more than the pin can stand for we have, for the safe bending-moment on a $2\frac{3}{4}''$ pin, moment of resistance

$$r = \tfrac{11}{14}.\ (1\tfrac{3}{8})^3 = 2{,}042$$

(Table I, Section No. 7) and from Formula (18) transposed, the safe bending-moment on pin

$$m = 2{,}042\ .15000$$
$$= 30630 \text{ pounds-inch}$$

or, only about one-half of the actual bending-moment. Had we used Table XXXIX instead of calculating arithmetically we should have passed along the horizontal line $2\frac{3}{4}''$ till we struck the dotted bending-moment curve for wrought-iron at 15000 pounds and then passed vertically to the *bottom.* This would be about two-fifths way between the vertical lines 30000 and the first one to its right, each space being evidently 1500 pounds, this would mean 30600 pounds-inch safe bending-moment on a $2\frac{3}{4}''$ pin.

Bending-moment from Table XXXIX.

It is evident, therefore, that we must re-arrange the rods, trying to get the span between loads E and E_1 shorter if possible.

Try new arrangement.

We now try the arrangement shown in Figure 188, placing load B at one end. The arm end B of pin now becomes a lever and we know from Formulæ (118 and 119) that the reactionary forces E and E_1 can no longer be equal.

From Formula (118) we have

$$E = \frac{32000 + 32000}{2} + \frac{4\frac{7}{16}}{3\frac{3}{8}}.24000$$
$$= 63555$$

From Formula (119) we have

$$E_1 = \frac{32000 + 32000}{2} - \frac{1\frac{1}{16}}{3\frac{3}{8}}.24000$$
$$= 24445$$

As a check the sum of the two should equal the whole strain $A\ E$ and we have in effect $63555 + 24445 = 88000$.

This arrangement must evidently be abandoned as a bad one, for the difference in the strains on E and E_1 is altogether too great to

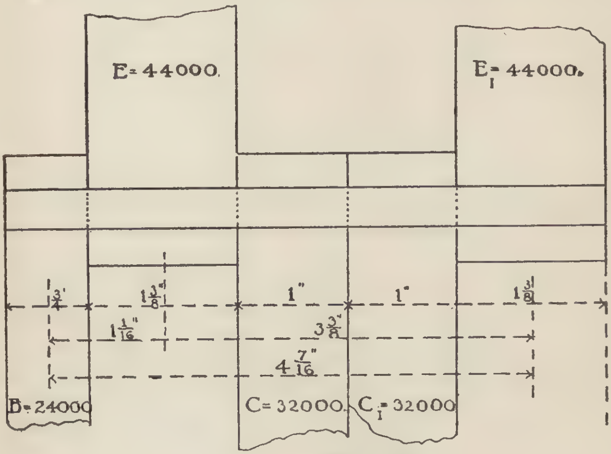


FIG. 188.

be overlooked. Besides, we can readily see that the moments at E and C_1 will exceed 24504 pounds-inch, the safe bending-moment on this sized pin.

For practice, however, we will figure out the bending-moments; they are: at E

(left side) $m_E = 24000.\ 1\frac{1}{16} - 63555.0$

$= 25500$ pounds-inch.

Check (use right side)

$$m_E = 32000.\ 2\frac{8}{16} + 32000.\ 1\frac{8}{16} - 24445.3\frac{3}{8}$$
$$= 25500$$

At C we should have:

(Left side) $m_C = 63555.\ 1\tfrac{3}{16} - 24000.\ 2\tfrac{1}{4}$

$= 21472$

Check (use right side)

$m_C = 24445.\ 2\tfrac{3}{16} - 32000.\ 1$

$= 21472$

At C_I we should have:

(left side) $m_{C_I} = 63555.\ 2\tfrac{3}{16} - 24000.\ 3\tfrac{1}{4} - 32000.\ 1$

$= 29027$

Check (use right side)

$m_{C_I} = 24445.\ 1\tfrac{3}{16} - 32000.\ 0$

$= 29027$

Had we used rule given on p. 51, Vol. I, we should have known that the greatest bending-moment was at C_I. In applying the rule to this case the end load B should be deducted from the nearer reaction to B.

We might next try dividing the force B in two halves of 12000 pounds each, (heads $\frac{3}{8}''$ thick), leaving one at B and placing the other to the right of E_I. This will restore equality to the forces E and E_I, but it will be found that even this arrangement will not do, as the bending-moment at C or C_I will still be found to be too large, namely, $= 27500$ pounds-inch.

Divide largest force.

After these numerous failures it is evident that we cannot well arrange the heads satisfactorily along the pin, unless we enlarge the pin, or else divide up the larger forces which cause us most trouble. We decide to do the latter and divide $A\ E$ into four parts of 22000 pounds each, with heads each $\frac{2{,}67}{4} = \frac{11''}{16}$ thick.

Forces will equalize themselves.

We now arrange the heads as shown in Figure 189. The correct way to calculate the bending-moment would, of course, be to consider the pin as a continuous girder running over four supports. This would make E_I and E_{II} much larger than E and E_{III}. Their heads would therefore have to be thickened so as not to crush the pin, or be crushed by it. It can readily be seen that were we to do this, the calculation would be almost interminable. Besides, practically, it would be expensive to use so many different sizes of rods, heads, etc. We must, therefore, assume that all the forces E, E_I, E_{II} and E_{III} are equal. This

they will be, too, for as soon as E_{I} and E_{II} tend to take more load, they will stretch under the added tension, and this stretching will bring the pin to bear more heavily on the ends and thus the strains will even themselves up.

The bearing of these heads against the pin we know are all right, also the shearing, as the greatest shearing under this arrangement

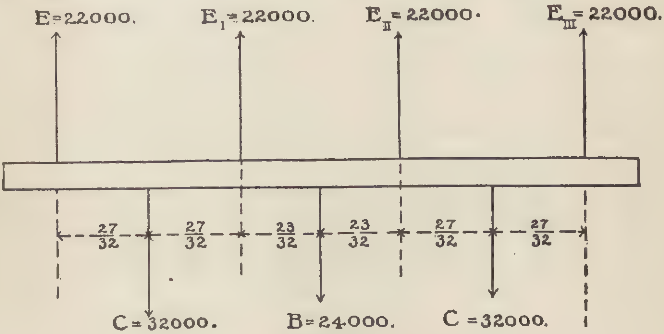


FIG. 183.

will be a single shear, between E and C (or E_{III} and C_{I}) and equal 22000 pounds, or considerably less than half the safe single shearing on this pin, which we previously found to be 48000 pounds.

The bending-moments on the pin will now be, at C:

(Left side)

$$m_C = 22000.\tfrac{27}{32} - 32000.0$$
$$= 18563 \text{ pounds-inch.}$$

Check (use right side)

$$m_C = 22000.\left(\frac{27 + 73 + 127}{32}\right) - 24000.\tfrac{50}{32} - 32000.\tfrac{100}{32}$$
$$= 18563 \text{ pounds-inch.}$$

At E_{I} we should have:

(Left side)

$$m_{E_{\text{I}}} = 22000.\tfrac{54}{32} - 32000.\tfrac{27}{32}$$
$$= 10125 \text{ pounds-inch.}$$

Check (use right side)

$$m_{E_{\text{I}}} = 22000.\left(\frac{46 + 100}{32}\right) - 24000.\tfrac{23}{32} - 32000.\tfrac{73}{32}$$
$$= 10125 \text{ pounds-inch.}$$

At B we should have:
(Either side)

$$m_B = 22000.\left(\frac{23+77}{32}\right) - 32000.\tfrac{50}{32}$$
$$= 18750 \text{ pounds-inch.}$$

At the ends, of course, there would be no bending-moment, for take end E we should have:

$$m_E = 22000.\left(\frac{154+100+54}{32}\right) - 32000.\left(\frac{127+27}{32}\right) - 24000.\tfrac{77}{32}$$
$$= 0$$

This arrangement (Figure 189) is therefore satisfactory, so far at least as the strains along the line $A\ E$ are concerned.

Examine along other lines.

We must now see if it will answer as well for the strains along the other lines (See Figure 182). The direction we shall now have to fear most, will be along the line $D\ A$ for force D must be placed entirely on the outside edge of pin (not having been located yet) and being quite large,

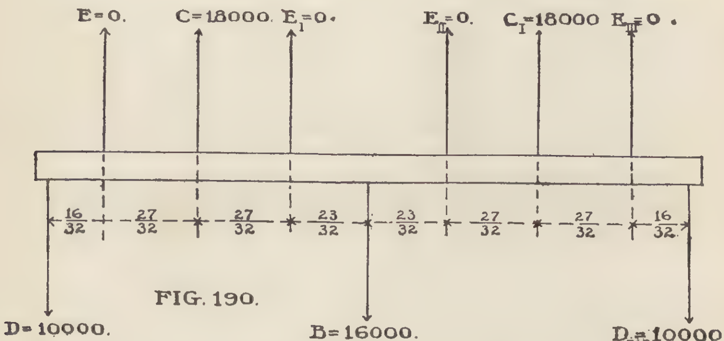


FIG. 190.

20000 pounds, may make us some trouble. In Figure 190 we have drawn the forces arranged the same as we settled on last (in Figure 189) but have added the two forces on the extreme ends D and D_I each $= \dfrac{D\,A}{2} = 10000$ pounds. It will be noticed that along this line ($D\,A$ Figure 182) the forces E and C are in the same direction. We have divided D into two parts for two reasons. Had we placed it entirely to one side, say to the right of E_{III}, the distance $E_{III}\ D_I$ would have been one-quarter of $\tfrac{5}{8}''$ larger or $= \tfrac{21}{32}''$; the leverage of

D_1 therefore at C_1 would have been $\frac{27+21}{32} = 1\frac{1}{2}''$ and the bending-moment

$$m_{C_1} = 1\tfrac{1}{2}.\,20000$$
$$= 30000 \text{ pounds-inch, too much for our pin.}$$

Besides were we to calculate C and C_1 by Formulæ (118 and 119) we should find all strain on C removed and C_1 more than doubled. This evidently would not do, without special provision to meet the unequal strain by increasing C_{11}, which would lengthen the pin, so that we prefer to divide $D\,A$, making each head of half the thickness, or $\frac{5}{16}''$ thick. The largest shearing will be between D and C (or D_1 and C_1) and equal 10000 pounds single cross-shear or about $\frac{1}{5}$ only of the safe resistance to shearing.

The bending-moments will be (Figure 190) at C.
(Left side)

$$m_C = \tfrac{48}{32}.10000 - 18000.0$$
$$= 13437 \text{ pounds-inch.}$$

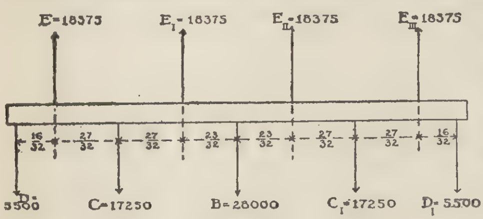
Check (right side).

$$m_C = \tfrac{143}{32}.10000 + \tfrac{50}{32}.16000 - \tfrac{100}{32}.18000$$
$$= 13437 \text{ pounds-inch.}$$

and at B
(Either side)

$$m_B = \tfrac{98}{32}.10000 - \tfrac{50}{32}.18000$$
$$= 9375 \text{ pounds-inch.}$$

The bearings, of course, are safe, as the thickness of head was originally determined by the largest strain on each rod along its own line. So that we are all right with our pin for strains along line $D\,A$.



.FIG. 191.

We now take up the strains on the pin along the line $A\,B$, Figure 182, which will be as shown in Figure 191.

The greatest shearing-strain here will be caused by B, and will be a double shear of 28000 pounds, or 14000 pounds on each area, perfectly safe on our size of pin ($2\frac{3}{4}''$). The moments will be :

At E

(Left side)

$$m_E = \tfrac{1}{2}.5500 - 18375.0$$
$$= 2750 \text{ pounds-inch.}$$

Check (right side)

$$m_E = 5500.\tfrac{170}{32} + 17250.\left(\frac{27+127}{32}\right) + 28000.\tfrac{77}{32} - 18375.\left(\frac{54+100+154}{32}\right)$$
$$= 2750 \text{ pounds-inch.}$$

At C:

(Left side)

$$m_C = 18375.\tfrac{27}{32} - 5500.\tfrac{43}{32}$$
$$= 8113 \text{ pounds-inch.}$$

Check (right side)

$$m_C = 18375.\left(\frac{27+73+127}{32}\right) - 5500.\tfrac{143}{32} - 17250.\tfrac{100}{32} - 28000.\tfrac{50}{32}$$
$$= 8113 \text{ pounds-inch.}$$

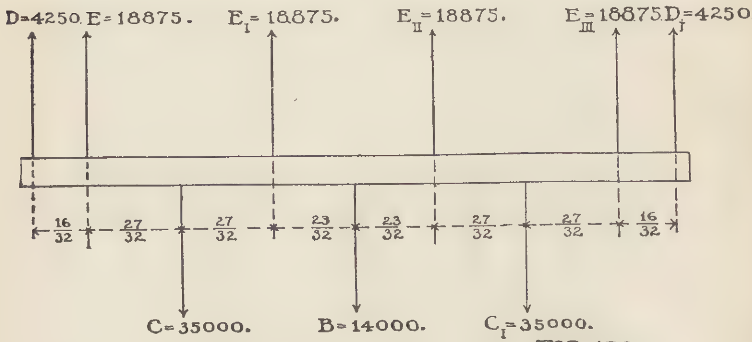


FIG. 192.

At E_1:

(Left side)

$$m_{E_1} = 18375.\tfrac{54}{32} - 5500.\tfrac{70}{32} - 17250.\tfrac{27}{32}$$
$$= 4422 \text{ pounds-inch.}$$

Check (right side)

$$m_{E_1} = 18375.\left(\frac{46+100}{32}\right) - 28000.\tfrac{23}{32} - 17250.\tfrac{73}{32} - 5500.\tfrac{116}{32}$$
$$= 4422 \text{ pounds-inch.}$$

At B:
(Either side)

$$m_B = 18375.\left(\frac{23+77}{32}\right) - 17250.\tfrac{50}{32} - 5500.\tfrac{88}{32}$$
$$= 14484 \text{ pounds-inch.}$$

So that the arrangement of heads along pin is all right so far as strains along line A B (Figure 182) are concerned.

We finally examine for the strains along line A C and have strains

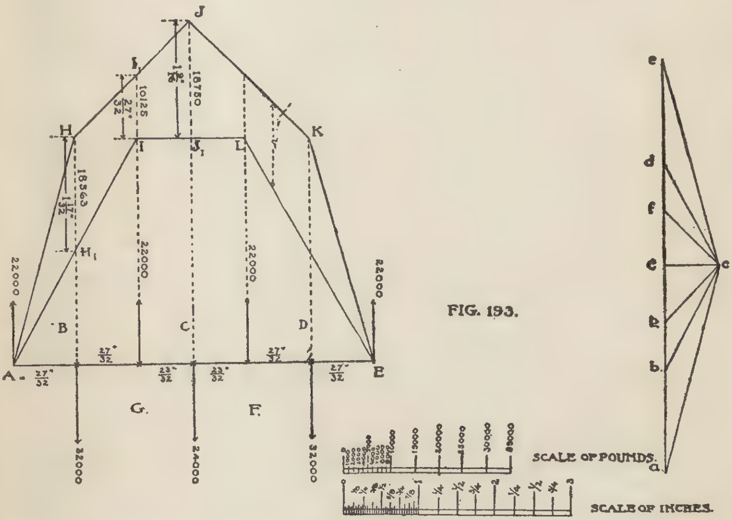


FIG. 193.

on the pin accordingly as shown in Figure 192. The greatest shearing here will be between E and C (or C_I and E_{III}) and will be

$$= 4250 + 18875 = 23125 \text{ pounds,}$$

single shear, still, less than one-half of the safe single-shear on the pin.

By calculation the moments will be found to be at the different points, as follows:

$$m_E = 2125 \text{ pounds-inch.}$$
$$m_C = 21637 \text{ pounds-inch.}$$
$$m_{E_I} = 11617 \text{ pounds-inch.}$$
$$m_B = 16648 \text{ pounds-inch.}$$

So that this arrangement of the different bars and strut along the pin is in every way satisfactory.

Graphical method more simple. The graphical method of obtaining bending-moments is frequently much more simple than the arithmetical method; in important calculations both should be used so as to check each other.

All the rules for formulæ given in Chapter VII for the calculation of transverse strains by the graphical method will apply equally well for pins; the only difference will be that where there are more than two forces on each side of the pin, the base line of the polygonal figure between reactions p and q will no longer be straight, but will be composed of several lines.

Thus, if we take the pin and forces shown in Figure 189, we should change the notation to that adopted for the graphical method, which would be as shown in Figure 193. That is force E (22000 pounds) of Figure 189 would be known as $A\ B$ in Figure 193; again force C_1 (32000 pounds) of Figure 189 would be called force $E\ F$ (*not* $F\ E$) in Figure 193.

Example solved graphically. Proceeding now to the calculation, we lay off along the vertical load line $a\ e$, the following forces:

$a\ b = A\ B = 22000$ pounds.
$b\ c = B\ C = 22000$ pounds.
$c\ d = C\ D = 22000$ pounds.
$d\ e = D\ E = 22000$ pounds.

and in the opposite direction, we lay off:

$e\ f = E\ F = 32000$ pounds.
$f\ g = F\ G = 24000$ pounds.
$g\ a = G\ A = 32000$ pounds.

which will, of course, bring us back to the starting point a, as the opposing forces must aggregate the same sum.

We now select our pole o. This we remember can be arbitrarily located, or else at a distance $o\ c = \left(\frac{k}{f}\right)$; in our case we will make the distance, say, 12000 pounds.

Pole distance. The distance (of pole from load line) being arbitrary we shall multiply the verticals v (inch-scale) in Figure 193, by this pole distance (pounds-scale) to obtain the bending-moments at the points of pin immediately below verticals. If the pole distance from load line had been made $= \left(\frac{k}{f}\right)$, then the

length of verticals v measured at inch-scale would have been the *required* moments of resistance of the corresponding points of pin below verticals; and each respective v multiplied by $\left(\frac{k}{f}\right)$ would be the bending-moment at each point. We will, however, make in our case the pole distance, arbitrary, *viz:* $c\,o = 12000$ pounds. We now begin at any point of line $A\,B$ and draw $A\,I$ parallel $b\,o$ till it intersects $B\,C$ at I; next draw $I\,L$ parallel $c\,o$ to intersection with $C\,D$ at L; and similarly draw $L\,E$ parallel $d\,o$; $E\,K$ parallel $o\,e$; $K\,J$ parallel $o\,f$; $J\,H$ parallel $o\,g$ and $H\,A$ parallel $o\,a$; the last line must intersect the first at point of starting A or some error has been made.

Strain diagram.

It will be noticed that the in lividual outlines of $A\,I\,L\,E\,K\,J\,H\,A$ cover the capital letters in Figure 193, corresponding to small letters from which their respective parallel lines started in strain diagram. Thus, for instance $A\,I$ covers letter B and is parallel to $b\,o$; similarly $I\,L$ covers C and is parallel to $c\,o$; $L\,E$ covers D and is parallel $d\,o$; $K\,J$ covers F and is parallel $o\,f$; $J\,H$ covers G and is parallel $o\,g$; similarly we can consider $E\,K$ as covering E and it is parallel $o\,e$; and $H\,A$ as covering A and it is parallel $o\,a$.

Curve of bending-moments.

We now measure the verticals through the figure $A\,I\,L\,E\,K\,J\,H\,A$, the longest, of course, will give the greatest bending-moment. This happens to be the central one $J\,J_1$ it measures $1\frac{9}{16}''$, therefore

$$m_J = 1\tfrac{9}{16}.12000 = 18750$$

which corresponds to m_B of Figure 189.

Similarly we should have:

m_1 (formerly m_{E_1}) $= \frac{27}{32}.12000 = 10125$ pounds-inch.

m_H (formerly m_C) $= 1\frac{17}{32}.12000 = 18563$ pounds-inch.

Had we analyzed the strains on pin as shown in Figure 192 graphically, our verticals would have measured,

at E:

$$v_E = \tfrac{8}{16}''$$

at C:

$$v_C = 1\tfrac{13}{16}''$$

at E_1:

$$v_{E_1} = \tfrac{31}{32}''$$

and at B:

$$v_B = 1\tfrac{3}{8}''$$

The corresponding bending-moments would have been:

at E:

$$m_E = \tfrac{3}{16}.12000 = 2250 \text{ pounds-inch.}$$

at C:

$$m_C = 1\tfrac{13}{16}.12000 = 21750 \text{ pounds-inch.}$$

at E_1:

$$m_{E_1} = \tfrac{31}{32}.12000 = 11625 \text{ pounds-inch.}$$

at B:

$$m_B = 1\tfrac{3}{8}.12000 = 16500 \text{ pounds-inch.}$$

which are very close to the correct moments, which we found arithmetically to be:

$$m_E = 2250 \text{ pounds-inch; } m_C = 21637 \text{ pounds-inch;}$$
$$m_{E_1} = 11617 \text{ pounds-inch; and } m_B = 16648 \text{ pounds-inch.}$$

Simplest method use curve of moments of resistance.

The simplest method of calculating pins, as a rule, will be — after calculating (or ascertaining from Tables) the safe bearing and shearing stresses of the pin, — to calculate the actual moment of resistance of the pin, see Table I, Section No. 7, fourth column. Now proceed graphically, being sure to make the pole distance in every case equal to the safe modulus of rupture $\left(\frac{k}{f}\right)$ of the material of pin. After this it will only be necessary to see that none of the verticals through the different polygonal figures (corresponding to $A\ I\ L\ E\ K\ J\ H\ A$ of Figure 193) — that none of these verticals measure at inch-scale *more* then the actual moment of resistance of the pin. If this is done the calculation and selection of the best arrangement becomes very simple and easy. After the final and best arrangement has been determined on, it would be well to calculate arithmetically the moments as per this final arrangement, thus checking the graphical solution.

Contractors claim of no bending-moment ridiculous.

The writer has frequently been told by contractors that owing to the friction due to the pressure of the nut and head, that it was impossible for any bending-moment to take place on a pin. As well it might be claimed that owing to the pressure of the walls, there is no bending-moment on a built-in beam. The safe modulus of rupture of a built-in beam can be assumed higher same as we do for pins, but the beam will break across if too heavily strained and so will the pin. Besides it must be remembered that the heads of eye-bars necessarily cannot fit the pins perfectly; and even if the argument

were correct, (which it is not,) that friction offsets the bending-moment, the least rusting of the joints would diminish the pressure between nut and head, thus destroying its value.

Again, contractors will admit that there exist bending-moments on pins, but will deny it in the case of rivets, though the cases are precisely analogous; this latter argument though, if sifted, will generally lead the contractor to admit that its real basis is the large number of rivets it frequently requires; and the less rivets he can get along with, the happier will your contractor be.

CHAPTER X.

PLATE AND BOX GIRDERS.

FIG. 194.

FIG. 195

WHEN it becomes necessary to cover floors or spaces of such large span, or to carry loads so heavy, that rolled-beams will not answer the purpose, girders are made up of plates and angles riveted together, and are known according to their section as "riveted plate girders" with single webs (Figure 194), "or riveted box girders" with two or more webs (Figure 195). As a rule, too, riveted girders of equal strength can be more cheaply manufactured than the heavier sections of rolled beams.

Single web girders.

In the case of the former the vertical plate or web has two angle irons riveted horizontally along its entire top edge and the same along its bottom edge, or four in all. In very light construction the free legs of the angles might answer for the top and bottom flanges, but, as a rule, a plate is riveted to these free legs, at right angles to the web, both top and bottom, thus forming the flanges of the girder. This plate need not necessarily extend the entire length, but it usually does. Where the thickness of flanges required is very great, say one inch or more, each flange is made up of two or more thicknesses or layers of plates.

In such cases only the layer nearest to the web is carried the entire length, the other layers gradually decreasing from the point of greatest bending-moment (usually the centre) towards the ends.

To carry all the layers to the ends in heavy work would be a great extravagance, the only advantage gained being a slight increase in stiffness, which can be very much more readily and economically gained by increasing the depth of web.

Box girders. In double web box girders only two angles are attached to each web, one at the top and one at the bottom, both on the outside surface of each web. To place angles on the inside surface is impracticable, as webs would have to be placed sufficiently far apart for the "holder-up" to crawl in, and the riveting would not only be weak, having to be done by hand, but it would weaken the flange by just so many additional rivet holes. In short girders with heavy loads, where shearing is the main danger, box girders with three webs are sometimes made; in that case the middle web has the usual four angles, but the two outside webs only two angles each.

Beyond the additional stiffness sideways, in resisting lateral flexure, there is no particular advantage in using a box girder. It is more clumsy to handle and to make, and not readily painted on all exposed surfaces, and besides is more extravagant of material in proportion to its strength. Where the flange is of great breadth and it becomes desirable to have two or more webs, the writer always prefers to use two or more single web plate girders, and to secure them together with bolts and separators, or by latticing the top and bottom flanges, together.

The angles need not necessarily be even-legged; nor need the web necessarily be of same depth throughout, nor of same thickness throughout. It will, however, greatly simplify the calculation to keep the web uniform throughout, and in most cases the extra labor involved in varying the thickness of web, would more than offset the cost of the unnecessary material at the centre.

Where girders are very deep, the web is made in sections or panels, as already explained. In such cases the web can readily and economically be thickened towards the ends.

Cambering girders. Whenever possible, the girder should be cambered up at the centre an amount equal to the calculated deflection. But as girders are usually made of straight plates, and machine riveted and punched, the cambering is rarely practicable. Should the girder, however, show any bending or cambering in transportation, the architect should be sure to have the cambered side placed on top.

The calculation of riveted girders is exactly the same as for iron beams, but has the additional element of the number and location of rivets to be looked into, also the stiffness of web and overhang of flanges.

The reactions, vertical shearing, bending-moments, actual and required moments of resistance, deflections, etc., can be calculated arithmetically by the rules given in Chapters I and VI; or graphically by the rules given in Chapter VII.

The rules fcr calculating riveted work were given in the previous chapter (IX).

Horizontal flange strain.

The only new matter is to find what the strain on the rivets will be. It will be readily seen that when a plate girder is loaded the tendency of the flanges and angle irons is to slide horizontally past the web (see Figures 120 to 125).

This tendency to slide is called the horizontal flange strain. The rivets connecting angles to web resist this tendency and there must be sufficient rivets to do this safely.

The total amount of this tendency to slide or horizontal flange strain between any selected point of girder and the *nearer* end of girder, is equal to the bending-moment at the selected point, divided by the depth of web of girder at the point, or

$$s = \frac{m}{d} \qquad (121)$$

Where $s =$ the total strain, in pounds, coming on all the rivets connecting *either* top or bottom flange to web, between any selected point of girder and the nearer end.

Where $m =$ the bending-moment in pounds-inch, at the selected point of girder.

Where $d =$ the total depth, in inches, of the web of girder at the selected point.

The above strain s will exist in both top and bottom flanges and will be resisted by all the respective rivets in *either* top or bottom flange that connect the angles to the web.

Number of rivets in web leg of angles.

It should now be ascertained which is the weakest resistance of each rivet, whether it be to bearing (compression), to shearing, or to bending, and this weakest resistance divided into strain s, as found by Formula (121) will, of course, give the number of rivets required along *each* edge of web between the selected point and the nearer end of girder.

Frequently many more rivets will have to be used than are required by calculation in order not to exceed the greatest pitch for rivets given in Formula (107).

To ascertain the number of rivets required (along either web edge) between any two points of girder, we will, of course, take the difference between the numbers required from each point to end of girder.

In the flange leg of angles usually fewer rivets can be placed than in the web leg, though many good engineers frequently make them equal in number. But this is really unnecessary, for even if the strains on the flange rivets were the same as those on the web rivets (which they are not) we should still have two rivets in the flange to one in the web on all single plate girders.

Number of rivets in flange leg of angles.

There seems to be considerable difference of opinion as to just how to figure the strain on the flange rivets. The best course would seem to the writer to be, to assume that each flange cover plate must transfer at each of its ends, by rivets, to the angle iron and parts of flange plates between it and the angle iron, an amount equal to the safe stress the plate is capable of exerting (that is, net area of cross section of the plate multiplied by either the safe compression stress $\left(\frac{c}{f}\right)$ or by the safe tensional stress $\left(\frac{t}{f}\right)$ as the case may be).

This amount should be transferred by sufficient rivets, between the end of each plate, and the point of girder at which the full thickness of the plate is required to make up the required moment of resistance. From this point to centre the rivets can be spaced according to the rule for greatest pitch, Formula (107), but when rivets are so spaced the pitch of the rivets immediately nearest the ends of any cover plate should be greatly decreased for a distance of three or four rivets at each end.

By the above method the amount of strain on rivets can be quite accurately computed.

Locating flange rivets.

The simplest method of locating rivets is to construct what might be called the curve of moments of resistance. This can be done as shown in Figure 151, Chapter VII (where $C\,D\,E\,F\,G\,C$ is the curve of moments of resistance), or we can calculate arithmetically the required moments of resistance at several points of girder, and lay out the curve as

shown in Figure 196, where $A\ B$ represents the length of girder, and $M\ C$, $I\ D$ and $N\ E$ the calculated required moments of resistance at points M, I and N.

The curve of moments of resistance is, of course, $A\ C\ D\ E\ B$, and its axis or base $B\ A$.

We now make $I\ H = a_{\text{I}}\ d$ see Formula (99); a_{I} being the net area of cross-section in square inches of two angles, and d the total depth of girder in inches.

$H\ D$ will now represent the total required thickness of flange plates, which we can find from Formulæ (36) or (98).

We will decide to divide it into three layers $H\ G$, $G\ F$ and $F\ D$. We draw the lines as shown and find that plate No. 1 can stop at K, though it would be better to run it full length, it is, however, needed of full thickness at J. Again plate No. 2, can stop at J, but is

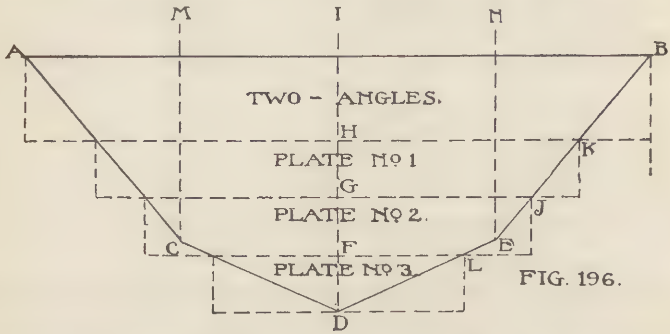


FIG. 196.

needed full thickness at L. The top plate, of course, will run from L to centre. The left half of girder, will of course be similar, the loads evidently being symmetrical each side of centre. In practice the plates rarely are stopped at the exact points calculated, but are usually extended beyond these points a distance equal to from once to twice the width of plate.

There must now be rivets enough between D and L to equal the efficiency of plate No. 3, between L and J to equal the efficiency of plate No. 2 and between J and K to equal the efficiency of plate No. 1. If there is not room to get them in the plates must be sufficiently extended to get them in, that is No. 3 must be lengthened beyond L and towards J; No. 2 must be lengthened beyond J towards K and No. 1 carried on towards end.

In laying out the rivets they should be as regular as possible, the best method is to lay out the total number of rivets required from centre to end, gradually decreasing the pitch towards ends, and then to make each of the plates No. 3, No. 2 and No. 1 of sufficient length beyond their respective point D (for No. 3) L (for No. 2) and J (for No. 1) to take in the number of rivets required. The length of plates may always be more than shown in Figure 196 without harm, but *never less.*

Regularity of spacing.

We should have then for the bottom flange:

Number rivets in end of each flange plate.

$$x = \frac{a.\left(\frac{t}{f}\right)}{v} \qquad (122)$$

Where x = the number of rivets required in each end of each flange plate between its ends and the nearer points to ends at which its full strength is required by the girder.

Where a = net area of cross-section of the flange plate, in square inches (less rivet holes), at its weakest section.

Where $\left(\frac{t}{f}\right)$ = the safe tensional stress, per square inch of the material. For top flange use $\left(\frac{c}{f}\right)$ and look out that rivets are not so far apart as to cause bending or wrinkling of plates.

Where v = the safe stress, in pounds, or least value of each rivet. That is the bearing, shearing or cross-breaking value of the rivet, whichever is the smaller.

The rivets in the flanges will, of course, be cantilevers, loaded with their respective amounts of $a.\left(\frac{t}{f}\right)$ or $a.\left(\frac{c}{f}\right)$ respectively, a being the area as given in Formula (122). The free end of cantilever will be of a length equal to the thickness of the respective flange plate, or equal to the thickness of leg of angle iron, whichever is the smaller should be used.

Value of rivets.

The bearing area will be the diameter of the rivet multiplied by the same smaller thickness.

In single plate girders the shearing of *rivets will* rarely determine their number, this will generally be more than the bearing or bending value.

Figure 197 shows clearly the way in which rivets are strained.

The web-rivet, No. 3, is *bearing* against web on the surface $E\,F$ and against angles on the two surfaces (sum of) $D\,E$ and $F\,G$

The rivet has two cross-shearing areas, at E and F. This rivet is a beam supported at $D\,E$ and $F\,G$ and loaded uniformily with its share of horizontal flange strain, which it is transferring to web.

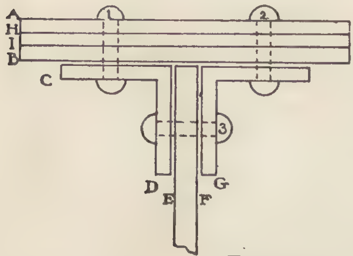


FIG. 197.

The flange rivets, Nos. 1 and 2, we will suppose are connecting the flange plate $A\,H$ to the angle iron. Their bearing then is against $A\,H$ and in the opposite direction against $B\,C$, the lesser should be used. Their shearing area is *either* along the line H or the line B according to which end is considered the cantilever, so that they have only one shearing-area practically in the calculation, instead of two as with the web rivets.

Then rivets Nos. 1 and 2 are cantilevers and are built in *either* from H to C and loaded uniformily on the free end $A\,H$, or built in from A to B and loaded uniformly on the free end $B\,C$, whichever projection $A\,H$ or $B\,C$ is smaller should be used. The load on the cantilever being as already explained equal to each rivet's share of an amount equal to the net-area, of top plate $A\,H$ multiplied by the safe tensional or compressive stress per square inch of the material.

There is, of course, a tendency of the plates $H\,I$, $I\,B$, etc., to slide past each other and past angles.

Plate tendency to slide.

This tendency will exist particularly at the centre of girder and in those parts of rivets which simply tend to hold the plates together after the plates have once transferred their strength and become a permanent part of the flange. But this tendency rarely amounts to much, unless the plates are very thick; and if the rivets are spaced according to rules given can be overlooked. If it is desired to calculate the strain on each rivet, due to this tendency of the flange plates to slide past each other, it can be done by the following formula, which assumes that at any right angled cross-section of flange through rivets there are always two rivet holes.

$$v = \frac{b}{d} \cdot (x - y)^2 \cdot \left(\frac{k}{f}\right) \qquad (123)$$

Where $v =$ the safe value or stress on any flange rivet, in

pounds, to resist the tendency of any two flange plates (or plate and angle leg) to slide past each other.

Where $b =$ the total breadth of flange plate, in inches.

Where $d =$ the total depth of girder, in inches.

Where $x =$ the distance, in inches, from the horizontal neutral axis of girder to centre of flange plate, further from neutral axis.

Where $y =$ the distance, in inches, from the horizontal neutral axis of girder, to centre of flange plate (or centre of flange leg) immediately next to other plate, but on the neutral axis side of same.

Where $\left(\frac{k}{f}\right) =$ the safe modulus of rupture, in pounds, of the material.

Splicing girder plates.

If any part of a girder, either web or flange, is spliced, made of two parts, the number of rivets *each side of splice*, and the amount of *additional* cover plates, etc., should be made sufficient to transfer the full strength of original plate across the joint.

In locating the rivets of a splice care should be taken not to weaken the original plate by holes not allowed for in the original calculation of moment of resistance of the section. There is no difficulty in splicing webs, as cover plates can be put on each side, and the strains in the web are comparatively small.

Web-splice.

These (web-splice) plates and their rivets *each side of* joint should be of sufficient strength to transfer the amount of the vertical shearing strain at the joint from one side of (spliced) web joint to the other side of joint. In the flange, however, it is more troublesome.

Flange-splice.

In heavy girders, however, (the only ones usually, where it is necessary or where it pays to splice the flange plates), it is best to carry the upper or outside layers of flange plates a longer distance from the centre (or point of greatest bending-moment) than required by calculation, thus gaining extra material in the flange, and more than required there by calculation, and then using this extra material to offset the loss suffered by making the additional rivet holes and by cutting or joining one of the flange plates at the point. For instance, Figure 198 represents the side-view of part of the top flange of a plate girder. $A\ B$ is the first flange plate running entire length of girder, A being towards end and B towards centre.

This plate has to be spliced. We have previously found that we can thin down our flange at the points *F, E, D* and *C.* We will decide to piece plate *A B* between *D* and *E* say at *G.* Of course the flange will thus be weakened at the point *G* by the entire loss of

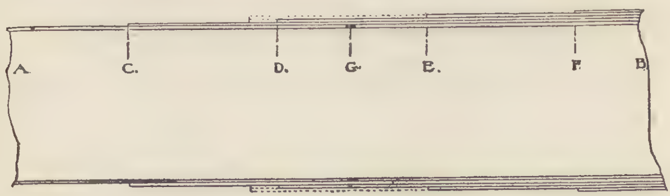


FIG. 198.

plate *A B* and if we attempt to regain this by cover plates it will lose the additional rivet holes. But by prolonging the upper plates as shown by dotted lines this loss can be made good and without any additional rivet holes.

For by the time the plate which originally ended at *E* has been extended to *G* the girder is considerably stronger than needed, that is, stronger by the amount of thickness of this extended plate, and the girder can therefore bear the loss suffered by the cutting of the lower plate. Providing, of course, the plates are of equal thickness. If there are not enough rivets between *G* and *D* to take up the strength of the spliced plate, the plate which ends at *D* will also have to be extended, as shown by dotted lines, till the number of rivets desired have been covered.

In many cases the extending of flange plates is sufficient to form the splice, but frequently an additional cover plate over the extended flange plate may simplify and cheapen the cost. The arrangement in each case will depend upon the number of rivets required, the respective thickness of plates and other local circumstances.

Angles in one piece.

As a rule the angles are made in one piece from end to end, as they can easily be obtained of great length, and are awkward to splice. Angles should be used as heavy as possible, but if very thick they are difficult to straighten, and besides reduce greatly the value of flange rivets, owing to the bending-moment.

In determining the thickness of web it has to have sufficient area

of vertical cross-section at all points to resist the vertical cross-shearing, and must be stiff enough not to buckle under its load, which will be equal to the vertical cross-shearing at each point of web.

As this vertical cross-shearing is always greatest at one or both supports, we should have for thickness of web:

Web-thickness. $$b = \frac{p}{d.\left(\frac{g}{f}\right)} \qquad (124)$$

Where $b =$ the required thickness of web, in inches, (should never be less than $\frac{1}{4}''$ thick).

Where $d =$ the depth of web, in inches, this should be the *net* depth d, that is depth *less* all rivet holes coming on any vertical section at or near reaction.

Where $p =$ the reaction, in pounds, at either end, (Formulæ 14 to 17). The larger reaction should be used, where they are unequal.

If web is not to be of uniform thickness throughout, use, in place of p, the amount of vertical shearing-strain, in pounds, at the point for which thickness of web is being calculated.

Where $\left(\frac{g}{f}\right) =$ the safe cross-shearing stress of the material, in pounds, per square inch.

Danger of buckling.

In many plate girders the web will be so thin in comparison to its depth, that there will be serious danger of the web buckling, particularly towards the ends where the vertical cross-shearing (except in case of single concentrated loads) is always greatest.

To avoid this danger the web is stiffened by riveting upright angle irons, or T-irons to same, between the flanges. The ends of these stiffeners should *always be planed* and bear truly against each flange.

Use of stiffeners.

At the very ends of girders there should always be stiffeners over the reactions, of sufficient strength, as columns, to carry the amount of reaction, less the amount of bearing of web on reaction. As the length of these columns will be only equal to the depth of girder, and the column will generally consist of the bearing amount of web, plus two angles, they, the stiffeners, can safely be considered as short columns and the full safe compression stress, per square inch, allowed on them. The number of rivets connecting any of the stiffeners to web, should equal the amount of cross-shearing being carried by the stiffener,

that is vertical cross-shearing at the stiffener, less the amount borne by web. This latter amount at the ends is the safe load on a column or section of web, equal to its bearing on reaction; between reactions a section of web equal to its depth is taken as assisting or being assisted by the stiffener, that is as acting together with the stiffener. While the web really receives its load from the flanges by pin-connected ends — rivets — it is, nevertheless, assumed by most engineers to have planed ends, presumably to avoid too many stiffeners, the whole calculation, as it is, being but very theoretical anyhow.

We should have, then, amount of strain on end stiffeners,

Strain on end stiffeners.

$$s = p - \frac{12000.\,b.\,x}{1 + \frac{0{,}0003.\,d^2}{b^2}} \qquad (125)$$

and amount of strain on intermediate stiffeners,

Strain on intermediate stiffeners.

$$s = y - \frac{12000.\,b.\,d}{1 + \frac{0{,}0003.\,d^2}{b^2}} \qquad (126)$$

Where $s =$ the total compression strain on stiffeners in pounds; the stiffeners should have sufficient area of cross-section to resist this strain, considered as short columns, and sufficient rivets connecting them to web to $= s$ in value.

Where $p =$ the *greater* reaction, in pounds, where the reactions are unequal; or *either* reaction where they are equal, see Formulæ (14 to 17).

Where $y =$ the amount of vertical cross-shearing, in pounds, at the point of girder, see Formula (11), at which stiffener is applied.

Where $x =$ the distance, in inches, that girder rests on (selected) reaction p.

Where $b =$ the thickness of web, in inches, at end or at point y, as the case may be.

Where $d =$ the depth of web, in inches, at end or point y, as the case may be.

To decide whether the web needs or does not need stiffeners, and if so, at what points, use the following formula.

Where stiffeners are necessary.

$$y = \frac{12000.\,b.\,d}{1 + \frac{0{,}0003.\,d^2}{b^2}} \qquad (127)$$

Where b and d same as in Formulæ (125 and 126). Should y be

larger than the greater reaction p no stiffeners are required, except at the very end.

Should y be *less* than either reaction, stiffeners will be required up to the point of web where the *vertical shearing* — (as found by Formula 11) — *just equals y*.

At this point place stiffeners a distance apart equal to the depth of web.

Stiffeners should always be placed under concentrated loads. At end of web place stiffeners and again just inside of reaction, and between end and point where y equals shearing place stiffeners, not less than the depth of web apart, and gradually diminishing the distance between them towards end; this distance should be regulated by the amount of increase in vertical shearing towards end.

Deflection of plate girders.

In regard to the deflection of plate girders, the same rules apply, as for beams, that is Formulæ (36) to (42), Table VII, and Formulæ (95) to (97). It should be noted, however, that owing to more or less imperfections in riveting, fitting of parts, etc., the plate girders will deflect very much more than if calculated by these rules, with a modulus of elasticity same as for perfectly rolled beams.

To allow for these imperfections in manufacture a lower modulus of elasticity should be used, to be varied according to the care exercised in manufacturing the girder. Experiments on riveted girders have given moduli of elasticity for steel as low as 5000000 pounds-inch.

This, however, is probably an extreme case. The writer would recommend that the following be used, where no experiments can be made:

Decreased modulus of elasticity.

For wrought-iron plate girders $e = 18000000$ pounds-inch.

For mild-steel plate girders $e = 20000000$ pounds-inch.

Where e = the modulus of elasticity, in pounds-inch, to be used in calculating the deflection of plate girders.

Before giving an example, Tables XLI, XLII and XLIII should be explained. They have been calculated to enable architects to lay out the required size of plate girders by their use, and without elaborate calculations. They will be found to be very accurate and valuable for preliminary estimates, quick designing of girders, and checking of final calculations.

Table XLI gives the value of the web in resisting the bending-moment. It should be remarked here, that some engineers do not include the web at all; others include only one-sixth of the web at top and bottom. This is practically reducing the web to the same level as if the top and bottom flanges were merely latticed together. The writer believes, that in house-work at least, it can and should be safely included, particularly as it does not greatly affect the final result anyhow. In box girders the two webs should be considered as one web of thickness equal to the sum of the two; except when calculating for buckling, when, of course, each web is taken separately.

Use of Tables XLI, XLII and XLIII.

Table XLII gives the value of the four angle-irons, for six different sizes of angles, and Table XLIII the value of *each inch of effective* width of flange. In all of the Tables the horizontal column of figures at the top indicates the length of span of girder, in feet; the vertical columns of figures to the left indicate the respective values in tons (of 2000 pounds each); while the figures on the curves indicate the *depth of web* of the plate girder.

The tables are calculated for a safe modulus of rupture $\left(\frac{k}{f}\right)$ or extreme fibre strain of 12000 pounds per square inch, and intended, of course, for wrought-iron.

For different fibre strains.

For those desiring to use a smaller or greater strain it will only be necessary to increase or reduce the actual load (respectively) in the calculation. Thus, if it is desired to use a fibre strain of 15000 pounds, this will be one-quarter more than allowed for in Tables. We therefore use but four-fifths of our load in the calculation and find by the Tables, what sized girder will safely carry four-fifths of our load with an extreme fibre strain of 12000 pounds. When we then add one-fifth of the original load (or a quarter additional to the calculated load) it will, of course, add also one-quarter or 3000 pounds to the extreme fibre strain. Or, we wish to use an extreme fibre strain, of only 10000 pounds. We add one-fifth to our load making it $\frac{6}{5}$ (or one and one-fifth) and find from Tables the size of plate girder to carry this increased load at 12000 pounds fibre strain; when now we deduct one-fifth of the original load (or one-sixth of the calculated load), we will, of course, at the same time diminish our extreme fibre strain one-sixth to 10000 pounds.

The use of the Tables is very simple and easy. For loads other than uniform, and for steel, the data at bottom of Table XLI shows their respective values as compared to those given in Tables.

It should be noted that the "greatest deflection" has been calculated for the most perfect work. For ordinary work this deflection will be increased, according to the quality of the workmanship, to one-half more than for perfect work.

In using the Tables, first settle the size of web, then of the angles, and finally the size of flange plates.

Example I.

Designing girders by Tables.

We will suppose that we have a wrought-iron plate girder of 60 *feet span, which is to carry a uniform load of* $89\frac{3}{8}$ *tons and two loads of* $44\frac{11}{16}$ *tons each, one concentrated near each end, and one quarter span from reactions. We are to use a web* 36″ *deep and flange* 21″ *wide. Design the girder parts by use of Tables.*

From the arrangement of loads W_{III} and W_{IV} (at bottom of Table XLI) we see their sum is equal to a uniform load, or

$$44\tfrac{11}{16} + 44\tfrac{11}{16} = 89\tfrac{3}{8}$$

that is, our two concentrated loads will have the same effect on the girder as a uniform load of $89\frac{3}{8}$ tons; our total load on the girder, therefore, will be equal to $178\frac{3}{4}$ tons uniform load, which we will assume includes the weight of the girder itself. We will decide to use four 6″ x 6″ x $\frac{7}{8}$″ angles, as the loads are very heavy, and $\frac{7}{8}$″ rivets throughout.

We now settle the thickness b of web, from formula given in right-hand corner of Table XLI, namely:

$$b = \frac{U}{8.d} = \frac{178\frac{3}{4}}{8.36} = 0{,}621$$

or, say, web should be $\frac{5}{8}$″ thick.

Value of web.

We now find the value of web in resisting the bending-moment from Table XLI. Pass down the vertical span line marked 60′ 0″ till we strike the curve marked 36″, this is two-thirds way between the horizontal lines marked on the left — (in the $\frac{5}{8}$″ thick vertical column, the second from left) — 7,5 and 10,0 respectively, or our web would safely carry about 9 tons. We next take Table XLII, remembering that we selected the 6″ x 6″ x $\frac{7}{8}$″ angles, the values for which are in the extreme left column.

Value of four angles. We again pass down the 60′ 0″ vertical line till we strike the 36″ curve, which is a little more than half-way between the horizontal lines marked in the extreme left — (6″ x 6″ x $\frac{7}{8}$″) — column 28,2 and 32,9 respectively; or, our four angles together will take care of about 30$\frac{3}{4}$ tons, this added to the web value (9 tons) makes 39$\frac{3}{4}$ tons of the 178$\frac{3}{4}$ tons to be cared for, or a balance of 139 tons to come on the flange.

The flange is to be 21″ wide, from this we must deduct the two $\frac{7}{8}$″ rivet holes,[1] or our effective width of flange would be = 19$\frac{1}{4}$″, therefore each inch must carry

$$\frac{139}{19\frac{1}{4}} = 7,22 \text{ tons.}$$

Value of flange. We now take Table XLIII, pass down the 60′ 0″ vertical line till we strike the curve 36″ and then pass to the left to find the above value 7,22 tons. We strike the curve on the fifth horizontal line from the top and passing to the left find that we cannot find any such value as 7,22 tons, in other words the flange will have to be thicker than two inches. The value of 2″ we find is 4,8 tons, leaving us 7,22 — 4,8 = 2,42 tons to care for in addition to the 2″ thickness; this, we find (still on the fifth horizontal line) is under the thickness marked 1 or our flanges will have to be exactly 3″ thick at the centre of girder.

In regard to the web, should we decide to make a box girder, each web should be at least one-half the calculated thickness or $\frac{5}{16}$″ thick. In practice it would be better to make them a little heavier, for such heavy girders, say about $\frac{3}{8}$″ thick each.

Example II.

Detailing a riveted girder. *A single web riveted plate girder is of* 59′,0″ *span. At four feet from left support, and thence every five feet to five feet from right support it carries a concentrated load of* 19500 *pounds, the third loads from each end being increased (by columns) to* 91000 *pounds. These loads include the allowance for weight of girder. The web must not be more than* 36″ *deep. Detail the girder.*

This girder is one of some twenty-five used by the writer in a large public building in New York City, hence the limitation as to depth of girder.

[1] Many engineers deduct in addition to size of rivet, 1-16″ for punching and 1-16″ for reaming, which in our case would make the rivet holes 1″ instead of $\frac{7}{8}$″.

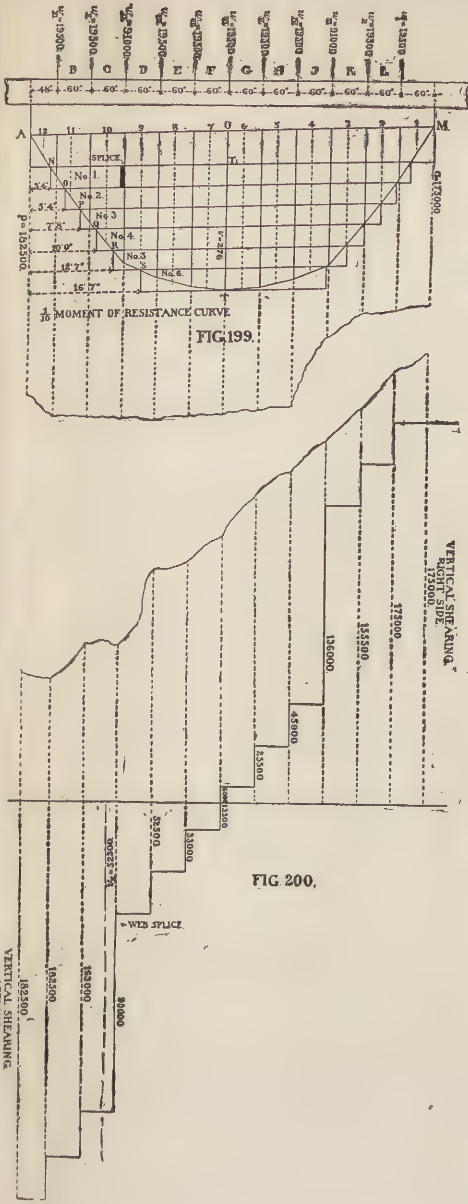
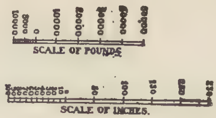


FIG. 199.

FIG. 200.



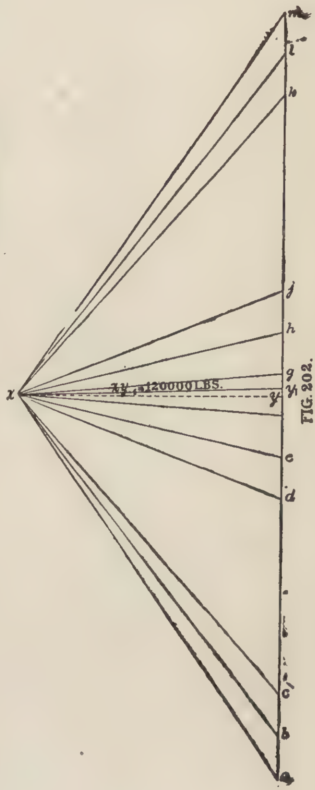


FIG. 202.

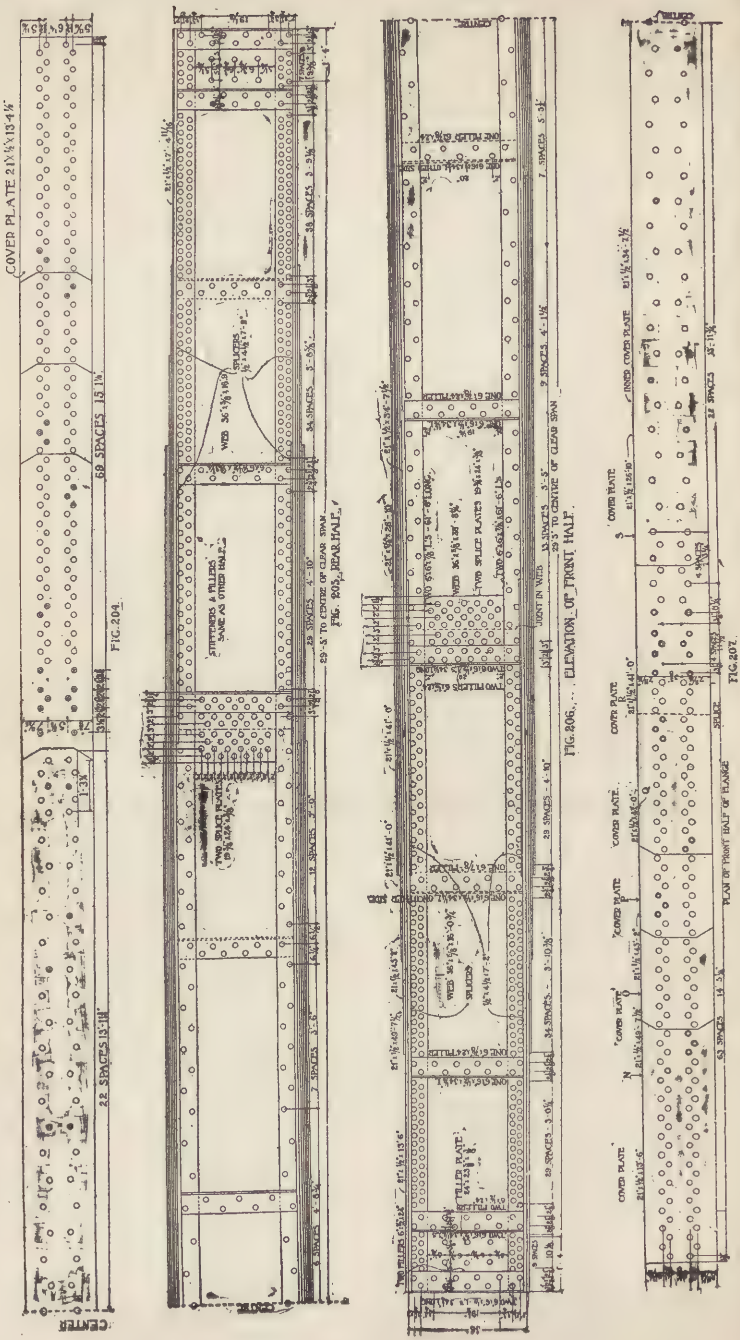


FIG. 204.

FIG. 205, REAR HALF.

FIG. 206. ELEVATION OF FRONT HALF.

FIG. 207.

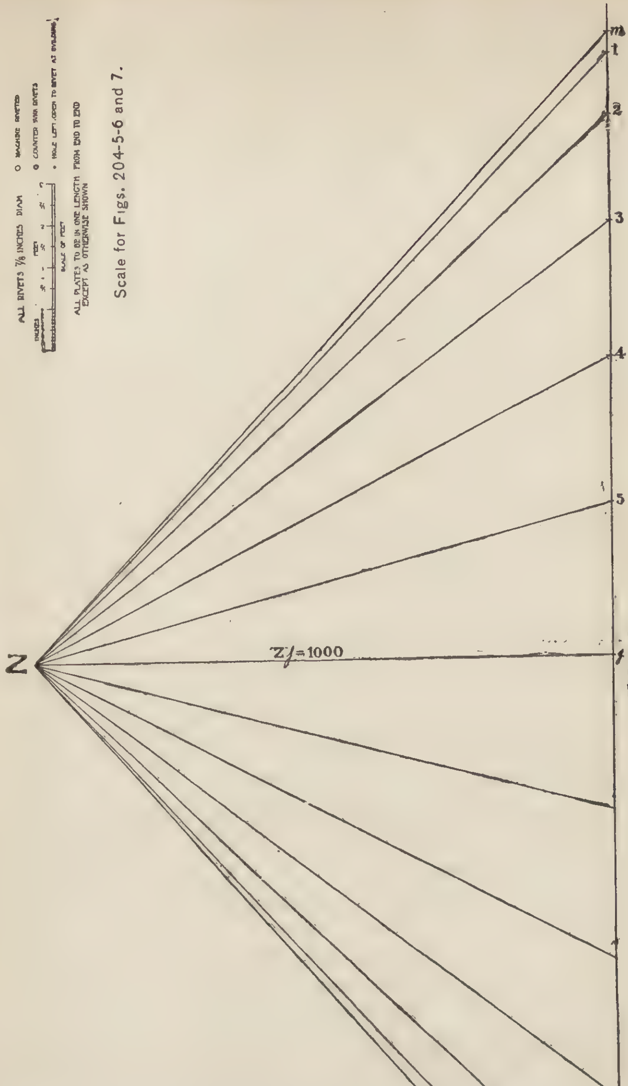


FIG. 203.

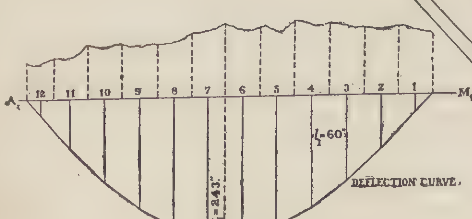


FIG. 201.

The working-drawings, as they were furnished to the contractors by the writer, are given in Figures 204 to 210 inclusive.[1] The only objection (on the score of cost) was to the length of some of the flange plates, but this could not be avoided, as the level of beams resting on the girders could not be disturbed, and there was not room enough between beams to get in the necessary length of cover-plates for splices.

The reader will readily see that this is practically the same example as the former one (*Example I*), so that we need not refer to the Tables for preliminary designing. We know then from the

Size by Tables. Tables that the girder, at the centre, will need to have a $36'' \times \frac{5}{8}''$ web, a $3'' \times 21''$ flange, four $6'' \times 6'' \times \frac{7}{8}''$ angles, and that we will use $\frac{7}{8}''$ rivets.

We will now see whether this is confirmed by calculation. We will first use the graphical method (see Chapter VII) on account of the large number of loads.

In Figure 202 (p. 121) we lay off our vertical load line $m\,a$, where $m\,l = l\,k = j\,h = h\,g$, etc., $= 19500$ pounds, and $k\,j = d\,c =$

Curve of moment of resistance. 91000 pounds at pounds-scale. We would select the pole distance $x\,y = \left(\frac{k}{f}\right) = 12000$ pounds, but that it will make the moment-of-resistance curve too deep for convenience. We will, therefore, decide to make the distance $x\,y = 10.\left(\frac{k}{f}\right) = 120000$ pounds at pounds-scale. We shall, therefore, have to multiply all the verticals through the moment-of-resistance curve in Figure 199 by ten to get their actual values. We draw the moment-of-resistance curve Figure 199 (see Chapter VII) and find its base line $A\ M$. As our loads are not symmetrical on the beam, being four feet distant at one end and five feet at the other, we draw in Figure 202 $x\,y_1$ parallel $A\ M$ of Figure 199, and find our reactions

$$y_1\,m = q = 175000 \text{ pounds and}$$
$$a\,y_1 = p = 182500 \text{ pounds.}$$

The greatest bending-moment will be at load w_{VI} where the greatest vertical v through the moment-of-resistance curve (Figure 199 on p. 121) measures 276 inches (by inch-scale). Not having

Greatest bending-moment. used the pole distance $x\,y = \left(\frac{k}{f}\right)$ in Figure 202 we

[1] It is to be regretted that some of the illustrations are necessarily very small; the reader is advised to use a magnifying glass.

must multiply this by ten to get the actual required moment of resistance which would be $= 2760$. For the same reason we cannot use Formula (98) to calculate the flange thickness and therefore refer to Formula (36) and have for thickness of flange at centre

$$= \frac{\frac{2760}{37} - 16{,}4}{19{,}25} = 3{,}02$$

Or we need, as found by Tables, a flange thickness of three inches at the centre of girder. In the above formula, it should be explained, 2760 was the *required* moment of resistance at the point (that is at load w_{v_I}) for which we were calculating flange thickness; 37 represented the approximate total depth of girder, allowing say for one half-inch plate to each end of girder; 19,25 was the net width of flange, after deducting two rivet holes; and 16,4 was the net area of cross-section (after deducting four rivet holes) of two $6'' \times 6'' \times \frac{7}{8}''$ angles.

Required flange thickness.

We will decide to use six half-inch thick plates in each flange and must next decide where to break them off. Accordingly we use Formula (99), and have value of two angles

Where to diminish flange thickness.

$$= 16{,}4 \,.\, 36 = 590{,}4$$

or of the whole *required* moment of resistance (2760) the angles furnish an amount $= 590$; now, as our moment of resistance curve is only of one-tenth the required depth, we divide this by 10 and make $O\,T_I = 59$ at inch-scale. We now divide $T_I\,T$ into six equal parts and draw the parallel lines to base $A\,M$, their intersections N, O, P, Q, R, and S with the curve are the points at which the respective plates can be broken off.

We shall, however, carry the first plate $N\,O$ the entire length, and as this plate is spliced inside of the curve, we shall have to carry plate No. 5 over the joint to make up for the lost section, and we shall have to carry both plates Nos. 4 and 5 sufficiently far to the left of the splice to get in the necessary number of rivets to equal the value of cut plate.

It will also be necessary to prolong some of the plates to get in the necessary rivets *beyond* their points of contact with curve. Thus between O and N we must get enough rivets to equal in value plate No. 1, or else prolong plate beyond N; between P and O enough rivets to equal plate No. 2, or else prolong plate beyond O; and so on till in the last

Number of flange rivets required.

plate No. 6 we must get enough rivets between T and S to equal plate No. 6. Now these plates are all of equal value $= \frac{1}{2}'' \times 19\frac{1}{4}'' =$ $9\frac{5}{8}$ square inches of cross-section, which multiplied by $\left(\frac{k}{f}\right) = 12000$ pounds gives the real strain s on the rivets, or

$$s = 9\tfrac{5}{8}.12000 = 115500 \text{ pounds.}$$

We will, therefore, lay out rivets enough, in each flange, between S and the end A to take this strain five times, gradually decreasing the pitch towards the end of girder. We will then carry each plate sufficiently far beyond the length required by curve, to get its respective number of rivets.

Spacing flange rivets.

Now the value of rivets ($\frac{7}{8}''$) in flange will be for shearing (single area) $= 4800$ pounds each. For bearing and bending the rivets will evidently get their value from the $\frac{1}{2}''$ plate, this being thinner than the angles, and we have bearing value $= 5250$ pounds per rivet. Either of the above can be found by calculation, or from Tables XXXV and XXXVIII.

Value of each flange rivet.

For bending we have from Table XXXVIII for a $\frac{7}{8}$ rivet, the safe bending-moment $= 990$ or say 1000 pounds-inch.

The actual greatest bending-moment will be, Formula (25)

$$m = \frac{u.\left(\frac{1}{2}\right)}{2} = \frac{u}{4}$$

and as the safe

$$m = 1000 \text{ pounds-inch,}$$

we have

$$1000 = \frac{u}{4} \text{ or}$$

$$u = 4000 \text{ pounds.}$$

The value of the rivets against bending — (4000 pounds each) — being their least value, will control the design. Each cover-plate therefore requires

$$\frac{115500}{4000} = 29 \text{ rivets, and from } S \text{ to end we shall require } 145$$

rivets in each flange. From S to T we require only 29 rivets, but they will have to be spaced more frequently to comply with the rule for greatest pitch, or Formula (107), accordingly the pitch of the latter should not exceed $= 16.\frac{1}{2} = 8$ inches.

Figure 207 shows a plan of the top flange of left half of girder. Plate No. 6 might have stopped at S, but is carried two rivets

further to avoid breaking under a beam, which rested on the girder at this point. The splice of plate No. 1 has been made just to the left of R, where plate No. 5 might have stopped; we must, therefore, carry plates Nos. 5 and 4 at least 29 rivets beyond the splice, which has been done. Plate No. 4 might have stopped two spaces nearer R, but for the splice; Plate No. 3 we stop at the right number of rivets to the left of Q, and plate No. 2 to the left of P. Plate No. 1, which might stop 29 rivets to the left of O, we decide to carry to the end.

Splicing the flange.

The countersunk rivets shown come under beam or column ends. The blank spaces were to bolt column plates to. The blank spaces for beams were marked on later from memoranda in the contractor's shop.

Having detailed our flange, which will be the same both for top and bottom flange, we will now consider the web.

Thickness of web.

The size of web we settle from Formula (124) and have for thickness b, assuming that there will be no more than six rivet-holes in any vertical section, or

$$d = 36 - 6.\tfrac{7}{8} = 30\tfrac{3}{4}'';$$

$$b = \frac{182500}{30\frac{3}{4}.8000} = 0{,}74$$

or nearly $\frac{3}{4}''$. The web, however, was made $\frac{5}{8}''$ as the above was required only at the one extreme end and through its rivet-holes. The effect of decreasing the breadth of web being, of course, to raise the actual shearing per square inch at this point to a little over 9000 pounds per square inch.

Where stiffeners are required.

We next decide where stiffeners are required; we use Formula (127) and have

$$y = \frac{12000.\frac{5}{8}.36.}{1 + \frac{0{,}0003.36^2}{(\frac{5}{8})^2}}$$

$$= 185000 \text{ pounds,}$$

Or we require stiffeners from the end to the point where the vertical shearing is less than 135000 pounds.

By referring to Figure 200, we find this would be about fifteen feet from the end.

The stiffeners, however, were placed more frequently, as shown in Figure 206, both for looks, and as there was some danger of heavier loads being placed on the centre of girders, which would, of course, increase the vertical shearing near centre. These stiffeners were made of $6'' \times 6'' \times \frac{1}{2}''$ angle irons, with $6'' \times \frac{7}{8}'' \times 24''$ filler plates

behind them, so as not to bend the stiffeners. The filler plates being cheaper than would be the cost of blacksmith work involved in bending these angles around the vertical legs of flange angles. Their upper and lower ends were "milled" off and made to bear firmly.

Now as to value of rivets through web, we should have for bearing

Value of web rivets.

$\frac{5}{8}''. \frac{7}{8}''. 12000 = 6562$ pounds; for shearing, being in double shear, twice the value previously found for single shear or, $2.4800 = 9600$ pounds; and for bending we have a $\frac{7}{8}''$ circular beam of $\frac{5}{8}''$ span. The safe bending-moment we previously found to be 1000 pounds-inch, the actual bending-moment is $\frac{u.l}{8}$, therefore

$$\frac{u.\frac{5}{8}}{8} = 1000 \text{ and}$$

$$u = 12800 \text{ pounds,}$$

Or the value against bending would be 12800 pounds. As the bear-

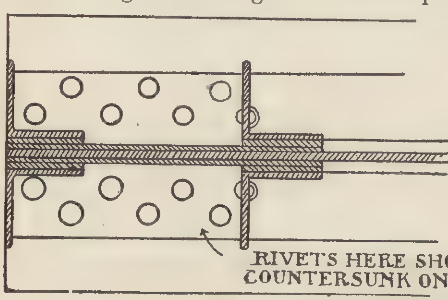


FIG. 208.

ing value (6562 pounds) is the smallest we will use that in determining the number of rivets in web.

For end stiffeners we use Formula (125)

Number of rivets in end stiffeners.

$$s = 182500 - \frac{12000.\frac{5}{8}.16}{1 + \frac{0,0003.36^2}{(\frac{5}{8})^2}}$$

$$= 122500 \text{ pounds.}$$

Or we should need

$$\frac{122500}{6562} = 18,6$$

Or we should need some 19 rivets in the end stiffener, we therefore decide to use a filler plate 24″ x 23$\frac{5}{8}$″ x $\frac{3}{8}$″ each side, which will not

only help stiffen the web, but affords us the room to get in the necessary number of rivets, without cutting more than six rivet-holes on any one vertical line. Figure 208 gives a plan of arrangement of web at end. We next take the stiffener located some 4′ 6″ from the end. The vertical shearing here (see Figure 200) is 163000 pounds.

Number of rivets in central stiffeners.

From Formula (126) we have therefore strain on this stiffener

$$s = 163000 - \frac{12000 \cdot \frac{5}{8} \cdot 36}{1 + \frac{0{,}0003 \cdot 36^2}{(\frac{5}{8})^2}}$$

$$= 28000 \text{ pounds.}$$

Or we should need

$$\frac{28000}{6562} = 4{,}3$$

or say five rivets, we must, however, locate them oftener, see Formula (107).

Splicing the web.

We next decide to splice the web at the point shown in Figure 206, and as shown in plan Figure 210.

The vertical shearing at this point (see Figure 200) is 163000 pounds, we need, therefore, *each side of joint*

$$\frac{163000}{6562} = 24{,}8$$

or say 25 rivets. Including those in the angles, which, of course, help splice the joint, we have 26 each side.

We next settle the size of splice plate by Formula (114). We shall have for its neat breadth

$$b = 24'' - 6 \cdot \tfrac{7}{8} = 18\tfrac{3}{4}''$$

and have for thickness of splice plates

$$h = \frac{163000}{18\frac{3}{4} \cdot 12000} = 0{,}72$$

As there are two plates, one each side of the web, we shall make each one-half the above, or say $\frac{3}{8}''$ thick, remembering, however, to fill out behind the angle with an additional $\frac{1}{2}''$ thick (filler) plate.

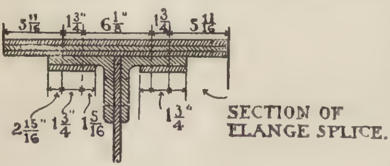


FIG. 209.

Number of rivets connecting web and angles.

We must next settle the number of rivets connecting the angle and the web.

The vertical through moment of resistance curve (Figure 199) at the centre measures 276″ and the axis $x\,y$ in (Figure 202) we made

$$= 120000 \text{ pounds,}$$

therefore, from Formula (93) the bending-moment at centre, or:

$$m_{\text{centre}} = 276.\,120000$$
$$= 33120000 \text{ pounds-inch.}$$

This divided by the depth will give the horizontal flange strain from centre to end of girder, see Formula (121), or

$$s = \frac{33120000}{36} = 920000 \text{ pounds.}$$

This again divided by the least value of web rivets, which we

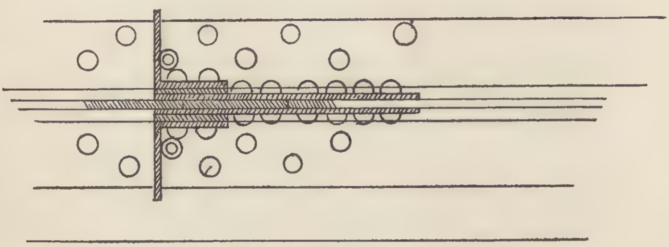


FIG. 210.

previously found to be 6562 pounds, gives the total number of rivets required, or

$$\frac{920000}{6562} = 140$$

In reality we have placed 143 rivets from centre to end, so as not to place the central ones too far apart. Again take a point just under the column (or w_{IX}) say fourteen feet from the left reaction. The vertical (Figure 199) measures 225″, therefore bending-moment at w_{IX}, or

$$m_{w_{\text{IX}}} = 225.120000$$
$$= 27000000 \text{ pounds-inch}$$

and horizontal flange strain

$$s = \frac{27000000}{36} = 750000$$

and required number of rivets

$$\frac{750000}{6562} = 114$$

In reality there are only 113 between this point and end, but that is near enough, as it spaces more evenly so. Again, take a point at the first load to the left, or w_{XI} which is four feet from left reaction. The vertical (Figure 199) measures 75″, therefore bending-moment,

$$m_{w_{XI}} = 75.120000$$

$$= 9000000 \text{ pounds-inch}$$

and horizontal flange strain to end

$$s = \frac{9000000}{36} = 250000$$

Therefore number of rivets required,

$$\frac{250000}{6562} = 39$$

In reality there are 42 rivets.

Length of bearing on reactions.

It will be noticed that in allowing for horizontal flange stress we take all the rivets to the very end of girder, this, of course, is right; although before right through for convenience we have considered the end as at the reaction. The amount the girder will run over the reaction will be determined by the crushing strength of the wall, or pier, or column it is supported by.

In our case we have a bearing 21″ x 16 = 336 square inches, and therefore load per square inch on masonry

$$= \frac{182500}{336} = 543 \text{ pounds}$$

per square inch. This was distributed onto the brickwork by heavy, ribbed, enlarged cast-iron plates.

The only thing remaining to be done now is to figure the deflection.

Deflection found graphically.

In Figure 199 we draw the vertical lines, 1, 2, 3, 5, 6, etc., through the moment of resistance curve, the distance between them (l_1) being practically 60″, the first one being a half distance. In Figure 203 we carry down these lengths in succession on line m 1, 2, 3, etc., to a. We select our pole z arbitrarily at a distance

$$z\,j = 1000''.$$

In Figure 201 we now construct the deflection curve. The longest vertical is in the centre of girder and

$$= v_1 = 243''.$$

The moment of inertia of the section of the girder at the centre will approximate very closely to 58000 (for exact amount see Table I,

section No. 14) and remembering that for built-up plate girders of wrought-iron we must use a modulus of elasticity equal to only 18000000 pounds-inch, we have the central deflection, in inches, of the girder (see Formula 97)

$$\delta = \frac{243.\,60.\,1000.\,120000}{18000000.\,59000} = 1{,}66''$$

The safe deflection, not to crack plastering, would be, (see Formula 28)

$$\delta = 59.\,0{,}03 = 1{,}77''$$

Deflection by Table XLI. Or, our girder is amply stiff. Were we to consider our load as equal to a uniform load of 357500 pounds we could use the approximate formula for deflection given in Table XLI, and should have had

$$\delta = \frac{59^2}{75.42} = 1{,}105''$$

We must add one-half to this for a modulus of elasticity of only 18000000 (the approximate formula being based on 27000000) and would have

$$\delta = 1{,}105 + 0{,}552 = 1{,}657''$$

or the same as by the graphical method.

Deflection found arithmetically. Or, we might have calculated the deflection by Formula (39) again considering the load as a uniform load, and should have had

$$\delta = \frac{5}{384} \cdot \frac{357500.\,708^3}{18000000.\,58000}$$

$$= 1{,}62''$$

Or, practically the same result, and showing how closely the different methods agree. Had we figured the girder arithmetically we should have obtained practically the same results throughout. We should have considered our load as a uniform load of 357500 pounds, which would give us equal reactions, of 178750 pounds each, an error of hardly 2 per cent.

Bending-moment arithmetically. The bending-moment at the centre would be, Formula (21),

$$m = \frac{357500.\,708}{8} = 31638750 \text{ pounds-inch.}$$

The required moment of resistance therefore, would be, Formula (18),

$$r = \frac{31638750}{12000} = 2636{,}5$$

The actual moment of resistance (by section No. 14, Table I) will be found to be at the centre

$r = 2740$ or considerably more than required.

The load, however, is not strictly equal to a uniform load, hence the discrepancy, we should, of course, use the result found graphically, which was based on the actual conditions.

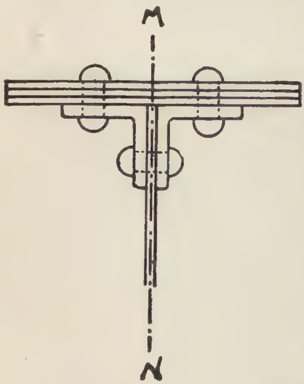


FIG. 211

In figuring the girder arithmetically the required moments of resistance at different points along the girder should be ascertained; after which the curve of moments of resistance can be laid out and the flanges, web, rivets, etc., of girder, calculated the same as already explained. If our girder were not braced sideways we should have to calculate for lateral flexure, using Formula (5). For the area a we should take the area of top flange at centre, plus two angles and the part of web between angles. For the square of the radius of gyration we should take the same parts around

Lateral flexure, top flange.

an axis $M \ldots N$ at right angles to flange, or as shown in Figure 211. We omit rivet-holes in this case, for ease of calculation, and as all parts are in compression.

We have then

$$a = 3.21 + 2.9{,}73 + 6.\tfrac{5}{8}$$
$$= 86{,}21 \text{ square inches.}$$

Now for ϱ^2, not finding the exact section in Table I, we must find the moment of inertia i and divide this by the area.

We have then

$$i = \frac{3.21^3}{12} + \frac{\frac{7}{8}.12\frac{5}{8}^3}{12} + \frac{5\frac{1}{8}.2\frac{3}{8}^3}{12}$$
$$= 2468$$

Therefore $\varrho^2 = \dfrac{2468}{86{,}21} = 28{,}6$

and from Formula (5), l being, of course, the span of girder in inches:

$$w = \frac{3.\,86{,}21\,.12000}{1 + \frac{4.\,708^2.\,0{,}000025}{9.28{,}6}}$$

$$= 2597800 \text{ pounds.}$$

One-third of this would be safe, therefore

$$\frac{w}{3} = 865933 \text{ pounds}$$

would be the safe stress in flanges not to cause lateral flexure, or the safe stress per square inch should not exceed

$$\frac{865933}{86{,}21} = 10094 \text{ pounds.}$$

The actual stress will, of course, equal the area 86,21 multiplied by the average fibre stress, per square inch. To find the average fibre stress, use the following Formula:

Average fibre stress in flanges.

$$v = \frac{2.\,x.\left(\frac{k}{f}\right)}{d} \qquad (128)$$

Where v = the fibre stress, per square inch, in flanges of plate girders.

Where x = the distance of the centre of gravity of part of flange being strained — (flange, angles and part of web between them) — from the centre of depth of web, in inches.

Where $\left(\frac{k}{f}\right)$ = safe modulus of rupture, in pounds, per square inch, or stress on extreme fibres, in pounds per square inch.

In our case this distance x would be found by rule given on p. 7, (Vol. I), and would be (see Figure 211)

$$x = \frac{21.\,3.\,19\tfrac{1}{2} + 12\tfrac{5}{8}.\,\tfrac{7}{8}.\,17\tfrac{9}{16} + 2\tfrac{3}{8}.\,5\tfrac{1}{8}.\,14\tfrac{9}{16}}{86{,}21}$$

$$= 18{,}63$$

Therefore the average fibre stress from Formula (128)

$$v = \frac{2.\,18{,}63\,.12000}{42}$$

$$= 10645 \text{ pounds.}$$

The actual total compressive stress on flange will therefore be

$$= 86{,}21\,.10645$$

$$= 917705 \text{ pounds.}$$

This result should, of course, be the same as our horizontal flange stress, previously found, and by referring back, we see that this was practically the same (920000 pounds.)

Our actual compressive stress we see therefore is about two and one-half times larger than the safe stress to resist lateral flexure as found above (383200 pounds.)

We should therefore, either brace the girder sideways — which was done by the beams in our case — or we should have to broaden the top flange.

Wrinkling of flange. We can readily see that there is no danger of wrinkling in so heavy a flange, but did we wish to calculate it, we would do so by Formula (4) or Table III.

A new deep beam. Since the publication of Table XX, the Homestead Steel Works of Pittsburgh (E) have begun rolling 24″ deep steel beams from 240 to 300 pounds per yard in weight.

The data in regard to these beams is as follows:

	Depth of beam (d)	24″	24″
	Weight per yard	300	240
	Width of flanges (b)	7,20	6,95
	Thickness of web	0,75	0,50
	Area of each flange	6,83	6,55
	Area of web	16,34	10,90
	Total area (a)	30,00	24,00
Neutral axis normal to web.	Moment of inertia (i)	2349,00	2061,00
	Moment of resistance (r)	195,75	171,75
	Sq. of rad. of gyration (ϱ^2)	78,30	85,88
	Transverse value (steel)	1958000	1718000
Neutral axis parallel to web.	Moment of inertia (i)	47,13	41,65
	Moment of resistance (r)	13,10	12,00
	Sq. of rad. of gyration (ϱ^2)	1,57	1,74
	Transverse value (steel)	131000	120000

CHAPTER XI.

GRAPHICAL ANALYSIS OF STRAINS IN TRUSSES.

THE same general rules which apply to beams and girders apply equally well to trusses; but as the latter are made up of a large number of parts, some sustaining the loads directly, others transmitting the consequent strains and thus helping indirectly to sustain the loads, it becomes difficult and often very complex to follow out all the strains arithmetically. For this reason the graphical method is generally used, and for the architect, who has many other things to remember, besides strains and stresses, will always be found to be the most convenient.

There are three steps necessary in designing a truss:

1st. Ascertaining the amounts of loads on each part, and their points of application

2d. Ascertaining the consequent strains on each member of truss.

3d. Designing the members and joints of truss.

In calculating trusses it is always assumed that all the members meeting at any joint are connected by a single pin, and are, therefore, at liberty to move around this pin, until they assume equilibrium towards each other, when of course, they will all counterbalance each other and remain stationary. All loads are, therefore, assumed to act directly on the joints, and are considered as vertical forces at these points (except where wind is allowed for separately). It will frequently happen, however, that the loads are not placed directly over the joints.

Transverse strain on members.

For instance, the load might be uniformly distributed over the entire rafter (or chord): in that case, we should have to assume one-half the load on each panel as coming directly (and vertically) on the joint (that is, each joint would act as a vertical reaction, made up of several parts), and afterwards when designing the truss members, we should have to add sufficient material to the rafter (or chord) to take care of the *transverse strain*, due to the uniform load.

Or, again, we might have a load w_I as shown in Figure 212. The amounts of this load coming on joints Nos. 2 and 3 would be figured exactly the same as reactions (Formulæ 14 and 15).

Joints act as reactions.

In these formulæ p would be load No. 2; q load No. 3; l would be the length of rafter from No. 2 to No. 3; m would be the length of C or from No. 2 to load; and n would be the length of D or from load to No. 3.

All these lengths can be measured either along the rafter, or horizontally between the vertical lines, the result will be the same.

Where loads are suspended from the lower chord, or what amounts to the same thing — rest on same (as ceilings for instance), the load

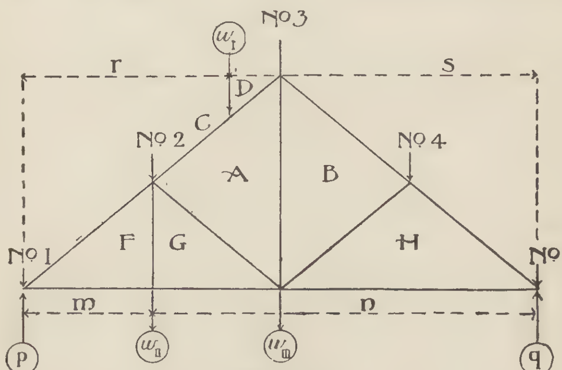


FIG 212

can be considered as hanging from the bottom chord as shown at w_{III} Figure 212.

Loads on lower Chord.

This, however, seriously complicates the strain diagram, it is better therefore to consider (which is also the fact) that the load w_{III} is transferred directly to No. 3 by the tie-rod A B. We will therefore in making our strain diagram add the amount of w_{III} to load No. 3, and *must remember later to add an amount of tension (equal to w_{III}) to rod A B over and above that found by strain diagram.* If we had a load w_{II} placed half-way between p and w_{III} we should have to make the tie-beam or lower chord sufficiently strong to bear this transverse load in addition to the tension existing in it. In this case, the point w_{III} and p would be the reactions for load w_{II}, and the rod A B would

transfer its share up to No. 3 in addition to load w_{III}. But as a rule it is more economical to transfer the load w_{II} up to joint No. 2 directly, by means of the tie-rod. When making the strain diagram, however, this rod should be omitted and the truss shown as at H. Otherwise it will be found that the corresponding points f and g of strain diagram would coincide. This would mean that there was no strain on F G due to the strains in truss; and this is a fact, as the only stress in the rod is in resisting the direct tension due to the load hanging from No. 2 by the rod.

Reactions at Ends.

When figuring the reactions p and q they should be figured by the Formulæ (14) to (17) inclusive. If all the loads are uniformly or symmetrically placed along the truss, each reaction will be just one-half of the total load. If not, then the truss is considered the same as if it were a beam and the reactions figured by the formulæ, the distances m, n, r and s are measured horizontally between verticals as shown; the distance l is, of course, the entire (horizontal) distance from p to q.

Allowance for Wind and Snow.

Wind can safely, as a rule, be assumed to act vertically on the truss and to simply add just so much to the calculated dead load. The writer generally adds for this climate (New York City) 30 pounds per square foot of surface of roof (measured on the slant, not horizontally). This will do for small roofs and approximate calculations of large roofs. This allowance will include the necessary allowance for snow, for, if the roof is steep, the snow will either slide off, or be blown off, and if the roof is flat the wind pressure will be very much smaller and the reduction in wind pressure will fully offset the weight of snow.

General Rule.

Of course taking the wind as a vertical dead load involves two errors: first, the wind is never on the entire roof, as it can manifestly act on one side only; secondly, the wind does not act vertically. Where wind pressure is calculated separately it is assumed to act at right angles to the surface of the roof and on one side only. In large trusses this should always be done, as it will frequently be found that stresses in certain members will be reversed.

Separate Diagram for Wind.

That is, members, which under a dead, vertical load show only tension or compression in the strain diagram may (with wind taken

Reversal of Strains. normal to roof and on one side only) be reversed and show, respectively, compression or tension.

Iron trusses, over eighty feet long, need some arrangement to allow for expansion and contraction. In roof trusses this is provided by anchoring down one end and leaving the other end free to move (horizontally) by placing it on rollers.

It will readily be seen that where there are rollers the effect on the truss will be very different, according to which side the wind is blowing from. In such trusses, therefore, it will be necessary to make three strain diagrams, one for vertical dead load (including snow but no wind); one for wind only on right side; one for wind only on left side.

The truss must then be designed to withstand the strains due to the dead load only; and enough added, where necessary, to withstand the additional or *different* strains due to either pressure. Where both strains are of the same nature they should be added together; where they are of opposite natures they will, of course, offset each other, but the member should be strong enough or stiff enough to withstand either *separately*.

Wind Pressure Depends on Inclination.

As the wind acts horizontally, it will on striking a roof of course cause a different pressure at right angles to the inclination of roof, than is its pressure against a vertical surface. This pressure will therefore vary with the inclination of the roof. To determine it, it is assumed that the greatest wind pressure per square foot of roof surface will never exceed forty pounds. For steep roofs with an inclination of 60° to 90° with the horizon, this is the pressure assumed.

For roofs forming smaller angles with the horizon a complicated trigonometrical formula is used. Its results are as follows:

TABLE XLIV.

TABLE OF WIND PRESSURES ON ROOFS.

Angle of Inclination of Rafters with Horizon.	Pressure or Load, in pounds, per square foot of Roof Surface.
10°	$9\frac{2}{3}$
15°	14
20°	$18\frac{1}{3}$
25°	$22\frac{1}{2}$
30°	$26\frac{1}{2}$
35°	30
40°	$33\frac{1}{3}$
45°	36
50°	38
55°	$39\frac{2}{3}$
60° to 90°	40

Approximate Rule for Wind. It will be noticed that approximately the pressures in pounds are about eighty per cent or four-fifths of the number of degrees of the angle of inclination.

Allowance for Snow. Where the wind is taken separately the allowance for snow in the strain diagram for dead loads should be fifteen pounds per square foot of roof surface.

Having once ascertained the amount of load on each joint, the strains on the different members of the truss are found by the general methods given at the end of Chapter I. (Pages 69 to 74, Vol. I.)

Particular attention is again called to the method of notation, and to the necessity of reading off the pieces in their proper order, and of reading around each joint in the same direction. The writer always uses the direction in which the hands of a watch would travel around each joint.

In calculating the strains each joint can be analyzed by a separate strain diagram, or all of the strain diagrams can be combined into one. As the latter method is much more convenient and less liable to error, it is the one always adopted.

Drawing of strain diagram. In laying out the strain diagram we begin with the joint with the least number of members and this usually is at one of the reactions, where we have only one strut and one tie. Having found these strains we pass to one of the joints at

their other ends, and so on. The reason for doing this is that it will be found impossible to draw the lines in the strain diagram representing any joint where there are more than two unknown strains. By beginning, therefore, with a joint of two members only, the strains on these can be found. We then can pass to joints of three members, containing at least one of the former strains and so on. Figures 213 and following ones give a large number of roof designs with their corresponding strain diagrams. We will analyze one or two of these and the student can puzzle out the rest.

Only a few will offer any particular difficulty, and these will be taken up and explained later.

Dotted lines in tension.

In all the figures dotted lines mean that the dotted member is in tension and full lines that the member is in compression. The numbers in Figures 213 to 228 give the amount of strain, in pounds, on each member due to one pound of load at each joint for roofs with inclination angles as shown in figures.

All that is necessary therefore, where roofs are designed similar to any of these figures, and with same inclinations and angles, to

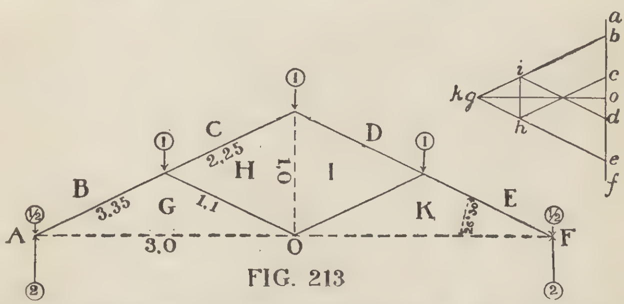


FIG. 213

ascertain the amount of load on the joints and then multiply the number or given strain on each member by the amount of load on each joint. This will give the actual amount of strain on each member.

For instance, we will say we have designed a roof truss similar to Figure 213 with the principal rafter at an inclination of 26° 30′; we will say the trusses are 10 feet apart and 48 feet span, and weight of roof including snow and wind 50 pounds per square foot

measured on the slant. By scaling the rafters we find they measure 27 feet each in length, therefore load on each joint

$= \frac{27}{2}.10.50 = 6750$ pounds.

We now refer to Figure 213 and have the strains, as follows:

Compression on rafter $B\,G = 3{,}35.6750 = +22612$ pounds.
Compression on rafter $C\,H = 2{,}25.6750 = +15187$ pounds.
Compression on strut $G\,H = 1{,}1.6750 = +7425$ pounds.
Tension on tie $G\,O = 3{,}0.6750 = -20250$ pounds.
Tension on tie $H\,I = 1{,}0.6750 = -6750$ pounds.

Had we drawn the strain diagram and made

$a\,b = e\,f = \frac{6750}{2} = 3375$ pounds at any scale, and at same scale made $b\,c = c\,d = d\,e = 6750$ pounds, we should find that at the same scale the respective lines would measure:

$b\,g = 22700$ pounds.
$c\,h = 15200$ pounds.
$g\,h = 7400$ pounds.
$g\,o = 20300$ pounds.
$h\,i = 6750$ pounds.

and to ascertain whether these strains were compression or tension we should follow the direction of each line at each joint and see whether it thrusts against or pulls away from the joint.

To draw the strain diagram we first select a convenient scale, by which we will measure all the loads and strains. We now draw our load line, making in (Figure 213) $a\,b =$ the load on $A\,B$ the foot of main rafter, we then make $b\,c =$ the load on joint $B\,C$, the next one on main rafter, $c\,d =$ the load on apex and so on; as we know the reactions will each be just one-half the load, we locate o half way between f and a, in other words we make $f\,o =$ reaction $F\,O$ and $o\,a =$ reaction $O\,A$.

To get the strains we begin at the joint $A\,B\,G\,O\,A$ at foot of main rafter, for here there are only two unknown strains, namely, the compression on $B\,G$ and the tension on $G\,O$.

Where to begin.

In strain diagram draw $b\,g$ parallel $B\,G$; and $g\,o$ parallel $G\,O$ till they intersect at g; then will $b\,g$ be the amount of thrust on the joint or the compression in $B\,G$, and $g\,o$ will be the amount of pull on the joint or tension in $G\,O$. To make sure of these we read off the lines following the proper succession, namely, $A\,B$, $B\,G$, $G\,O$, $O\,A$; referring now to strain diagram we read $a\,b$, this is down or a

vertical load; next *b g*, this is down or towards the joint, therefore compression; next *g o*, this is to the right or pulling away from the joint, therefore tension; and finally *o a* (which brings us back to the point of starting *a*) and being upward is, of course, the direction of the reaction. As our strain diagram is a closed figure *a b g o a*, we know the joint is in equilibrium. Had we failed to get back to the

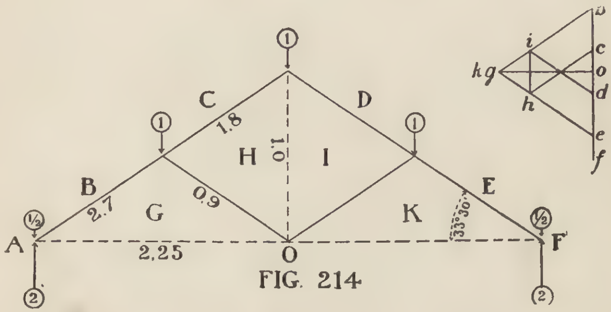


FIG. 214

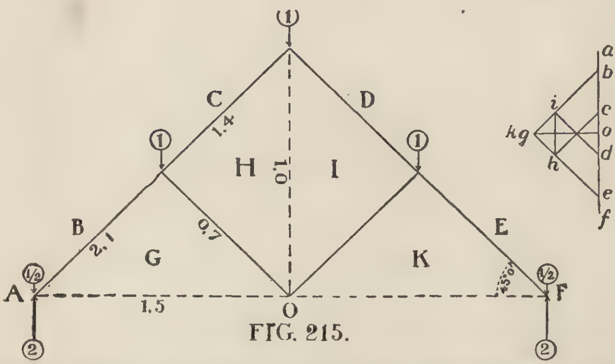


FIG. 215.

point of starting *a*, we should have known there was some missing member to the truss at this joint.

Superfluous members.

On the other hand, if we had had two letters coming onto the same point — (which would be the case with letters *F* and *G* were we to draw strain diagram for Figure 212) — we should know that the member or line between these two letters was superfluous so far as aiding the general truss is concerned. We now (in Figure 213) pass to the next joint with

only two unknown strains. This is evidently the joint at half the height of rafter, we have just found the amount of compression on GB — (note that we now read GB and not BG as before, for around this new joint the hands of a watch would travel in the direction GB) — and the only unknown strains are the compressions on CH and on HG. In strain diagram we draw ch

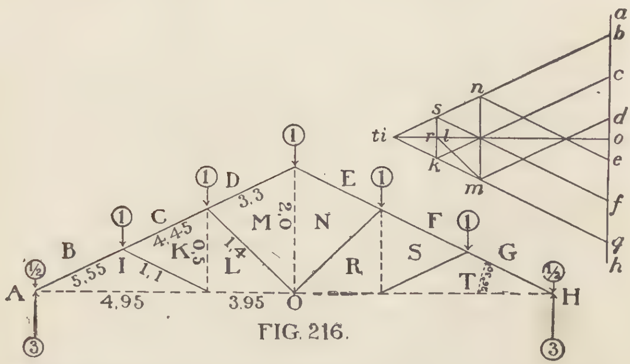


FIG. 216.

parallel CH and hg parallel HG till they intersect; ch will then be the amount of compression on CH and hg the amount of compression on HG. We now read off the pieces in succession BC,

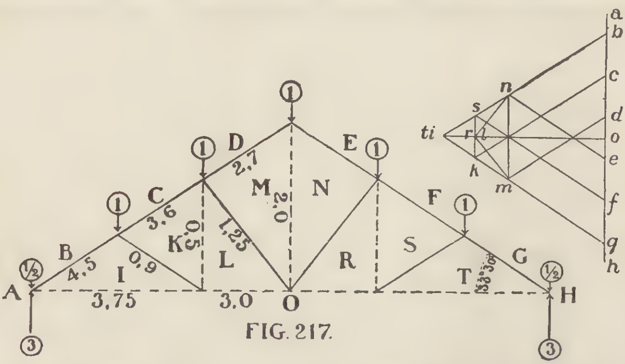


FIG. 217.

CH, HG and GB; and in strain diagram; bc, downwards, therefore a vertical load; ch downwards towards joint, therefore

compression; $h\,g$ to the left, towards joint, therefore compression; and $g\,b$ upwards towards the joint, therefore compression; as our figure is a closed one, we having arrived back at the point of start b, the joint is in equilibrium. We next examine similarly the joint at apex and at centre of horizontal tie. The joints to the right will be

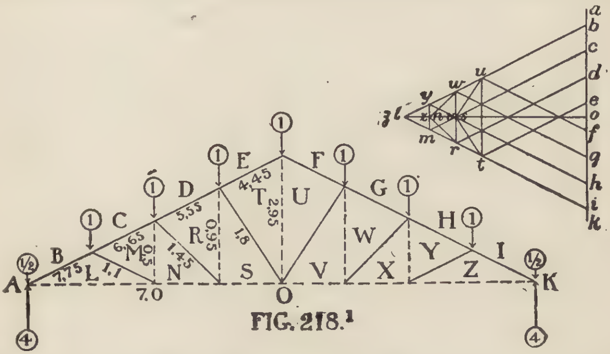


FIG. 218.[1]

similar to the corresponding left-hand joints, as the truss is uniformly loaded. Figures 214 and 215 are similar to 213, the only difference being in the increased pitch of the rafter.

When we analyze Figure 216 we first take the joint $A\,B$ at foot

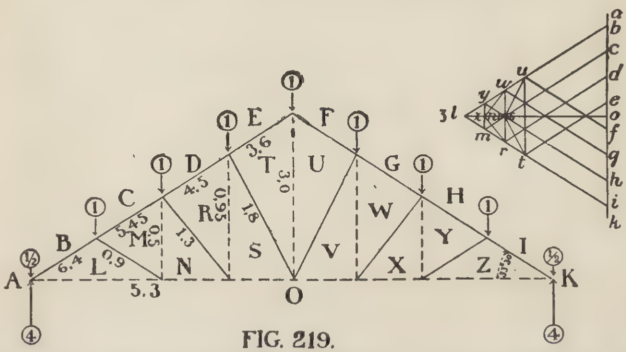


FIG. 219.

of rafter; next the joint $B\,C$ immediately above; next the first joint along tie-rod; next the second joint along rafter and so on. Figures 217, 218[1] and 219 are similar. Figures 220, 221 and 222 will

[1] Angle of rafter (or angle Z) in Figure 218 is 26° 30′.

present no difficulty; nor will Figures 226, 227 and 228, but in the latter three we find that we cannot follow the above rule of passing from a joint along rafter to one on tie-rod; for we begin at joint $A B$

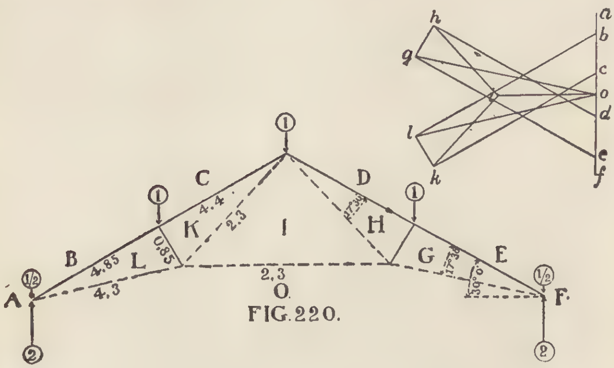


FIG. 220.

(Figure 226) and find the strains $B S$ and $S O$; we next pass to joint $B C$ and find the two unknown strains $C R$ and $R S$; were we now to pass to the joint at the tie-rod, we should find it impossible to continue, for there would be three unknown strains, namely, $R N$,

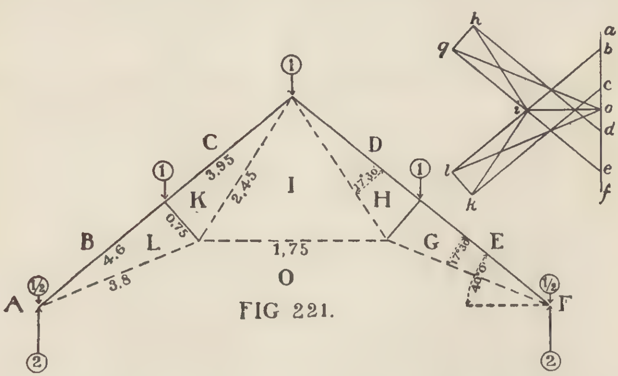


FIG 221.

$N M$ and $M O$; but if we pass first to joint $C D$ we find the two unknown strains $D N$ and $N R$, and then going back to the joint at tie-rod, there remain only two unknown strains $N M$ and $M O$ which we can now find.

Figure 223, we will find, presents a difficulty at the joint $C\ D$ and this truss, in effect, cannot be analyzed by the same method as the others.

Analysis of Figure 223.

We begin at joint $A\ B$ and find the unknown strains $B\ L$ and $L\ O$; we pass to the joint $B\ C$ and find the strains $C\ M$ and $M\ L$; we now pass to first joint along tie-rod and find the strains $M\ N$ and $N\ O$; we now pass to joint $C\ D$ but find three unknown strains $D\ S$, $S\ R$ and $R\ N$; we try the second joint along tie-rod, but this, too, has three unknown strains $N\ R$, $R\ U$ and $U\ O$; so have all the other joints, we are, therefore, completely stopped. We reason, however, that

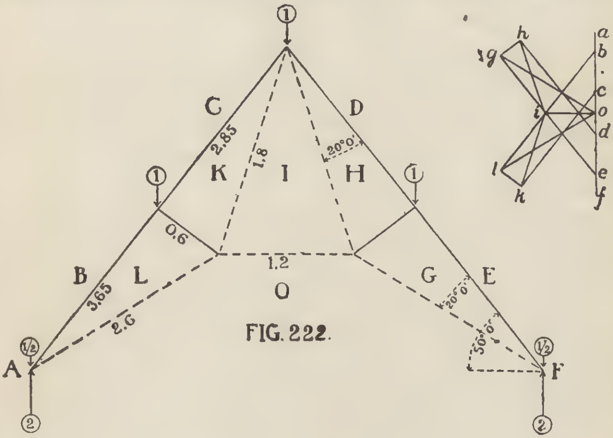


FIG. 222.

the duties of $L\ M$ and $M\ N$, are apparently the same as those of $T\ S$ and $S\ R$, namely, to truss transverely the long (half) lengths of rafter. As the loads are the same on all joints we will assume that

$$S\ R = M\ N \text{ and } T\ S = L\ M.$$

We can now continue our work, for at joint $C\ D$ there only remain two unknown strains $D\ S$ and $R\ N$. Passing to the second joint along tie we can continue, for, $N\ R$ being now known, there only remain the two unknown strains $R\ U$ and $U\ O$.

This method of reasoning is correct, where loads are uniform and the rafter divided into four equal panels; were this not the case, we should have to find the strain on $R\ N$ first, and by one of the methods shown in Figures 223*a* and 223*b*.

In the former we assume (temporarily) that the intermediate panels or joints do not exist.

We take care to reproportion our loads on the joints, as properly

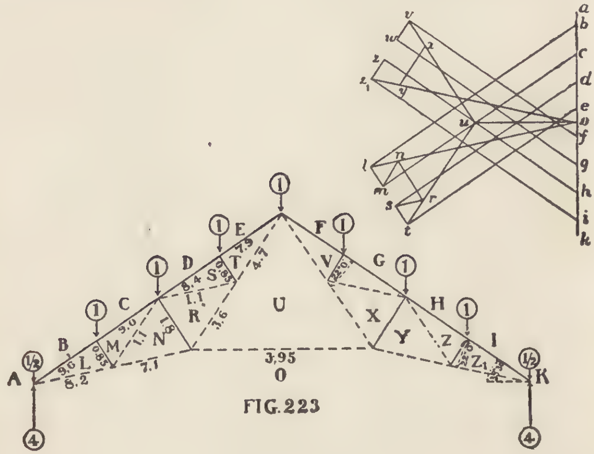


FIG. 223

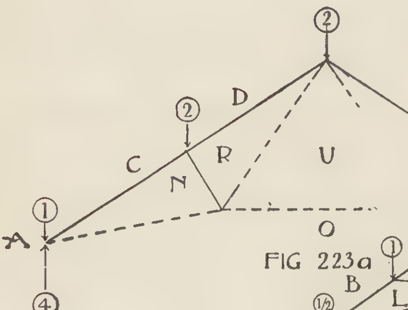


FIG 223a

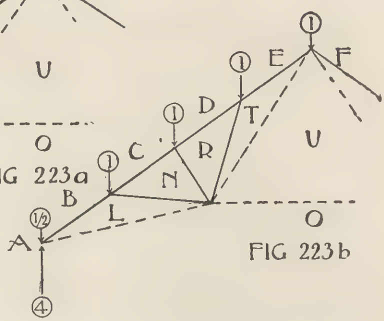


FIG 223b

there are now only two panels to main rafter, where before there were four.

We now find the strain on $R\ N$ without any difficulty by drawing a separate strain diagram similar to the one shown in Figure 220 and

having found the strain on $R\ N$ we proceed with our original strain diagram annexed to Figure 223.

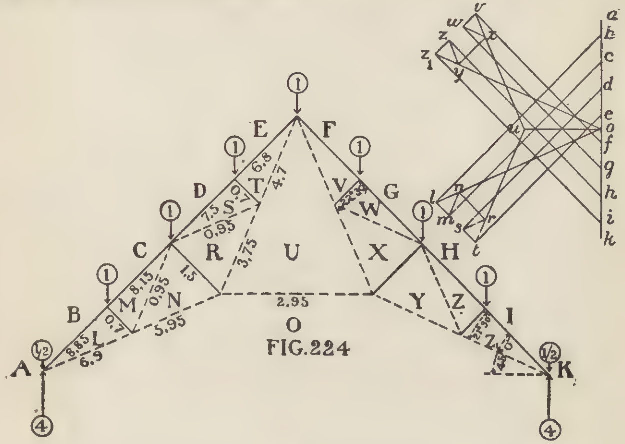


FIG. 224

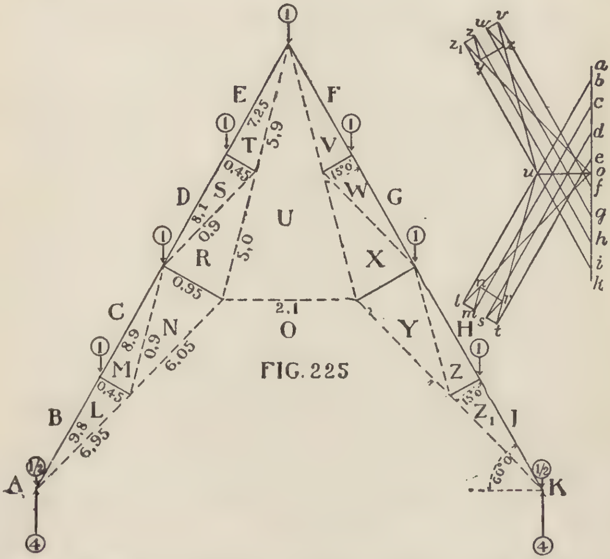


FIG. 225

Or, we will assume (temporarily) that Figure 223 is altered to

have three struts as shown in Figure 223*b*, taking care to keep *R N* the same, and the divisions of rafter the same, the loads on joints will therefore not be changed. We now find *R N* without difficulty by drawing a separate strain diagram on the principle of that shown

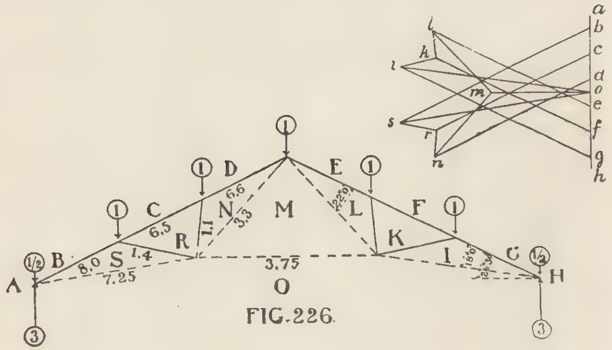


FIG. 226.

in Figure 226; and having found *R N* proceed with our original strain diagram without difficulty.

Figures 224 and 225 are similar to Figure 223, excepting the inclination of main rafters, and angles between rafters and tie-rods.

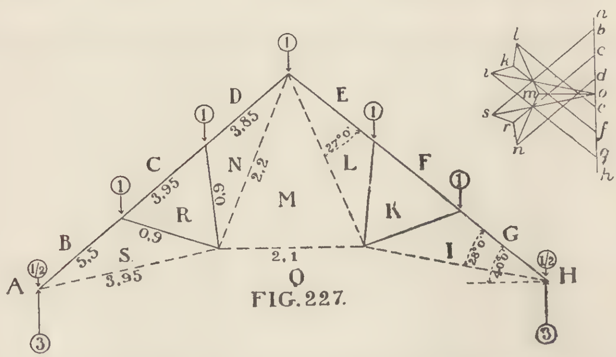


FIG. 227.

Figures 213 to 219 are best adapted to wood, or wood and iron construction; while Figures 220 to 228 are best adapted to iron construction, they being on the principle of a pair of inclined trussed

beams tied together at one point and there taking up the horizontal thrust due to the inclination. Sometimes, if more convenient for

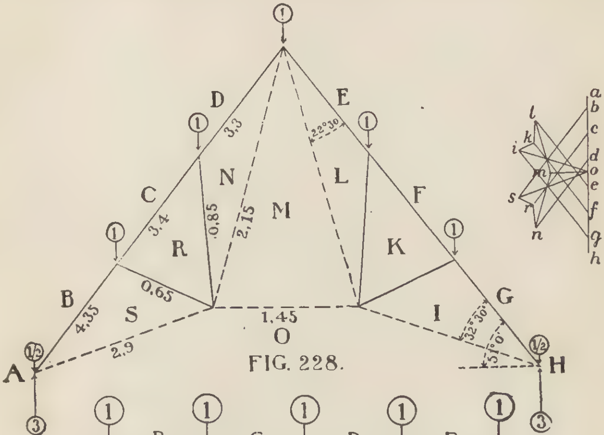


FIG. 228.

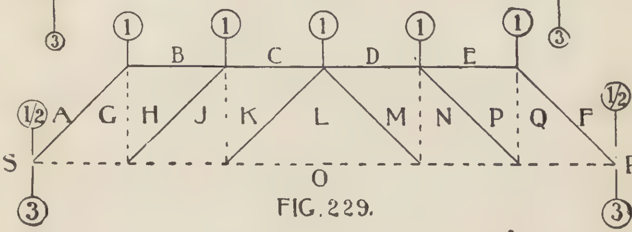


FIG. 229.

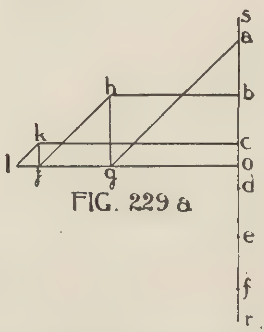


FIG. 229 a

securing roof beams, etc., the principals are made of wood, the balance of truss usually being of iron.

There are, of course, many different truss designs, based on those

given in Figures 213 to 228 and they can be similarly analyzed.

Howe Truss. Figure 229 shows a "Howe" truss with six bays, it will present no difficulty in analyzing.

The central vertical member is not needed unless the weights are placed along the bottom chord, in which case it will be needed to

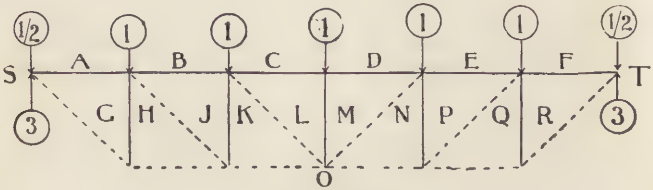


FIG 230

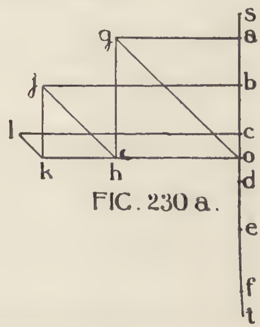


FIG. 230 a.

transfer the load to the top, and all the other verticals will be increased over the strains shown in diagrams by the amount of their respective loads on bottom chord.

This truss is frequently drawn and used upside down from that shown in Figure 229, in which case all the strains will be reversed (see Figure 230), those in compression in Figure 229 becoming tension in Figure 230; and those in tension in Figure 229 becoming compression in Figure 230. It will also be found that the central vertical member is needed, if the loads are placed along the top chord. If the loads (in the reversed truss, Figure 231) are placed along the bottom chord the central vertical will not be needed, and as the loads will be taken up to the top chord by means of the slanting ties, it is better in this case to make a strain diagram with the load

arrows in their right place and as shown in Figure 231. The end loads of this and subsequent trusses have been omitted, as they do

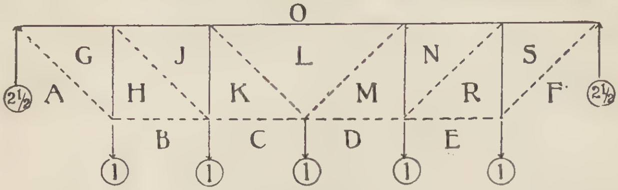


FIG. 231

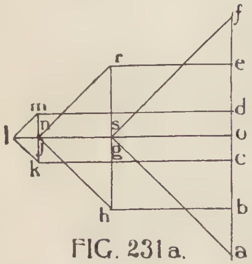


FIG. 231 a.

not affect the strain diagrams, and unnecessarily increase the number of letters used. It will be noticed that the entire strain diagram and each part, is the reverse of what it was in Figure 229. There is, however, no difficulty in analyzing this truss.

In Figures 232 and 233 we have two examples of the "Warren" truss.[1] Here, too, it will be seen that the reversing of the truss reverses all the strains. If the loads were along the bottom chords we should have to draw the

Warren Truss. arrows in their proper places. Figure 235 gives an example. In Figure 234 we have an example of a "lattice" truss. This cannot be analyzed unless we divide it into two reversed

Lattice Truss. Warren trusses as shown in Figures 232 and 233. We analyze each of these separately, and then imagine them laid over each other, adding together the separate strains, where they cover.

If there were vertical members in Figure 234, we should analyze it by dividing it into two reversed Howe trusses, like those in Figures 229 and 230. In this case our widths of panels would be the same as in the original truss, and we should use all the verticals in both trusses, the loads therefore to be assumed on each of the joints of the divided or part trusses should be *only one-half of the original loads.* The "Whipple" truss is on the Howe truss principle

Whipple Truss. with verticals bisecting the diagonals. This truss can be analyzed same as the Warren latticed

[1] The true "Warren" truss usually has the diagonals drawn at 60° inclination.

truss, by dividing it into two halves, each having every other vertical and every other diagonal, and consequently a full load on each joint. Arched trusses are usually built up of a series of panels formed on the Howe or Warren or latticed truss principles, the only difference being that their top and bottom chords instead of being horizontal,

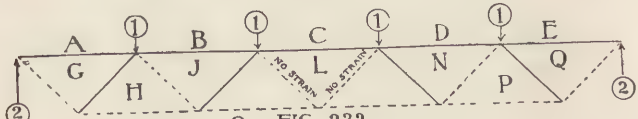


FIG. 232.

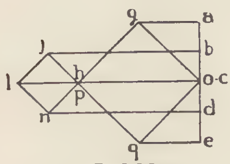


FIG. 232 a.

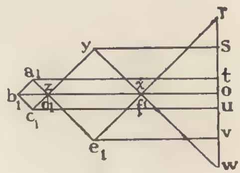


FIG. 233 a

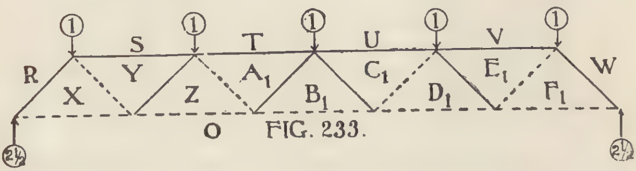


FIG. 233.

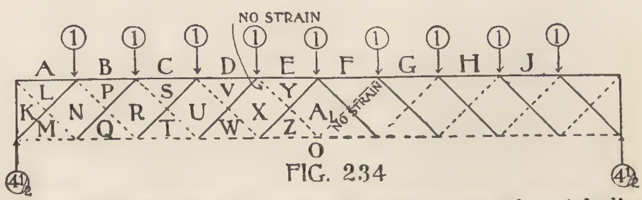


FIG. 234

are made up (or assumed to be made up) of a series of straight lines at different inclinations. They are analyzed without difficulty, where they do not have horizontal tie-rods or abutments to take up their horizontal thrusts.

Arched Trusses.

Figure 236 is an example of an arched truss built on the Warren principle. It will be noticed that the struts $B\ F$ and $F\ O$ are shown as if coming to a point. If this were done in practice the truss in all probability would not have

sufficient bearing. We should, therefore, enlarge the foot of truss, by means of heavy plates and angles, but our calculation will have to

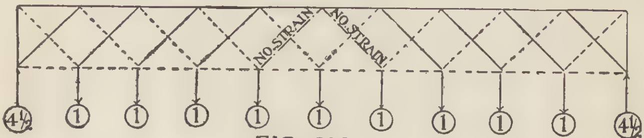


FIG. 235

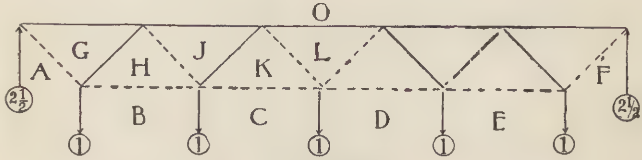


FIG. 235 a

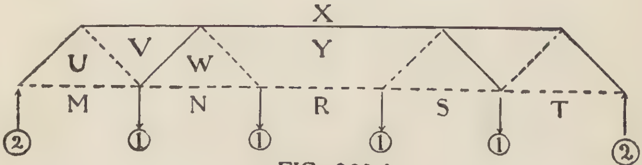


FIG. 235 b.

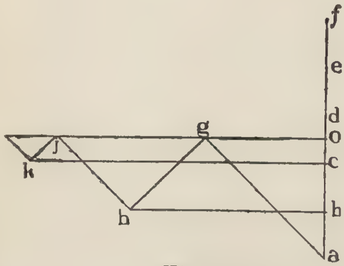


FIG. 235 a₁.

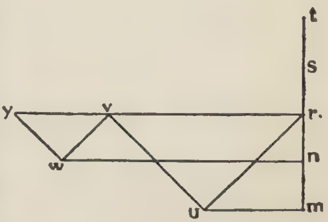


FIG. 235 b₁.

be made as drawn, or we will find the analysis of the truss impossible

Figure 237 shows a latticed arched truss, built upon the Howe principle. This we separate into two reversed Howe trusses, Figures

238 and 239 and find no difficulty in analyzing the strains. We must remember, however, to use only half loads at each joint.

In separating the truss we note that in Figure 239, we have the additional tension member *J I* (see Figure 229); and in Figure 238 the members *K J* and *I H* are in tension instead of in compression as they were in Figure 230. These changes are due to the fact that the upper and lower chords are broken instead of straight lines.

Had the truss been a Warren latticed truss, we should in separating find that we had only half the number of joints and hence would use the full load on each.

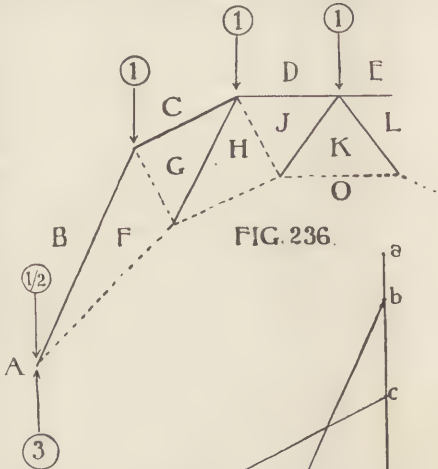


FIG. 236.

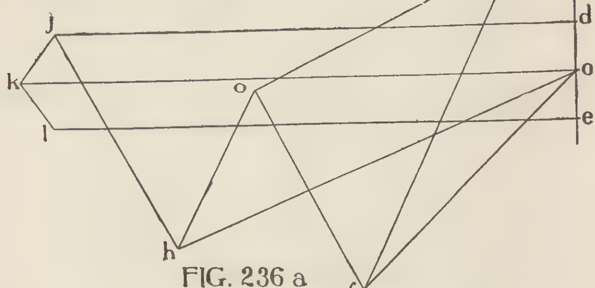


FIG. 236 a

Where there is a horizontal tie or abutment to take up the horizontal thrust of an arched truss, we must find the amount of this thrust by the rules given at the end of Chapter I and at the beginning of Chapter V (both in Vol. I). Having found this we draw an arrow at the foot of the truss in the proper direction to represent this tie (if a rod) or thrust (if an abutment) and then proceed to analyze the truss without difficulty.

In Figure 240 we have what is known as a " scissors " truss. It can be built in either wood, or iron, or a combination of both. For small spans it is a cheap truss where a vaulted ceiling is needed. If we attempt to analyze this truss in one strain diagram, as is done in Figure 243, we first take the lower joint *A B* and find the two unknown strains *B G* and *G O*.

Scissors Truss.

At the next joint *B C* there remain three unknown strains, we therefore abandon it for the present and pass to the joint at the

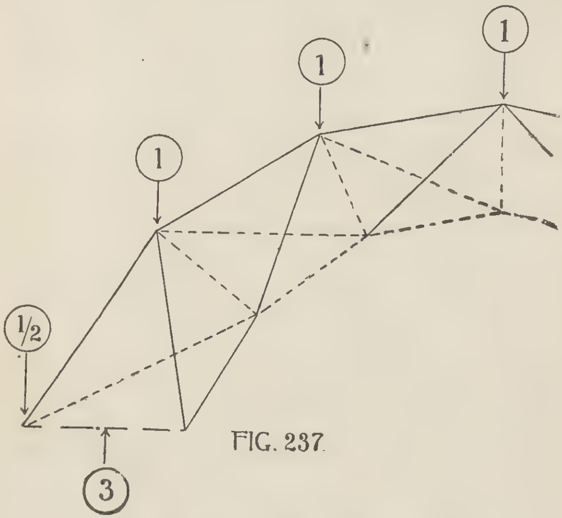


FIG. 237

apex, here there are but two unknown strains *D J* and *J C* and we find them by drawing in our strain diagram Figure 243 the lines *d j* and *j c* parallel to them.

Having now found the point *j* in the strain diagram there is no difficulty with the rest.

This truss is really a combination of two trusses; we might build it by dividing it horizontally along the line *J H* as shown in Figure 240. We should then have a bottom truss with two inclined struts *B G* and *E K*, and one horizontal strut *J H* and two ties *G O* and *K O*, the loads on the joints *B J* and *J E* will each be increased by half of the apex load. Over this truss we should have another truss, with two inclined struts *J C* and *D J* and a horizontal tie

JH. This truss transfers its load to the lower truss. Now the remarkable thing that we have discovered is that the member *J H* is at one and the same time both in compression and tension. If we analyze the top truss separately (Figure 241) and also the bottom truss (Figure 242) we find that the compression greatly exceeds the

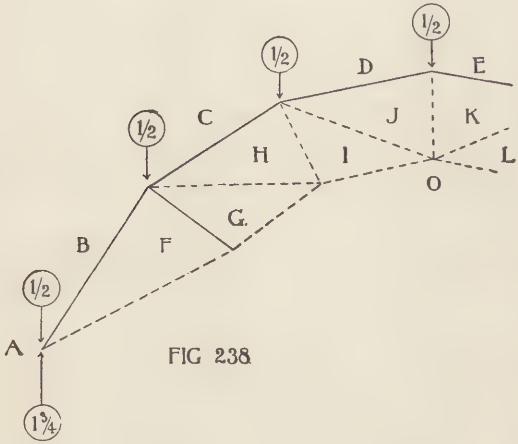


FIG 238

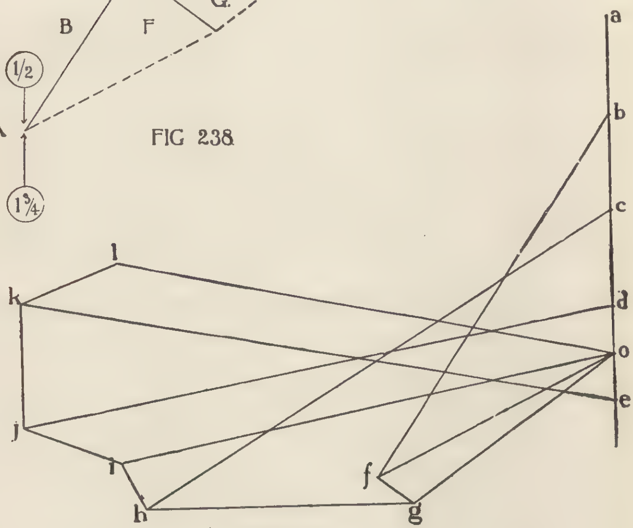


FIG. 238 a.

tension; the member, however, should be designed to resist both, having straps at the ends, sufficient to take up the tension. The actual stress in the member itself will, of course, be the difference between the two, and by referring to our single strain diagram (Figure 243) we see this clearly, for *h j* is the difference between *h o* the total

compression and $j\ o$ the total tension. Usually there is a bolt at the joint $H\ O$ connecting the two ties. This is done, as, when the wind

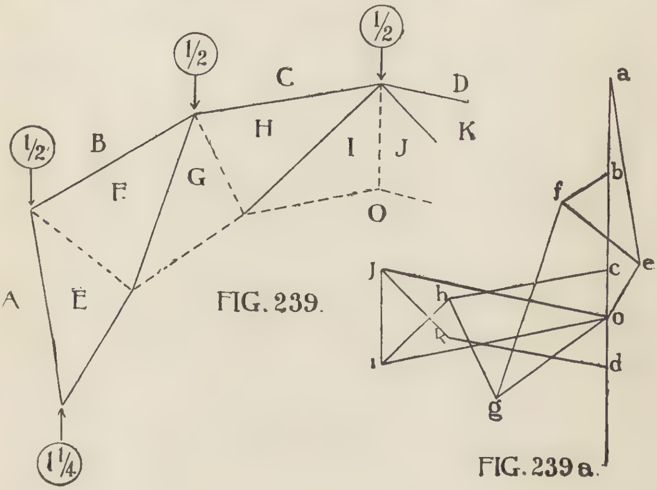


FIG. 239.

FIG. 239 a.

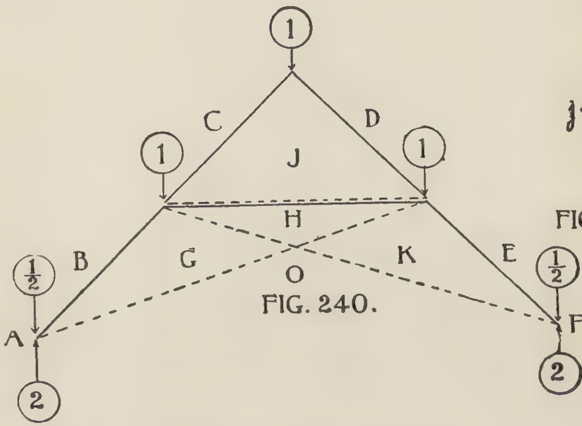


FIG. 240.

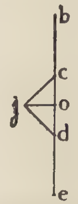


FIG. 241

blows from one side only, the tie pointing to that side becomes a strut and is stiffened by being reinforced at this joint.

Hammer-beam Truss.

Figure 244 represents what is known in Gothic architecture as a "hammer-beam" truss. We can analyze this truss in two ways, first (Figure 244) assuming that the wall takes up the thrust of the truss, due to the absence of any horizontal tie; or second (Figure 246) we can consider the truss as composed of two inclined trussed rafters united at their upper joints and so taking up their own thrust.

The latter method will require a very much heavier truss.

In the latter case the semicircular member in the central panel becomes a tie, but the two lower quarter-circle members do not act at all, while in the first assumption the reverse is the case, the two lower quarter circle members acting as struts, while the semicircular member has no duty. In either case, however, these members come

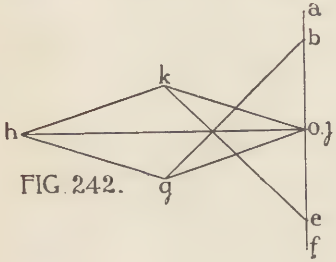


FIG. 242.

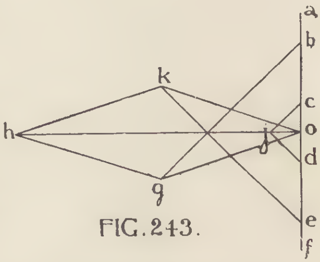


FIG. 243.

into play under wind-pressure calculated from one side only; it should be remarked here that when drawing the strain diagram all curved members must be assumed to be straight — (as shown at *L O* in Figure 244 and at *Y O* and *T O* in Figure 246). Of course, the stress to resist the strain thus found will be greatly increased when the curved member is called upon to do the same duty as a straight member. The increase in stress is equal to that produced on a beam of the length of the curved member with a *bending-moment* at its centre equal to the original strain multiplied by the (longest) versed sine of the circle, or:

Increased stress in curved members.

$$m = s.x \qquad (129)$$

Where m = the (cross) bending-moment, in pounds-inch, existing in a curved strut or tie at its centre due to longitudinal strain.

Where $s =$ the calculated longitudinal strain that would come on the strut or tie if it were straight, in pounds, per square inch.

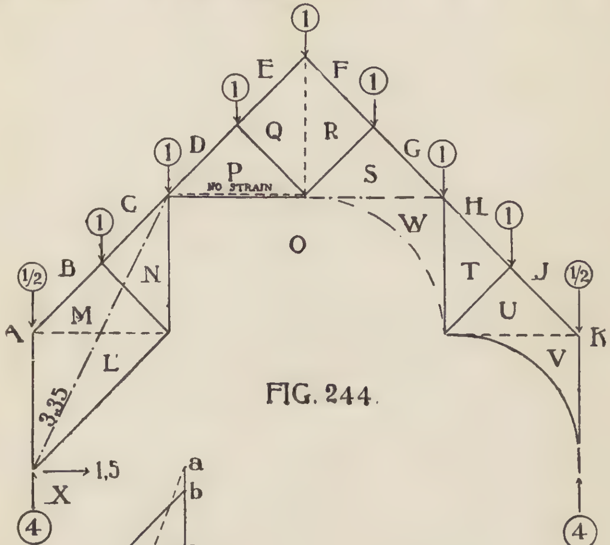


FIG. 244.

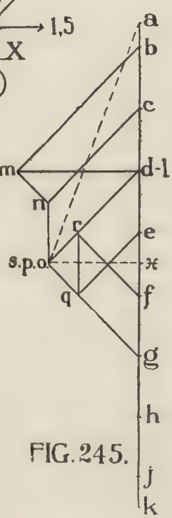


FIG. 245.

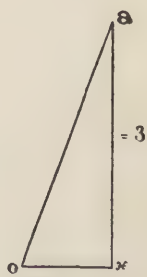


FIG. 247

Where $x =$ the length, in inches, of the longest versed sine of the curve (at the centre.)

If the longest versed sine is not at the centre, the bending-moment m will be at the point where the longest versed sine is located. Of course, the curved member must be designed to resist both the

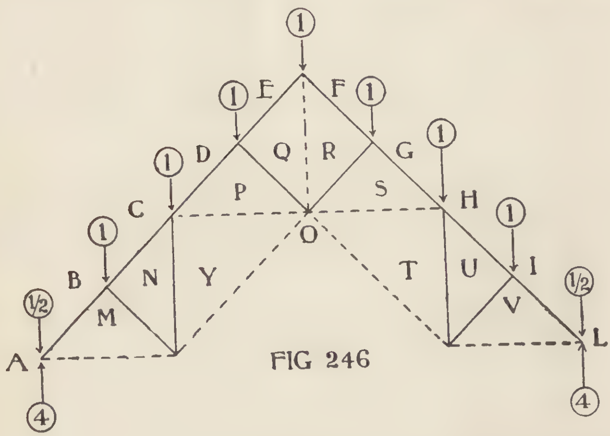


FIG 246

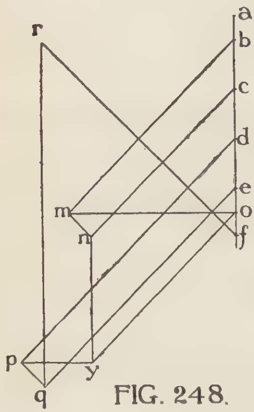


FIG. 248.

longitudinal strain and the additional strain due to the bending-moment.

Example I.

Example of curved strut.

A Georgia pine strut is 10 *feet long (measured on a straight line) but is curved, the versed sine at centre being* 20 *inches. It resists a compression strain of* 15000 *pounds. What size strut is required?*

The bending-moment at centre will be Formula (129)

$$m = 15000 . 20 = 300000 \text{ pounds-inch.}$$

The safe modulus of rupture $\left(\frac{k}{f}\right)$ for Georgia pine (Table IV) is 1200 pounds. Inserting these values in Formula (18) we have the required moment of resistance to this bending-moment:

$$r = \frac{300000}{1200} = 250.$$

From Table I, Section No. 2, we have for a rectangular cross-section

$$r = \frac{b.d^2}{6} \text{ therefore}$$

$$\frac{b.d^2}{6} = 250, \text{ and}$$

$$b.d^2 = 1500$$

If now we make $d = 12$ inches, we should require a width

$$b = \frac{1500}{144} = 10,4 \text{ inches.}$$

It should be noted that d must always be assumed in the direction of the versed sine, that is resisting the bending-moment.

We must now add sufficient material to resist the longitudinal compression. As the strut will evidently be nearly square, 12 inches will be our least diameter, inserting therefore values in Formula (3) we should have (assuming the ends to be secured by bolts only):

$$15000 = \frac{a.750}{1 + \frac{120^2.0,00067}{12}} \text{ or,}$$

$$a = 36 \text{ square inches.}$$

Now as $a = b.d$ and d having been fixed at 12 inches we should have,

$$b = \tfrac{36}{12} = 3 \text{ inches.}$$

Adding this to the above we should require a strut,

12 inches $\times$ (10,4 + 3) = 12 $\times$ 13,4

In practice we should probably make it 12 inches $\times$ 12 inches.

Example II.

Example of curved tie.

The lower chord of a wrought-iron arched truss is made of a channel iron. At one panel the length between bearings is 5 *feet; the tension* 36000 *pounds, the versed sine of the curve* 6 *inches. What size channel is required?*

The bending-moment m will be

$$m = 36000.6 = 216000 \text{ pounds-inch.}$$

Inserting values in Formula (18) we have required moment of resistance

$$r = \frac{216000}{12000} = 18$$

The additional area required to resist the direct tension will be

$$a = \frac{36000}{12000} = 3 \text{ square inches.}$$

We now consult Table XXI. We take the neutral axis normal to

web, as this will, of course, be the position of channel in the truss and in resisting the bending-moment.

We select for a trial the $10\frac{1}{2}$ inch — 105 pounds per yard channel; its area a is 10,5 square inches, and its moment of resistance $r = 24{,}64$. Now of the area 3 square inches resists tension leaving us 7,5 square inches or $\frac{7{,}5}{10{,}5} = \frac{5}{7}$ of the whole amount to resist cross-bending. The amount of r available to resist cross-bending will, therefore, be $= \frac{5}{7}.r = \frac{5}{7}.24{,}64 = 17{,}6$.

This is not quite equal to the required r (18) but is near enough for all practicable purposes, we should, therefore, use a $10\frac{1}{2}$ inch — 105 pounds per yard channel.

We will now return to our hammer-beam truss. Assuming then that the wall is capable of resisting the thrust we should analyze our truss somewhat similarly to the way we did with the scissors truss.

Truss with abutment.

On the left side of truss (Figure 244) we omit the circle corresponding to $O\ W$ entirely, and draw the one corresponding to $O\ V$ straight. We now can assume that we can divide the truss horizontally at $P\ O$ which will give us a truss above $P\ O$ similar to Figure 213 and a truss below it consisting of an inclined trussed strut running from bottom joint X to joint $C\ D$ with a horizontal strut $P\ O$ and an outward thrust $O\ X$ to be resisted at its foot. To obtain this outward horizontal thrust $O\ X$ we will temporarily consider the inclined trussed strut as a straight line running from joint X to $C\ D$. The load will evidently be three-quarters of the load coming on the entire rafter, if then in Figure 247 we make $a\,x =$ this load or $= 3$ in our case, and draw $x\,o$ horizontally and $o\,a$ parallel to the straight line (representing the inclined strut in Figure 244) we shall have the amount of the horizontal thrust $x\,o$. We can now make separate strain diagrams for the upper and lower trusses, in which case we will find that for the upper truss there exists a tension just equal to $o\,x$ in the member $P\ O$ and for the lower truss there exists a compression just equal to $o\,x$ in the member $P\ O$. In other words there is no stress in $P\ O$. This is confirmed by drawing the combination diagram (Figure 245) where p and o fall on the same point.

That these strains just equal each other is due to the design of the truss and its uniform loading. Were the design of the parts less symmetrical, or the load not symmetrical, or the wind on one side only, the two strains would not be found to be equal.

In Figure 246 where we consider the wall as incapable of taking up the thrust, and the truss has to take care of it itself, it will be found that this horizontal member (now called $P\ Y$) is in tension.

Truss without abutment.

The drawing of the strain diagram (Figure 248) presents no difficulty, but it will be seen that nearly all the strains on the truss are very much larger.

The actual thrust of the truss Figure 244 on the wall will be parallel to and equal to $a\ o$ of Figure 245. To resist it there must be weight enough to deflect this line sufficiently not to overturn the wall. This subject was thoroughly treated in Chapter V, Vol. I, under the heading "Arch with Abutment."

Wind load-line normal to roof.

The analysis of strains due to wind pressure, when taken separately on one side of the roof only, and at right angles to same, is treated the same as for dead loads, excepting that the load line is no longer vertical, and is drawn therefore parallel to the wind arrows (that is, at right angles to the main rafter on which wind blows). Where the roof is a broken one the wind load line is broken also, its different parts being proportioned to the amounts and parallel to the directions of the wind on the different parts.

Wind reactions unequal.

It will be readily seen that the left and right reactions due to the wind cannot be equal, and the calculation of these reactions offers the only difficulty.

Let us take a truss similar to the one shown in Figure 240 and overlooking the steady or dead load assume that the wind is blowing from the right hand side. Its effect will be as shown in Figure 249.

The pressure on the apex or $A\ B$ will be equal to half the length of rafter $B\ G$ multiplied by the product of the distance between trusses and the wind pressure per square foot, as given in Table XLIV. It is indicated by the arrow $A\ B$. The pressure in the middle joint — indicated by arrow $B\ C$ — will be this same (latter) product multiplied by the sum of half the length of $B\ G$ plus half the length of $E\ C$. The pressure at the foot — or arrow $C\ D$ — will be the same product multiplied by half the length of $E\ C$. The reactions at p and q which resist this wind pressure will evidently be in the opposite direction to the wind, or slanting, as shown. If, therefore, the truss and dead load is not heavy enough to sufficiently deflect these reactions, there would need to be bolts or buttresses at

p and q to keep the truss from sliding-off the wall. This will be explained at the end of the chapter in speaking of spires.

We now have the amount and direction of wind pressures and

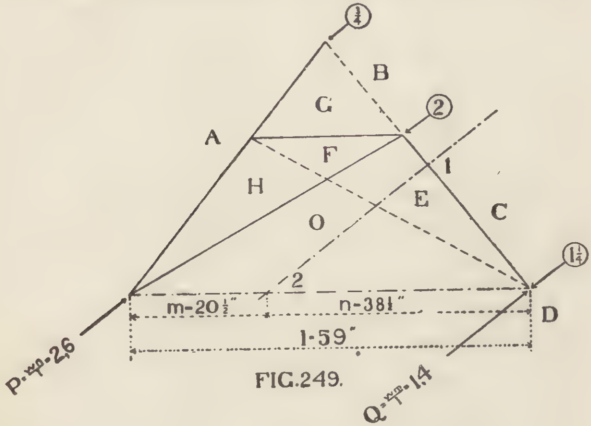


FIG.249.

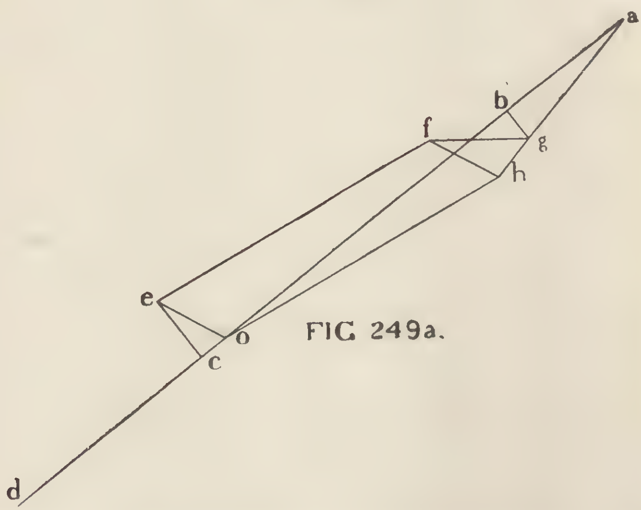


FIG 249a.

direction of the reactions, but we do not know the amounts of the reactions, as they evidently cannot be equal.

If we imagine the neutral axis of the entire wind pressure, that is the entire wind pressure concentrated at the point 1 at the centre of the length of rafter, the effect, so far as reactions are concerned will be the same.

We now draw the horizontal line $P\,Q$ connecting the feet of rafters, prolong the centralized wind pressure (or neutral axis) arrow 1 till it intersects $P\,Q$ at 2, then will the reactions P and Q be inversely as the divisions of line $P\,Q$; that is, if $P\,Q = l$ and $P\,2 = m$ and $2\,Q = n$ and the whole wind pressure $= w$, we should have

Amount of wind reactions.
$$p = \frac{w.n}{l} \tag{130}$$

and

$$q = \frac{w.m}{l} \tag{131}$$

Where $p =$ the amount of (slanting) left reaction, in pounds, due to wind pressure.

Where $q =$ the amount of (slanting) right reaction, in pounds, due to wind pressure.

Where $l =$ the length, in inches, measured horizontally between centres of feet of truss.

Where m and $n =$ respectively, in inches, the lengths into which the horizontal line is divided by the prolongation of the centre line (or neutral axis) of wind pressure, m being nearer p and n nearer q.

The analogy between these formulæ and Formulæ 14 to 17 should be noticed. The analyzing of the strains by means of the diagram Figure 249a will present no difficulty. We draw $a\,b$ parallel and equal the pressure $A\,B$ at the apex, $b\,c = B\,C$ pressure at central joint and $c\,d = C\,D$ pressure at foot. Along this load line $a\,d$ we now lay off in opposite directions the reactions, namely,

$d\,o = D\,O$ or reaction q

and

$o\,a = O\,A$ or reaction p.

We first find the strains on the foot joint $O\,A$ by drawing $a\,h\,o\,a$, then skip to the apex joint and draw $a\,b\,g\,a$; the rest presents no difficulty.

It will be noticed that the effect of the wind is to reverse the strains on two pieces.

The upper part of the rafter on the side on which the wind is

blowing is now in tension, whereas in Figure 240 it was in compression; further the tie $H\ O$ now becomes a thrust member opposing the wind, where before it was in tension.

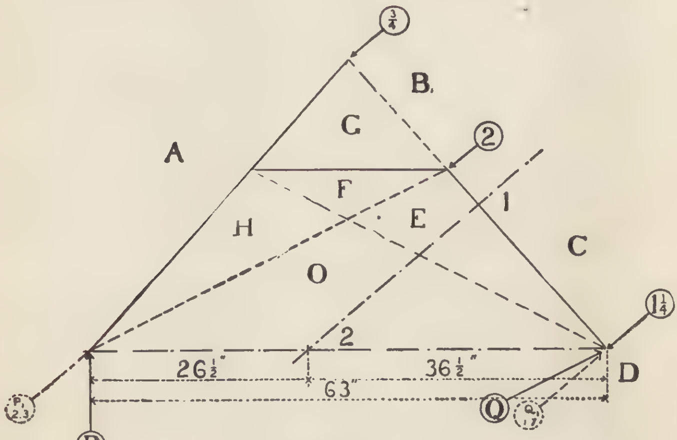


FIG. 250.

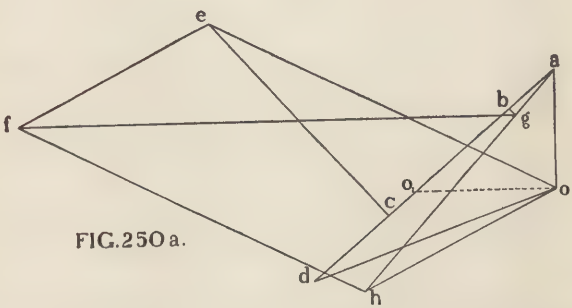


FIG. 250 a.

Had we drawn the wind on the left side of truss the calculation would have been similar. The direction of p and q would have been from right upwards to the left; p would have been the larger reaction and we should have found the strains same as in Figure 240, except on $A\ G$ which would become tension and on $H\ F$ (or $O\ E$) which would now become compression. It is not necessary to make

more than one wind diagram, therefore, where both feet of the trusses are bolted down.

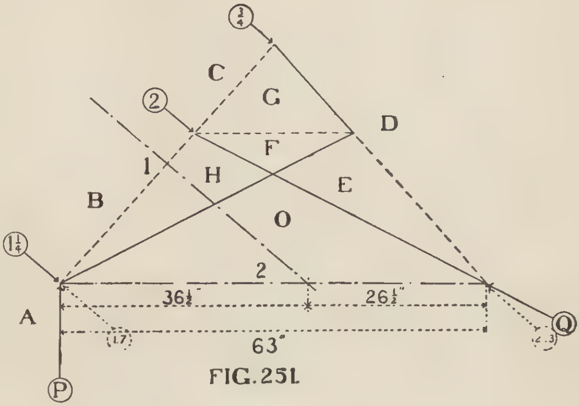


FIG. 251.

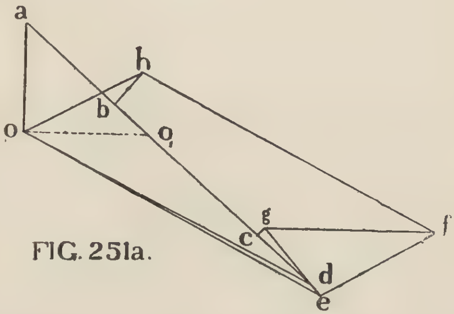


FIG. 251a.

Truss with rollers. If, however, one foot only were bolted down and the other foot were placed on rollers, to allow for expansion and contraction, we should have to make two diagrams, with the wind from left and right respectively, as it makes a decided difference to the strains as shown in Figures 250 and 251. In Figure 250 the wind is from the right with rollers under the left or opposite foot of truss; in Figure 251 the wind is from the left and the rollers are under the left (or same) foot of truss. It will be readily seen that in both cases the (left) roller foot will not oppose any tendency to slide due to the wind, the right

foot must therefore be bolted down to resist this tendency in both cases. We can readily see therefore that in Figure 251, the member HF or OE must be a brace to keep the truss from sliding towards the right foot, whereas in Figure 250, it must be a tie to hold back the truss from sliding away from the right foot. The strain diagrams will show this to be the case. To obtain the reactions we proceed as before, so far as the central wind pressure ($\frac{1}{2}$) and lines P 2 and 2 Q are concerned, but it is evident that the reaction P will be vertical, while the reaction Q will no longer be in direct opposition to the wind as shown by dotted line, as it is deflected from this line, owing to the horizontal (sliding) strain to be taken up from the other foot.

Wind opposite rollers.

We therefore draw in Figure 250a the load line $a\ b\ c\ d$ as before, also the reactions $d\ o_1 = q$ and $o_1\ a = p$. Through a draw the vertical projection $a\ o$ of $a\ o_1$ — (that is draw $o_1\ o$ horizontally and $a\ o$ vertically) — then will $o\ a$ be the amount of vertical reaction p and $d\ o$ the direction and amount of vertical reaction q. The rest of the strain diagram is easily drawn, remembering to take the foot joint first and then to skip to the apex joint.

Wind on roller side.

To analyze Figure 251, it must be treated in exactly the same way and it offers no difficulty. We notice that Figure 250 has strains similar to those in Figure 240 excepting the upper part of rafter on the wind side, which is now in tension. In Figure 251, however, things are greatly changed. The upper end of rafter becomes a tie, both ties become struts and the short strut $G\ F$ (or $J\ H$ in Figure 240) now becomes a tie.

Wind and load in one diagram.

Wind strains and steady loads can be calculated from one strain in diagram, as shown in Figure 252 and Figure 252a. This is a combination of the conditions obtaining in Figures 240 and 249 and the strain diagram of their respective strain diagrams, Figures 243 and 249a. By simply following around the arrows, as there shown (first obtaining the amounts of $A\ B$ and $K\ O$) the diagram offers no difficulty whatever. If an actual example were figured out, first separately, as shown in Figures 240 and 249, and then in combination, as shown in Figure 252, the result would be the same, remembering to make all compressive strains positive and all tension strains negative and to obtain the arithmetical result of the consequent additions or subtractions. A practical example will be given in the next chapter.

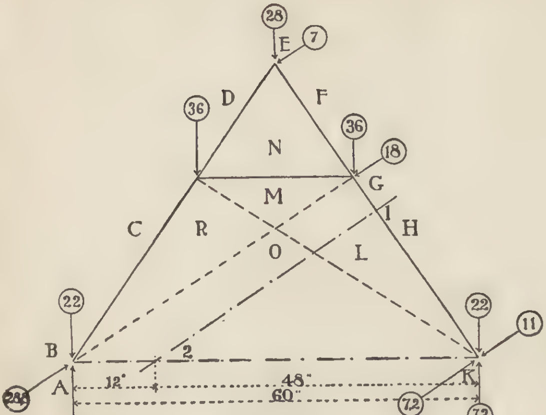


FIG 252.

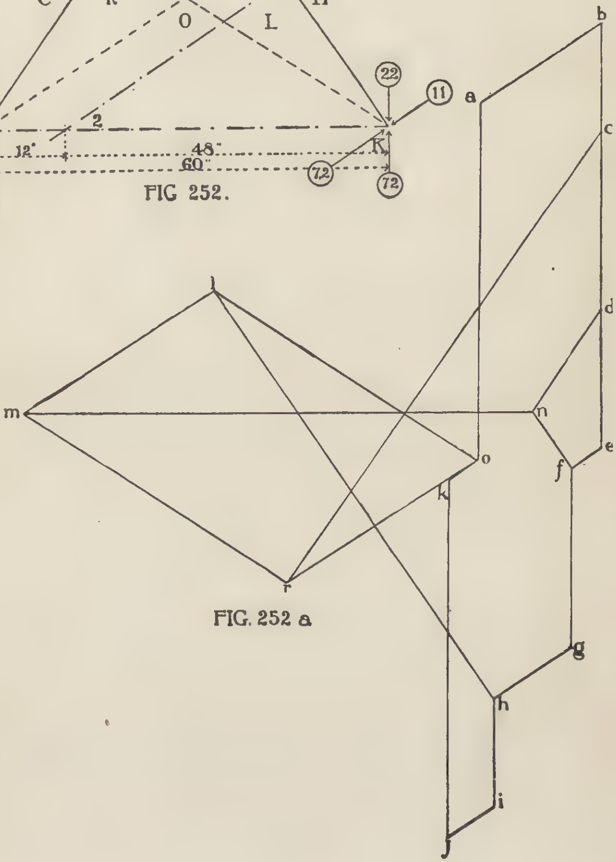


FIG. 252 a

Where the roof rafter or top chord of a truss is not in one straight line, but is made-up of a series of differently inclined lines, as in a

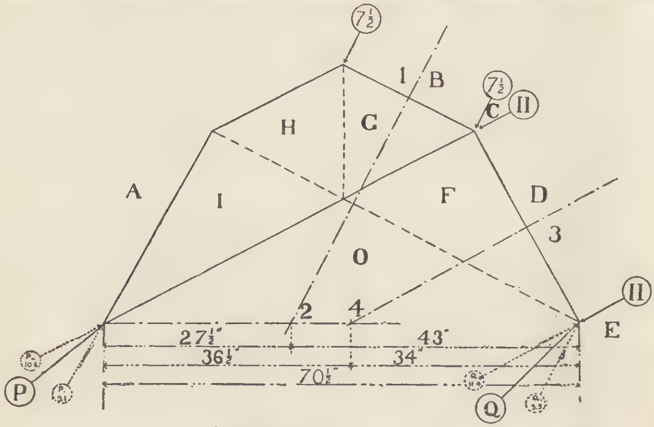


FIG. 253

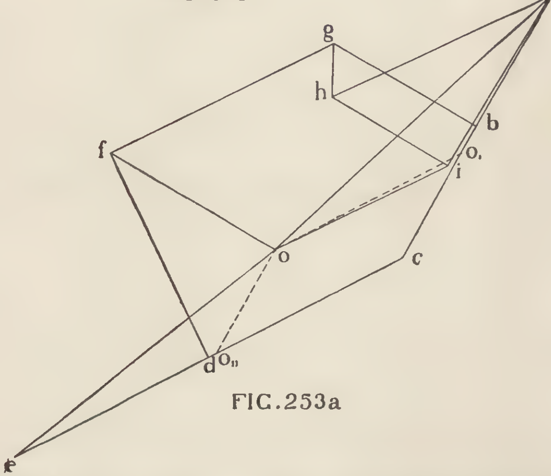


FIG. 253a

broken roof, or in a circular or arched truss, the wind pressure will, of course, be at different angles too, each at right angles to its respective surface. We can in such a case, work out separately the reaction due to the wind pressure on each surface, and then obtain the resultant

Wind on broken roof.

(reaction) of all these lesser ones, as explained on p. 71, Vol. I, or we can combine them all into one diagram. In Figure 253, we have a mansard roof with wind pressure from the right. We draw the neutral axis (or central lines) of wind pressure on each rafter prolong them to their intersections with the horizontal and can thus figure out the respective reactions due to each. In the strain diagram Figure 253*a* we now draw $c\,e$ parallel and equal to total wind pressure (3) on rafter $F\,D$; and $a\,c$ parallel and equal to total wind pressure (1) on rafter $B\,G$. Draw $e\,a$ and it will be the direction and amount (sum of) the actual reactions. Now to get each reaction separately make $e\,o_{II} \doteq$ to reaction q of wind pressure (3) on $F\,D$; and $c\,o_{I} =$ to reaction q of wind pressure on $G\,B$; of course $o_{II}\,c$ and $o_{I}\,a$ will equal the reactions p due to the respective wind forces. Or we might divide $e\,c$ and $c\,a$ so that

$$e\,o_{II} : o_{II}\,c = P\,4 : 4\,Q$$

and

$$c\,o_{I} : o_{I}\,a = P\,2 : 2\,Q$$

We now draw $o_{II}\,o$ parallel $c\,a$ and if we have drawn correctly $o\,o_{I}$ must be parallel $e\,c$. Having found o the rest of the diagram presents no difficulty. We make $a\,b = A\,B$ or pressure at apex; $b\,c = B\,C$ pressure at joint C due to wind on $G\,B$; $c\,d = C\,D$ pressure on same joint C due to wind on $F\,D$ and finally $d\,e = D\,E$ pressure at foot and proceed with the other lines.

Were we to make a strain diagram for a steady load on all joints we should find similar strains on all members except $I\,H$. This is a compression member of the truss, but becomes subjected to tension when the wind blows from the right.

Similarly $G\,F$ would become tension if the wind were from the left.

Wind tendency to overturn roofs.

We will consider the tendency of wind to overturn roofs, and this can best be done by calculating one or two practical examples of steeples. Before doing this, however, the student should be warned to *always* arrange for *wind braces*, that is, diagonal ties between the trusses and in a plane at right angles to the trusses. The object of these, is to make all the trusses (that is the entire roof), act as one mass and thus keep the wind from blowing over each truss individually, and thus collapsing the roof. The arrangement of these ties varies with circumstances. They are usually placed immediately under the roof surface, that is, from foot of

Wind braces.

one rafter to apex of next rafter on each side. If the rafters are long they are placed diagonally in the individual panels. Sometimes a longitudinal truss is made the entire length of ridge, by wind bracing from both sides of each apex to centre of each neighboring main tie.

Example III.

A wooden, slate-covered steeple, 27′ 6″ high, covers a 24′ square brick tower. The walls are 16″ thick at the top and the steeple is anchored down four feet into the walls. Is this sufficient to keep it from blowing over?

Wooden steeple.

It makes no difference just how the anchoring is done, it can be either by means of iron anchors bolting the plate down to the masonry below, or by means of wooden trestle work built inside and against the walls, which will also force the steeple to lift the walls from their bearings, before the steeple can topple over.

The wind pressure on one side of the roof will be the area of this side multiplied by 40 pounds per square foot (see Table XLIV) and it will act or can be considered as centralized at the centre of gravity of the side, which, being a triangle would be at one-third its height, or at D in Figure 254. The length of rafter will be 30′, therefore area of one side $= 30.12 = 360$ square feet, and wind pressure $F\,D = 360.40 = 14400$ or say 15000 pounds total wind pressure which we consider as centralized at D and normal to rafter. Resisting this, we have the dead load, that is the weight of steeple and of the masonry as far as steeple will have to lift it.

Taking the weight of steeple at 20 pounds per square foot (making, of course, no allowance for snow) and weight of brickwork at 112 pounds we have weight of (four sides of) steeple,

$$= 4.360.20 = 28800 \text{ pounds.}$$

Weight of masonry

$$= 4.\,(24 - 1\tfrac{1}{3}).4.112 = 54208 \text{ pounds,}$$

or total dead (vertical) load $= 83000$ pounds.

We now draw $c\,a$ parallel and $= 15000$ pounds, the wind pressure; and $a\,b$ vertically and $= 83000$ pounds, the dead load. Draw $c\,b$ and from intersection G of $F\,D$ with the main vertical neutral axis of the dead load draw $G\,K$ parallel $c\,b$ till it intersects the horizontal

joint line $A\ B$, which we draw four feet below the tops of walls or at the anchor level. We now use Formulæ (44) and (45) to obtain the extreme edge strains at A and B.

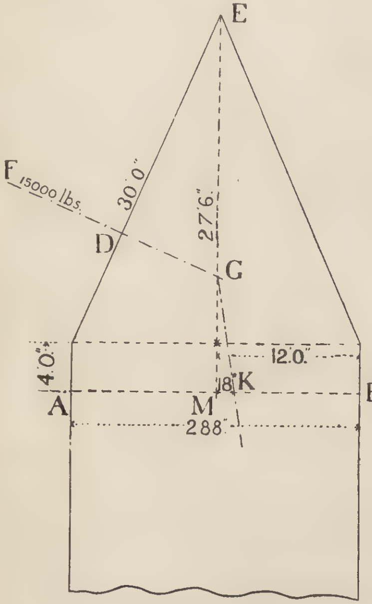


FIG. 254.

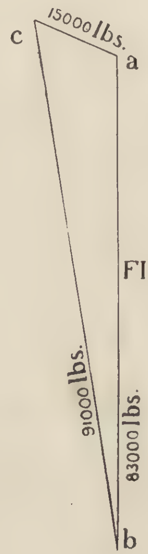


FIG. 254 a.

We have these values:

$p =$ the pressure $= c\,b = 91000$ pounds.

$x = M\,K = 18''$ (by measurement)

$d = A\,B = 24.12 = 288''$

$a =$ area of wall at $A\,B = 4.16''.\,(288 - 16)$

$= 17408$ square inches.

We have then for pressure at nearer edge B

$$= \frac{91000}{17408} + 6.\frac{18.91000}{17408.288}$$

$= +\ 7,21$ pounds compressive pressure, per square inch; and at A

$$= \frac{91000}{17408} - 6.\frac{18.91000}{17408.288}$$

$= +\ 3,25$ pounds compressive pressure, per square inch.

If the latter value had been a negative one, we should have had to rely on the quality of the mortar not to tear apart at A and thus allow the steeple to fall. It would be better, however, in such a case to carry the anchoring process further down, and thus gain more dead vertical load to resist the wind pressure.

Example IV.

A square stone steeple has 12″ stone sides at the top; 19 feet from the top vertically, the length of side is 12 feet. Is the steeple safe against wind pressure at this point?

Stone steeple. This example is intended to show that we can examine any point of steeple similarly to the manner of examining the base.

In Figure 255, $A\,D$ measures 12 feet $= 144''$; $M\,C = 19$ feet and $C\,D$ scales 20 feet, hence area of each side $= 6.20 = 120$ square feet, and weight of each side approximately $= 120.150 = 18000$ pounds and weight of four sides or vertical load $= 4.18000 = 72000$ pounds.

The wind pressure will be

$= 120.40 = 4800$ pounds and will be centralized at B or one-third the height of $A\,C$.

We draw $B\,G$ normal to $A\,C$ till it intersects $C\,M$ at G.

Make $a\,b = 72000$ pounds and vertical, draw $c\,a = 4800$ pounds and parallel $B\,G$ and draw $a\,b$ which scales 74000 pounds. Draw $G\,K$ parallel $c\,b$ and we find K is $4''$ distant from centre of joint M. The area of joint is $= 4.11.144 = 6336$ square inches. We have, therefore, pressure at nearer edge of joint D,

$$= \frac{74000}{6336} + 6 . \frac{4.74000}{6336.144}$$

$= + 13,65$ pounds, per square inch.

and at edge A

$$= \frac{74000}{6336} - 6 . \frac{4.74000}{6336.144}$$

$= + 9,75$ pounds per square inch.

In a similar manner we might examine any other joint of the steeple.

It will be found that at the very top or near it the greatest danger exists. The finial frequently exposes a large surface to the wind and almost at right angles to the vertical dead load, deflecting this

line much more in proportion. Then, too, the mortar is so much exposed that it cannot be relied on. For this reason there is placed usually a vertical iron tie-rod from the finial to some point below, frequently even below the springing line of the steeple. This arrangement is all right, provided the rod is properly put in. The writer has seen ponderous rods 2″ diameter or more and perhaps, fifty or sixty feet long, intended as tie-rods, that had become so loosened by contraction and expansion that they could be easily

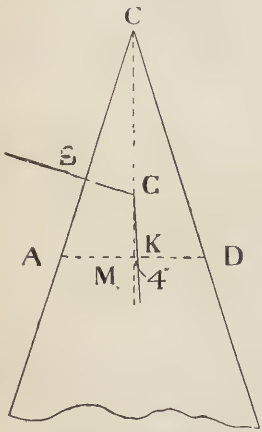


FIG. 255.

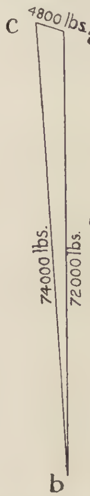
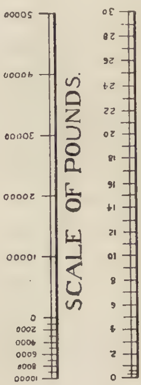


FIG. 255a.



SCALE OF POUNDS.

SCALE OF FEET.

FOR FIGS. 254 & 254a AND 255 & 255a.

swayed back and forth by the hand. Hence their only service was their own dead weight.

Vertical tie-rods in steeples.

The way to put in such rods is to leave the lower end free to move vertically, that is up and down, but secure it against lateral movement and then to attach to the lower end a heavy weight, proportioned according to circumstances.

Wind allowances for different surfaces.

In figuring the allowance for wind it is customary to take only one-half the usual allowance for circular or surfaces slanting to the direction of wind. This is done because they offer less resistance than a flatly opposed surface, allowing the wind to slip by more readily. In a circular steeple we should take as our area the half circumference of base multiplied by the length on the rafter line and

multiply this by only one-half the pressure for wind as given in Table XLIV. This usually would be 20 pounds per square foot.

For an octagonal roof we should assume full pressure on the central surface and only half pressure on the two side surfaces. Where the octagon is a regular one, this would amount practically to assuming double pressure on one surface only.

There is no danger of a steeple blowing over diagonally, as the sides would in such a case present slanting surfaces to the wind, allowing it to slide off readily; and besides the base line, being the diagonal of the square, would be so much longer.

On vertical (wall surfaces) the writer usually allows only for a maximum wind pressure of 30 pounds per square foot, for, as a rule, they (or part of them) are low down near the ground and therefore not exposed to the full force of the wind.

CHAPTER XII.

WOODEN AND IRON TRUSSES.

IN Figure 256 we have the design of one-half of an ordinary king-post truss, with three bays to each principal rafter. Figure 257 gives the strain diagram and Figure 258 the design of the truss in detail. The length of principal rafter is 25 feet 6 inches, therefore

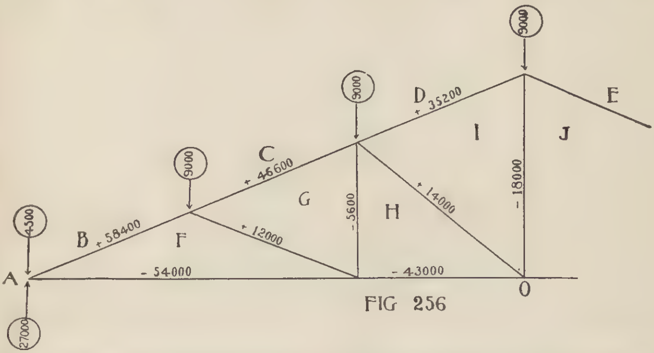


FIG 256

length of each bay 8 feet 6 inches. The trusses in the building from which this example has been taken had to be spaced far apart — 19 feet from centres — on account of ceiling and skylights; this, however, is not an economical arrangement, as it increases not only the sizes of the truss members, but of all the purlins as well. The weight assumed was 55 pounds per square foot of roof surface, being made up as follows:

Large spaces extravagant.

Slate,	7 pounds.
Plaster,	8 pounds.
Boards and construction,	10 pounds.
Wind and snow,	30 pounds.
Total	55 pounds.

The slate was the ordinary roofing slate; if the slate used had been called for of even thickness and $\frac{1}{4}$ inch thick the allowance would have been 10 pounds.

The weight of "Boards and construction" was estimated and not checked off, in this case. In important constructions and particularly in ironwork, this should be carefully weighed up.

The load on each joint of our truss was therefore:

$19.8\frac{1}{2}.55 = 8882$ pounds,

or say 9000 pounds on each joint, excepting, of course, the joints at reactions which carry only half-loads.

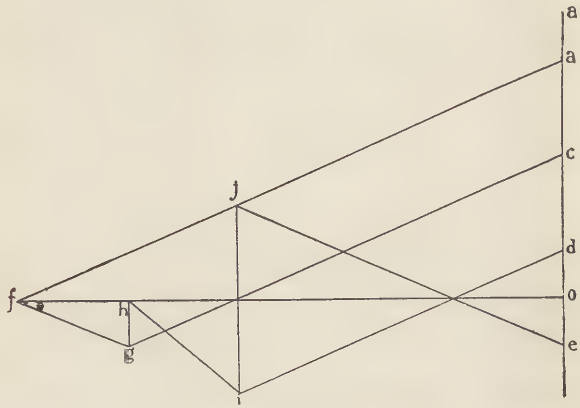


FIG. 257.

The loads on the truss being symmetrical, each reaction will be one-half the total load or 27000 pounds.

There is no difficulty in drawing the strain diagram, or in finding the stresses in each member, the latter are marked on Figure 256, the positive + sign denoting compression, and the negative — sign tension.

Detailing wooden truss.

We now proceed to detail the truss. The principal rafter will evidently, though not necessarily, be made in one piece from foot to apex, the largest strain — at the bottom panel — will therefore determine its size. This is 58400 pounds compression. The rafter is evidently a series of columns each 8 feet 6 inches long, or comparatively short columns; for the struts brace the rafter against yielding downward;

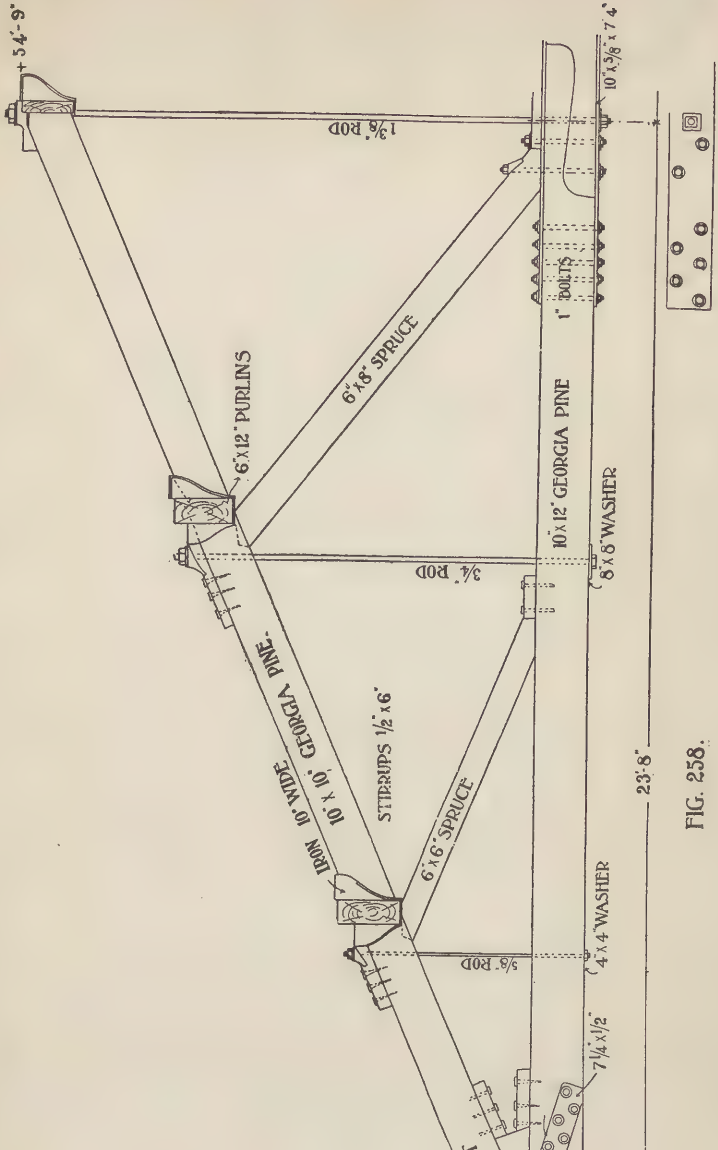

+54'-9"
10"x5/8"x7'4"
1 3/8" ROD
6"x12" PURLINS
6"x8" SPRUCE
1" BOLTS
10"x12" GEORGIA PINE
8"x8" WASHER
3/4" ROD
IRON 10" WIDE
10" x 10" GEORGIA PINE.
STIRRUPS 1/2"x6"
6"x6" SPRUCE
23'-8"
4"x4" WASHER
5/8" ROD
7 1/4"x1/2"
1" BOLT
8"x1/2"

FIG. 258.

the loads and tie-rods keep it from yielding upwards; and the purlins keep it from yielding sideways. The safe compression per square

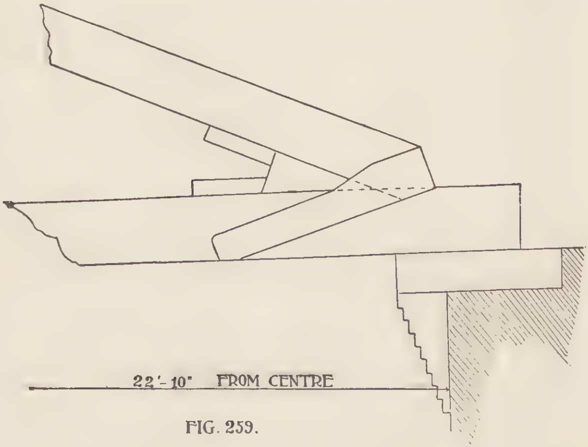


FIG. 259.

inch on Georgia pine (see Table IV) is 750 pounds. Without bothering with the complex Formula (3) for columns, we will assume that we can safely use

Principal rafter.

$$\left(\frac{c}{f}\right) = 700 \text{ pounds in our case,}$$

we require then in the main rafter an area of cross-section

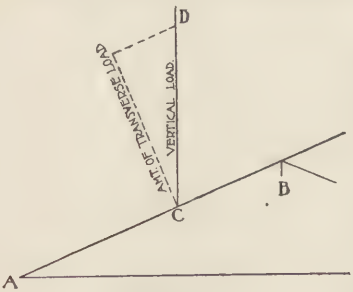


FIG. 260.

$$a = \frac{58400}{700} \doteq 83{,}4 \text{ square}$$

inches,

or say a timber $8\frac{1}{3}$ inches x 10 inches. This we increase to 10 inches x 10 inches to allow for cutting away at bolt-holes.

If the purlins had not been placed directly over the joints we should have to increase this size to provide for the transverse strain in each panel. In such a case we should have a beam 8 feet 6 inches long, supported at both ends. The transverse load on the beam would not be the full

vertical load, but only its resultant normal to the rafter, as shown in Figure 260.

Vertical load on slanting beam.

If $C D$ were drawn at any scale equal to the vertical transverse load on rafter $A B$, draw $E D$ parallel rafter, and $E C$ normal to same, then will $E C$ measured at same scale as $D C$ represent the transverse load on the rafter $A B$, the latter being considered as a beam of length $A B$ supported at A and B.

Returning to our truss; if it were not supported sideways by purlins we should have to greatly increase the size of rafter, to guard against lateral flexure, see Formula (5).

Tie-beam.

The stress in tie-beam is 54000 pounds tension, the safe $\left(\frac{t}{f}\right)$ for Georgia pine (Table IV) is 1200 pounds, therefore required area

$$a = \frac{54000}{1200} = 45 \text{ square inches.}$$

In a wooden truss the principal members are naturally of same thickness, therefore we should require $4\frac{1}{2}$ inches x 10 inches to resist tension. To this must be added the necessary area for bolt-holes, transverse strain due to ceiling hanging to tie-beam, etc. In our case the beam was pieced, as shown at centre, and it was therefore rather heavily increased and made 10 inches x 12 inches.

Struts.

The struts were of spruce; their size will, as a rule, be determined by the area of their bearing against the principals — so as not to indent these — rather than by their length as columns, though both should be tried.

By making our larger strut 6 inches x 8 inches we get a bearing against the rafter of about 6 inches x 10 inches, or say 60 square inches, this would make a compressive stress *across the grain* on the Georgia pine rafter of

$$\frac{14000}{60} = 233 \text{ pounds per square inch.}$$

Nature of compression large blocks.

Table IV gives 200 pounds as safe across the grain on Georgia pine, but we can pass the above as safe. It should be remembered that the action of crushing stress in short blocks greatly resembles that of an explosion. A short block or cube of stone, if compressed, will fly off in the four directions normal to the four free sides, with sudden and great force, as if there had been an internal explosion. If the block is large

the central fibres will tend to explode outwardly, while, of course, the fibres nearer the edge will tend to explode inwardly and as a result the central fibres, meeting with the opposition of the external fibres, will resist more compression than they would if they were free. This is confirmed by actual experiment, where it is found that large blocks of the same material will resist crushing proportionately to a very much greater extent than will the small blocks.

Where, therefore, we are proportioning the sizes of struts — or later the sizes of washers — for their bearing area, *across* the wood, we will bear the above in mind, and if the safe values for crushing across the grain, as given in Table IV, require unusual dimensions, we will increase the amount of the safe value per square inch, as our judgment may dictate.

Now trying the strut as a column of spruce 6 inches x 8 inches, or 48 inches area, and 9 feet, or 108 inches long, we should have from Formula (3) the safe load:

$$w = \frac{48.650}{1 + \dfrac{108^2 .\, 0{,}00033}{\dfrac{6^2}{12}}}$$

$$= 13700 \text{ pounds,}$$

which is near enough to be safe.

In calculating the other strut we should find that as a column 6 inches x 6 inches and 6 feet 3 inches long it can safely carry 14400 pounds, the actual strain being 12000 pounds. For pressure across the grain against the rafter and main tie-beam we have an area of about 6 inches x 8 inches against each or 48 square inches, therefore actual compression per square inch across the grain = 250 pounds, which they can safely stand, as these parts are of Georgia pine and the crushing area quite large.

Tie-rods.

We next proportion the sizes of tie-rods, which will be of wrought-iron. The safe tensional stress of wrought-iron being 12000 pounds per square inch we require areas as follows:

For the principal rod,

$$= \frac{18000}{12000} = 1\tfrac{1}{2} \text{ square inches.}$$

For the side rods,

$$= \frac{5600}{12000} = \tfrac{7}{15} \text{ or say } \tfrac{1}{2} \text{ square inch.}$$

The rods being circular the principal rod would need to be $1\frac{3}{8}$ inch diameter and the side rods $\frac{13}{16}$ inch diameter or say $\frac{3}{4}$ inch.

We also place a small $\frac{5}{8}$ inch diameter rod in each end panel to keep the tie-beam from sagging. The rods will all have to have their ends upset, or else they will not have enough sectional area between the threads. In practice, however, it would be cheaper, where the rods are so small and short, to enlarge their diameter and pay for the extra material, rather than to save this and have to pay for expensive blacksmith's work.

Upset ends.

Screw ends should only be upset where the material saved fully counterbalances the labor.

We next proportion the washers, they bear against Georgia pine and across its grain and should, therefore, be proportioned at about 200 pounds per square inch.

Washers.

For the principal rod we will need, therefore, a bearing area for each washer of

$$\frac{14000}{200} = 70 \text{ square inches,}$$

or say 8 inches x 8 inches.

For the side rods we will need

$$\frac{12000}{200} = 60 \text{ square inches,}$$

or about the same, we will therefore to save expense make them all alike.

The lower washers are made to bear horizontally against both head (or nut) and tie-beam, but the upper washers have to be modelled to bear against the slanting side of rafter, and horizontally against nut. It will also be noticed that the lower end of the washer is "toed-in" to the rafter to keep the washer from sliding.

The wooden blocks which are screwed onto the rafter at each washer were made to allow for cutting away, to provide horizontal surfaces on which to rest the bridle irons which carried the purlins.

It will be noticed that the principal tie-beam is pieced at the centre. The cut or halving was made slanting, so as to force each half to bear on and pull against the other half; and all sharp edges, where sudden increase in strains would take place were avoided by rounding off, as shown. Wrought-iron plates were placed over and below the cut and sufficient bolts placed each side of the cut, not to crush the wood.

Halving long timbers.

We now must still design the foot-joints.

The bearing area required must first be found; we can safely put 200 pounds per square inch on Georgia pine, across the grain, and our load at each reaction being 27000 pounds, we need

Bearing area.

$$\frac{27000}{200} = 135$$ square inches of bearing area, and as the timber is 10 inches wide, it would have to rest on its bearing for a length of $13\frac{1}{2}$ inches.

In this case, however, we made it 24 inches, because the principal rafter bearing on the tie-beam so far from the support, it would have been apt to bend it. The left end rested on an iron column and was bolted to it as shown in Figure 258. In Figure 259 is shown the right end, which rested on and was bolted to a wall. The latter was corbelled out under the truss and capped with blue stone.

We next provide at each foot an inclined bolt, to hold the rafter down to the tie-beam and keep it from jumping out of the strap, which it might do, if subjected to heavy transverse strains. We next "toe-in" the rafter to the tie-beam as shown, taking care to get enough bearing area and area ahead of the toe to keep it from pushing or shearing its way out through the tie-beam, or from being sheared off itself. In our case, however, the toe is made small, the only reliance we place on it being to steady the rafter sideways. The rafter transfers all the load vertically across the tie-beam, we shall therefore need the same length of bearing area on the tie-beam, $13\frac{1}{2}$ inches, as the latter needs on the wall.

Bolt at foot.

We more than get this by means of the hardwood block, driven in tightly with its edge grain against the rafter and tie-beam and held in position by screwed on blocks.

We next calculate the strap. The strains coming on it are 54000 pounds tension and resisting this 58400 compression. The lesser will, of course, be the one it must resist. The width of strap required not to crush the rafter must first be determined. The strap bears almost along the grain of the rafter, but not quite, we will therefore reduce slightly the safe allowance for compression along the grain as given in Table IV, and allow, say, 670 pounds per square inch, we then require $\frac{54000}{670} = 80$ square inches, or the rafter being 10 inches wide,

Calculating strap.

a strap 8 inches wide. The thickness of strap should be sufficient not to shear off, we have two shearing areas one at each angle or $\frac{54000}{2} = 27000$ pounds on each. The safe shearing stress on wrought-iron being 8000 pounds per square inch (see Table IV) we need an area at each angle $= \frac{27000}{8000} = 3\frac{3}{8}$ square inches. The strap being 8 inches wide would therefore need to be about $\frac{7}{16}$ inch thick. It was made $\frac{1}{2}$ inch, however, as in this case, it was doubtful whether the ironwork would be of a high character. Now each side of the strap will have to withstand a tensional strain of 27000 pounds. While the safe tension on wrought-iron in Table IV, is 12000 pounds, in this case it was assumed at 9000 pounds only, or the strap needed a sectional area $= \frac{27000}{9000} = 3$ square inches.

The strap being $\frac{1}{2}$ inch thick, would therefore need to be 6 inches wide, to which must be added $1\frac{1}{4}$ inch to allow for bolt holes, making the strap $7\frac{1}{4}$ inches x $\frac{1}{2}$ inch. We now settle the number of bolts, which we will make 1 inch diameter.

Number of bolts.

The safe shearing on a 1 inch bolt will be $= \frac{11}{14}.8000 = 6300$ pounds. Each bolt has two shearing areas, one at each end and will therefore resist 12600 pounds. The total strain being 54000 pounds, we need $\frac{54000}{12600} = 4{,}3$ or say five bolts to resist shearing.

For bearing we have an area 1 inch diameter by $\frac{1}{2}$ inch thick against the strap at each end of each bolt, or just one square inch bearing area against iron to each bolt, which will equal a safe resistance to compression of 12000 pounds per bolt, we need therefore $\frac{54000}{12000} = 4{,}5$ or say five bolts to resist the compression of the strap.

The bolts also bear against the Georgia pine tie-beam and tend to crush it along the grain, the safe resistance of the wood being 750 pounds per square inch. Each bolt bears against 1 inch x 10 inches or 10 square inches of wood, and will safely bear therefore $10.750 = 7500$ pounds. We need therefore, to avoid crushing the wood $\frac{54000}{7500} = 7{,}2$ or say eight bolts, which is the number shown in drawing.

We will now similarly analyze an iron truss. This, the same as the above wooden truss, is not intended so much as a guide in design-

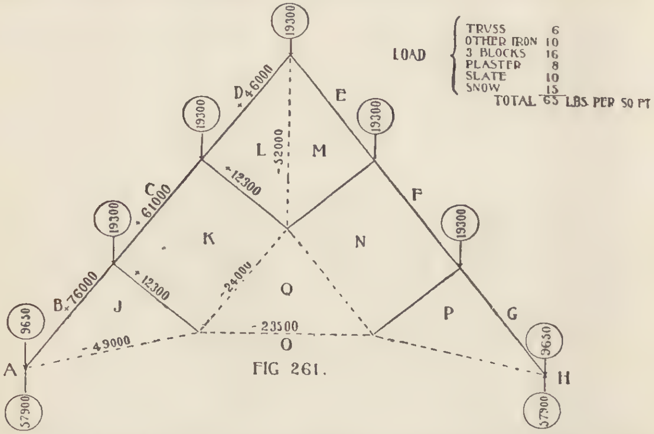


FIG 261.

ing, as it is as a guide in analyzing the details of each joint. Every truss should be designed with reference to its special duty and position, and joints should be designed in each case to be made up as simply as possible.

Iron truss.

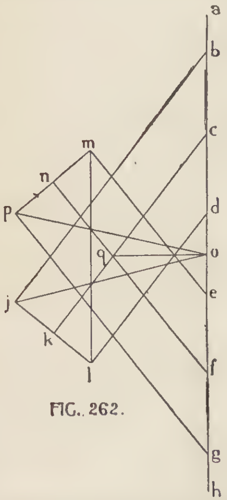


FIG. 262.

The designer should not hesitate for fear of criticism as to novelty, to make his truss or its joints of any shape that may be most convenient, bearing always in mind, that the simpler the parts and the nearer to standard sizes, the cheaper will be the execution of the work.

Almost any design can be made by the use of wrought-iron or steel, or of cast-iron.

In the two former, care should be taken to analyze carefully the work required of each rivet. Wrought-iron is gradually taking the place of cast-iron, even for shoes and such parts of trusses, as the expense o riveting them up out of different parts, is apt to be cheaper than the cost of the patterns required for castings.

In Figure 261, we have the design of the axial lines of an iron roof truss recently erected in New York City.

Vertical load. The vertical load on this truss was assumed at 65 pounds per square foot and made up as follows:

Weight of truss	= 6 pounds.
Weight of other iron	= 10 pounds.
Weight of 3 inch blocks	= 16 pounds.
Weight of plaster	= 8 pounds.
Weight of slate	= 10 pounds.
Weight of snow	= 15 pounds.
Total (per square foot)	= 65 pounds.

Figure 262 gives the strain diagram for this load; the rafter panels were each 17 feet long, and the trusses placed 17 feet 6 inches from centres, so that the load on each panel was $17.17\frac{1}{2}.65 = 19337$ or say 19300 pounds.

The wind-pressure was calculated separately. In Figure 263 it is supposed to blow from the left. The pressure normal to the roof

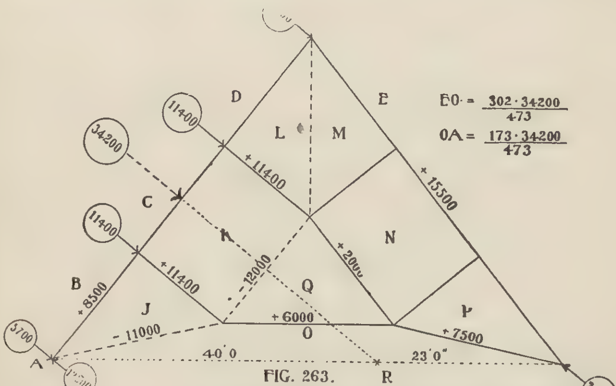


FIG. 263.

WIND PRESSURE ONLY 38.4-LBS.-SQ.FT. FOR ANGLE 53°

surface per square foot for a roof of this inclination (51°) will be, say, 38,4 pounds, (see Table XLIV.)

We should have then on each panel:

$17.17\frac{1}{2}.38,4 = 11424$ or say 11400 pounds.

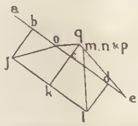
Wind pressure. The total wind-pressure therefore was $= 34200$ pounds. By prolonging the central axis of the wind

pressure $C R$ till it intersects the horizontal $A H$ at R we obtain the reactions due to wind pressure. As $A R$ measures 40 feet and $R H$ 23 feet, we know that the reaction at H will be

$$= \tfrac{40}{63}.34200 = 21700 \text{ pounds,}$$

and at A

$$= \tfrac{23}{63}.34200 = 12500 \text{ pounds.}$$



STRAIN DIAGRAM FOR WINDLOAD
FIG. 264.

The strain diagram Figure 264, can now be easily constructed. We notice that there are no strains in $M N$ nor in $N P$ due to wind; also that the latter reverses the strains in $Q N$, $Q O$ and $P O$, they all having to resist compression. In Figure 265 are tabulated the strains due to both vertical load and wind pressure, the last column giving the actual result, that is, their sum or difference, as the case may be. In

TABLE OF STRAINS OF ROOF.

Name of piece.	Standing load.	Wind load.	Result.
B. J.	+ 76000	+ 8000	+ 84500
G. K.	+ 61000	+ 8500	+ 69500
D. L.	+ 46000	+ 8500	+ 54500
E. M.	+ 46000	+ 15500	+ 61500
F. N.	+ 61000	+ 15500	+ 76500
G. P.	+ 76000	+ 15500	+ 91500
J. K.	+ 12300	+ 11400	+ 23700
K. L.	+ 12300	+ 11400	+ 23700
L. M.	— 52000	— 15000	— 67000
M. N.	+ 12300		+ 12300
N. P.	— 12300		+ 12300
R. Q.	— 24000	— 12000	— 36000
G. N.	— 23500	+ 2000	— 21500
J. O.	— 49000	— 11000	— 60000
Q. O.	— 23500	+ 6000	— 17500
P. O.	— 49000	+ 7500	— 41500

Fig. 265.

Figures 266 and 267 we reach exactly the same results, by combining both vertical load and wind pressure in one diagram.

In designing the truss, we must remember that were the wind

blowing from the right, the corresponding members of each half of the truss would exchange their respective strains, we must therefore design each such pair of (right and left) members to resist the larger strain.

Detailing the truss.

Figure 268 gives the drawing of truss and detail of joints.

In designing the truss we find that the heaviest strains exist when the wind blows with the exception of the horizontal central tie $Q\,O$ which has its largest tension, when there is no wind; we select accordingly, the heaviest strains for each member and proceed to design the parts.

Take heaviest strains.

For the tie-rods we require areas, as follows:

Size of rods.

Vertical rod $L\,M = \frac{67000}{12000} = 5{,}58$ square inches or a diameter of say $2\frac{5}{8}$ inches.

Inclined rod $K\,Q = \frac{36000}{12000} = 3$ square inches or a diameter of say 2 inches.

Lower inclined rod $J\,O = \frac{60000}{12000} = 5$ square inches or a diameter of say $2\frac{1}{2}$ inches.

Horizontal rod $Q\,O = \frac{23500}{12000} = 2$ square inches or a diameter of say $1\frac{5}{8}$ inches.

By placing sleeve nuts (or turn buckles), as shown, we can tighten up the truss at any time.

We next design the struts to stand 23700 pounds compression each. They are 13 feet 4 inches long or 160 inches long. After several trials we decide to use two 3 inches x 3 inches x $\frac{1}{2}$ inch tees, placed one inch apart, back to back. The weakest way in compression will evidently be with the neutral axis parallel to the web. From Table XXIV, we find their square of radius of gyration for this axis to be $\rho^2 = 0{,}42$ and their area of cross-section $= 2{,}75$ square inches each or $5\frac{1}{2}$ square inches for the two.[1]

Size of struts.

[1] It should be noted that no matter how many tees were placed along the *same axis in the same direction* we should have the same ρ^2: for while each additional one would increase the area, and the moment of inertia i, and the moment of resistance r, the square of the radius of gyration ρ^2, being simply the quotient of the inertia divided by the area, would remain constant, no matter how many tees were used.

We have then for the safe compressive stress in these tees to resist the 23700 pounds compression, (see Formula 3):

$$w = \frac{12000.5\frac{1}{2}}{1+\frac{0,00005.160^2}{0,42}} = 23322 \text{ pounds.}$$

which is near enough. It should be noted that we consider the strut as a column with pin ends. The ends have got separately forged

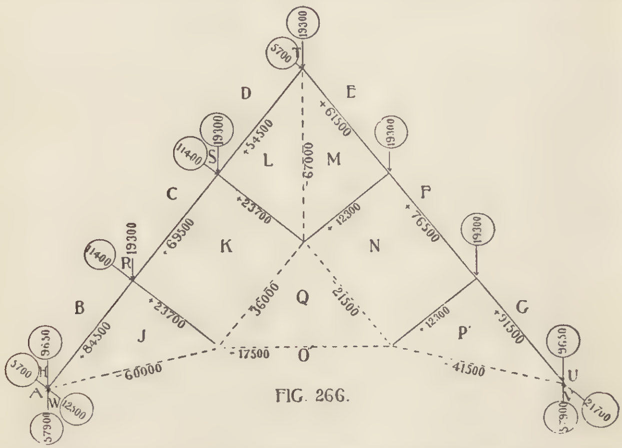


FIG. 266.

pieces, bearing on the pins and riveted between the ends of struts with eight $\frac{5}{8}$ inch rivets.

The rivets were determined, as follows:

Rivets in strut ends. Bearing area of each rivet $\frac{5}{8}$ inch x 1 inch $= \frac{5}{8}$ square inch or $= \frac{5}{8}.12000 = 7500$ pounds, value per rivet.

Shearing area of each rivet $= 0,3068$ square inches, there being two shearing areas to each rivet, we have 0,6136 square inches resisting shearing, or shearing value of each rivet $= 0,6136.8000 = 4908$ pounds.

For safe bending-moment on each rivet, we have from Table I, section No. 7 and from Formula (18) transposed:

$$m = \frac{1}{1}\frac{1}{4}.\left(\frac{5}{16}\right)^3.15000 = \frac{1}{42}.15000 = 360 \text{ pounds-inch.}$$

Remembering to use the larger value $\left(\frac{k}{f}\right) = 15000$ pounds for bending-moment on pins or rivets, we could have read the same

results for bearing, shearing and bending moment directly from Tables XXXV and XXXVIII.

The actual bending-moment on all the rivets would be Formula (21):

$$m = \frac{23700.1}{8} = 2962 \text{ pounds-inch.}$$

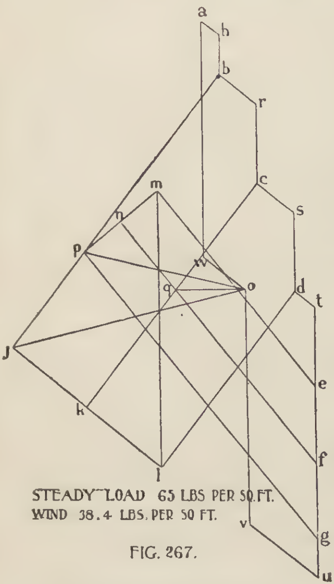


FIG. 267.

We require therefore, to resist bending

$$\frac{2962}{360} = 8{,}2 \text{ or say eight rivets.}$$

This being more than required for either bearing or shearing determines the number of rivets to be used.

Calculating the pins. We next decide on size of pin used. It is evident that the joints will be similar as regards arrangement.

The largest rod will be central on pin, each side of it will be another rod and outside of this the strut. We can also readily see that the joint at foot of vertical rod $L\ M$ will be the most severely taxed, and as it is usual to use the same size pin throughout a truss we will calculate for this joint. It is, also, evident that we need calculate only for the strains in the vertical line, as these will be the heaviest.

In order to reduce the bending-moment on the pin, we reduce the head or eye-part of vertical rod to 2 inches thick, this will make a compression of $\frac{67000}{2} = 33500$ pounds per inch thickness and require a diameter of pin $= \frac{33500}{12000} = 2{,}8$ or say $2\frac{13}{16}$ inches diameter, (which was the nearest regular size of pin made by the mill who had the contract).

The same result could have been read directly from Table

XXXVI. For the rod $K\,Q$ we require a width of bearing

$$= \frac{36000}{2\frac{15}{16}.12000} = 1 \text{ inch.}$$

For each strut we require a width of bearing

$$= \frac{23700}{2\frac{15}{16}.12000} = 0{,}67 \text{ or say } \tfrac{3}{4} \text{ inch.}$$

The single shearing area of pin being about $6\frac{3}{4}$ square inches or $= 6\frac{3}{4}.8000 = 54000$ pounds, there can evidently be no danger from that quarter.

We now calculate the bending-moment, we first obtain the vertical resultants of all the strains, by laying off each strain along its respective line of action at a certain scale, and then measuring the length of its projection along the vertical line. The vertical projection of the 36000 pounds-tension is 28000 pounds pulling downwards; the projection of the 21500 pounds-tension is 16500 pounds pulling downwards; the projection of the 23700 pounds-compression is 15000 pounds pushing downwards; the projection of the 12300 pounds-compression is 7500 pounds-compression pushing downwards; the sum of all of these is $28000 + 16500 + 15000 + 7500 = 67000$ pounds downward. Resisting this we have the 67000 pounds of upward pull.

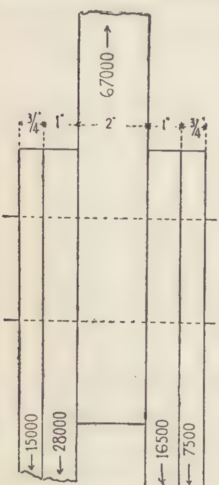


FIG. 269.

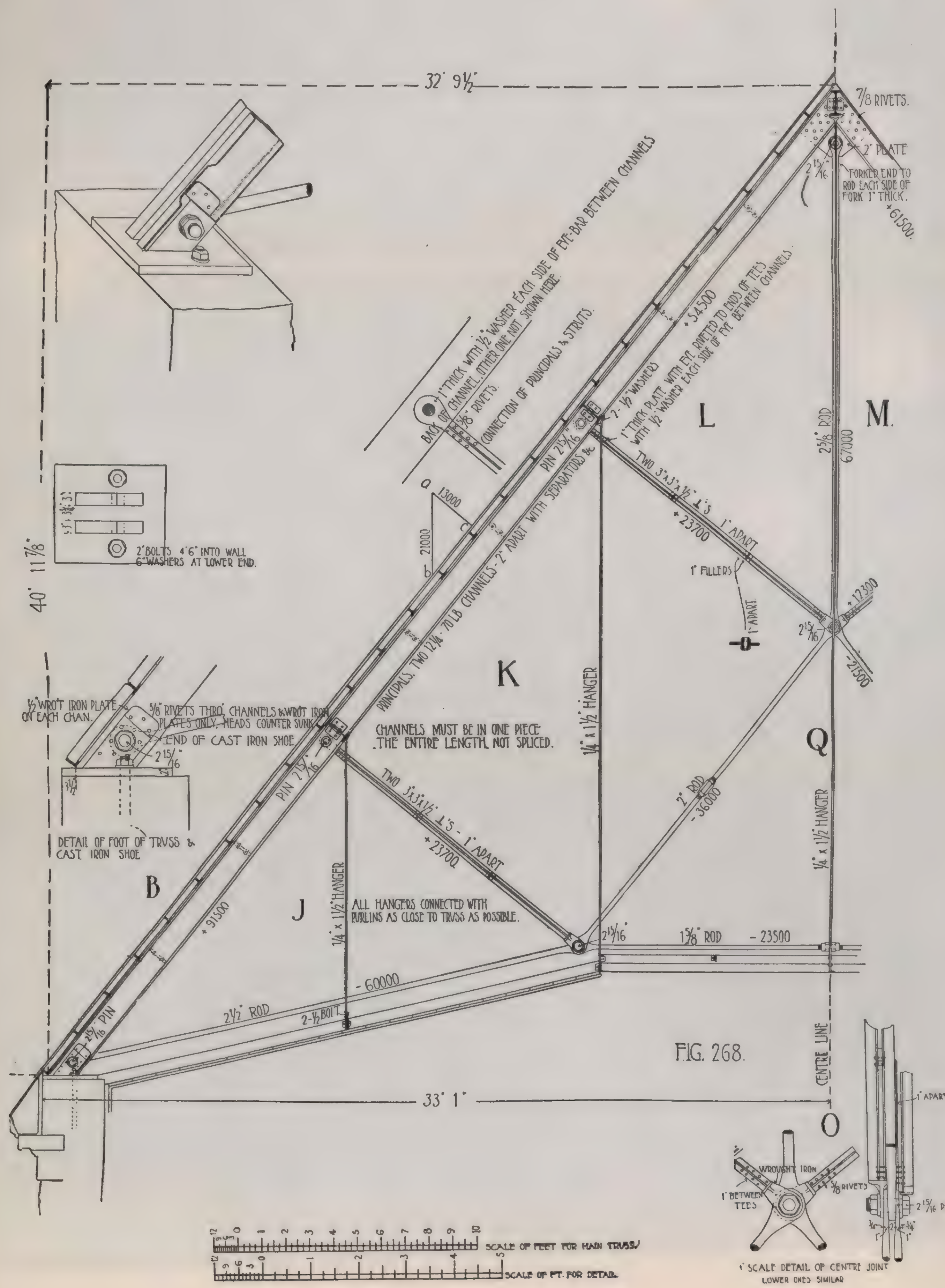
We now lay out, Figure 269, the pin, and find that we have a double lever arrangement; the fulcrum being the 2 inch wide 67000 pounds strain. To the left of this we have a lever $1\frac{3}{4}$ inch wide loaded with two loads, one 28000 pounds one inch wide and one beyond 15000 pounds $\frac{3}{4}$ inch wide. To the right we have lighter loads, the heaviest bending-moment will therefore be on the left side, or:

$$m = \tfrac{1}{2}.28000 + 1\tfrac{3}{8}.15000$$
$$= 34625 \text{ pounds-inch.}$$

The safe-bending-moment on a $2\frac{15}{16}$ inch pin is:

$$m = \tfrac{11}{14}.(1\tfrac{15}{32})^3.15000 = 37500 \text{ pounds-inch.}$$

The pin is therefore safe. The same result could have been read off directly from Table XXXIX.



32' 9½"
40' 11⅞"
33' 1"
⅞ RIVETS.
2" PLATE
FORKED END TO ROD EACH SIDE OF FORK 1" THICK.
+61500.
1" THICK WITH ½" WASHER EACH SIDE OF EYE-BAR BETWEEN CHANNELS
BACK OF CHANNEL, OTHER ONE NOT SHOWN HERE.
⅝" RIVETS
CONNECTION OF PRINCIPALS & STRUTS.
2-½" WASHERS
1" THICK PLATE WITH EYE RIVETED TO ENDS OF TEES WITH ½" WASHER EACH SIDE OF EYE BETWEEN CHANNELS.
+54500
2" BOLTS 4'6" INTO WALL
6" WASHERS AT LOWER END.
PRINCIPALS, TWO 12¼" 70 LB CHANNELS 2" APART WITH SEPARATORS &c.
PIN 2 15/16"
TWO 3"x3"x½" T'S 1" APART
+23700
1" FILLERS
1" APART
+12300
-21500
CHANNELS MUST BE IN ONE PIECE THE ENTIRE LENGTH, NOT SPLICED.
¼" x 1½" HANGER
½" WROT IRON PLATE ON EACH CHAN.
⅝" RIVETS THRO' CHANNELS & WROT IRON PLATES ONLY, HEADS COUNTER SUNK.
END OF CAST IRON SHOE.
DETAIL OF FOOT OF TRUSS & CAST IRON SHOE
2" ROD
-36000
TWO 3"x3"x½" T'S - 1" APART
+23700
ALL HANGERS CONNECTED WITH PURLINS AS CLOSE TO TRUSS AS POSSIBLE.
+91500
1⅝" ROD
-23500
-60000
2½" ROD
2-½" BOLT
2 15/16" PIN
2⅝" ROD
67000
B
J
K
L
M
Q
O
CENTRE LINE
FIG. 268.
WROUGHT IRON
⅝ RIVETS
1" BETWEEN TEES
2 15/16" PIN
1" APART
1" SCALE DETAIL OF CENTRE JOINT
LOWER ONES SIMILAR
SCALE OF FEET FOR MAIN TRUSS.
SCALE OF FT. FOR DETAIL.

FIG. 268.

Back of
Foldout
Not Imaged

Size of Main Rafter.

We next design the main rafter; as this is to be in one length, we design it only for lower panel (91500 pounds-compression) and the other panels will be, of course, too strong. Besides the compression we will have a transverse strain or bending-moment on rafter, per square foot, as follows:

Wind	$=38{,}4$
3 inch blocks,	$=16$
Slate	$=10$
Iron tees (say)	$=\ 5$
Total	$=69{,}4$ pounds,

or allowing for weight of rafter, say, 70 pounds per foot. The rafter lengths being 17 feet and $17\frac{1}{2}$ feet apart, we have total uniform load $u=17.17\frac{1}{2}.70=20825$ or say 21000 pounds. We now draw any-

Transverse strain on rafter.

where along rafter a vertical line $ab=21000$ pounds; then draw ac normal to rafter; it scales 13000 pounds, therefore the actual transverse load on rafter is 13000 pounds.

The required moment of resistance to resist this load will be Formula (18)

$$r=\frac{13000.(17.12)}{8.12000}=27{,}6$$

By reference to Table XXI, we find we require one $12\frac{1}{4}$ inch 80 pounds channels to take care of the transverse strain. We also note that the area of channel is 8 square inches, which is about what we need *additional* for resisting the compression. We will decide then to use two $12\frac{1}{4}$ inches 80 pounds channels.

The square of the radius of gyration is $\varrho^2=21{,}10$

Compression on rafter.

The area of the two will be 16 square inches, but as just one-half of their total moment of resistance r is needed to resist transverse strain, we will have only 8 square inches left of the total area to resist compression. The column is 17 feet or say 204 inches long. One end will be a pin end, the other a milled or planed end with fair bearing. We have then for safe compressive load Formula (3):

$$w=\frac{8.12000}{1+\frac{204^2.0{,}000033}{21{,}1}}=90300 \text{ pounds},$$

or near enough to pass as safe. If the transverse strain had required

say a moment of resistance of $r = 35$ we should have had left to resist compression of the total area 16 square inches only,

$$= 16 - \left(\frac{35}{2.27,7}\right).16$$

$=$ (about) 6 square inches instead of 8, which (in such a case), would not be enough and would require a heavier channel.

Bearing of rafter webs on pins.

The thickness of our channel web we see from Table XXI, is 0,39 inches, or two webs $= 0,78$. The safe bearing on pin will therefore be $= 0,78.2\tfrac{1}{16}.12000 = 27495$ pounds. The web of channel will therefore need thickening at the shoe pin, but not at the two others. At the apex the pin does not bear on channels, but is connected indirectly by a 2-inch thick plate.

Reinforce plates to web.

At the shoe the channels have planed ends and rest directly on the shoe, the strain therefore against their webs, will be 60000 pounds, due to the rod trying to tear the pin out. The webs take care of 28200 pounds of bearing, leaving $60000 - 28200 = 31800$ pounds, to be transferred by rivets to thickening or reinforce plates. Of this each side takes care of

$$\frac{31800}{2} = 15900 \text{ pounds.}$$

The thickness required for each plate, is therefore

$$\frac{15900}{2\tfrac{1}{16}.12000} = 0,42 \text{ or say } \tfrac{1}{2} \text{ inch each.}$$

The number of rivets required must next be settled. We use $\tfrac{5}{8}$ inch rivets:

The channel being thinner than the plate determines the bearing value for each, or

Rivets in reinforce plates.

$$= 0,39.\tfrac{5}{8}.12000 = 3000 \text{ pounds,}$$

or

$$\frac{15900}{3000} = 5,3 \text{ or say six rivets required for bearing.}$$

The rivets are in single shear, their area $= 0,3068$ and their value $= 0,3068.8000 = 2454$ pounds, and we require $\frac{15900}{2454} = 6,5$ or say seven rivets.

For bending-moment each rivet is a single lever held by the half-inch plate, projecting 0,39 inches and uniformly loaded on free end

with its share of the 14100 pounds carried by each channel web, the total bending-moment will therefore be from Formula (25).

$$m = \frac{14100.0{,}39}{2} = 2750 \text{ pounds-inch.}$$

The safe bending-moment on a $\frac{5}{8}$ inch rivet we previously found to be 360 pounds-inch, and require therefore

$$\frac{2750}{360} = 7{,}7 \text{ or say eight rivets.}$$

The disposition of rivets around the pin, however, requires nine rivets.

We now go to the apex point. We use here a 2-inch plate, its bearing on pin must be all right, as we made the other end of vertical rod 2 inches thick. The upper end of rod we make forked, each side 1 inch thick, and 2 inches between to admit plate.

Pin plates at apex.

The plate is so large that there is evidently no danger of the pin shearing out.

For the rivets we can see it will require a large number and therefore decide to use larger or say $\frac{7}{8}$ inch rivets.

Number of rivets.

The bearing value on two webs of each rivet will be $= \frac{7}{8}.2.0{,}39.12000 = 8190$ pounds.

The (double) shearing value of each rivet will be

$$= 2.0{,}6013.8000 = 9620 \text{ pounds.}$$

The safe bending-moment on each rivet will be

$$= \tfrac{11}{14}.\left(\tfrac{7}{16}\right)^3.15000 = 987 \text{ pounds-inch.}$$

If now we consider that the two rafters have planed ends and butt fairly against each other, taking up the thrust from each, the rivets need only take care of the vertical down pull 67000 pounds, *all* of the rivets taking a share. In that case we should need for bearing $\frac{67000}{9360} = 7{,}2$ rivets, for shearing less, and for total bending-moment, remembering that the channel backs are 2 inches apart and that the rivets are beams supported at both ends and uniformly loaded, from Formula (21)

$$m = \frac{67000.2}{8} = 16750 \text{ pounds-inch.}$$

Therefore number of rivets required to resist bending-moment $= \frac{16750}{987} = 16{,}9$ or say nine each side of joint.

The plate, however, requires ten each side for even distribution.

If the channels do not butt fairly the plate will have to transfer the thrust from one to the other. This thrust will be equal to the *horizontal resultant* of compression on rafter at apex which is 54500 pounds. Its horizontal projection measures 34000 pounds. The rivets each side of joint must take care of this strain and the plate be large enough not to crush under it. We need not calculate any in this case, however, as this strain is just about one-half of the strain for which the total number of rivets were proportioned.

Designing shoe-plate.

We next design the shoe. It is a flat cast-iron plate 2 inches thick, and 28 inches by 24 inches with two flanges each 3 inches thick to receive the pin. As the channels bear directly on the plate, the only strain on the flanges will be due to the pin trying to shear its way out, the strain being 60000 pounds. Resisting this there are two areas to each of the two flanges, each area 3 inches x 3 inches, or a total area $=4.3.3=36$ square inches, the actual stress is therefore

$$\frac{60000}{36}=1667 \text{ pounds per square inch, which is safe.}$$

Bearing area.

Include wind reaction.

The wall must carry the whole load of truss, each reaction is 57900 pounds due to vertical load. To this must be added the vertical resultant (or projection) of the largest wind reaction 21700 pounds. Its vertical projection measures about 14000 pounds making the total vertical reaction 71900 pounds. The area of plate is $28 \times 24=672$ square inches, therefore compression per square inch on brick-work

$$=\frac{71900}{672}=108 \text{ pounds, which is safe.}$$

Rockers or rollers.

Had this truss been of larger span, we should have had to place one shoe either on a rocking saddle, or else on rollers. If the latter there should be sufficient rollers, and they should be of large enough diameter not to *indent* the plate, and to roll back and forth freely. It is usual to put upward flanges all around the bottom plate and downward flanges all around the upper plate to hold the rollers in place between them. The flanges should be less than radius of rollers, so as not to meet. The foot of girder must be secured against yielding sideways.

The size of rollers is determined by the following formula:

Formula for rollers.

$$w=l.750.\sqrt{d} \qquad (132)$$

Where w = the safe load, in pounds, on *each* roller.

Where l = the length, in inches, of each roller.

Where d = the diameter in inches, of each roller, if of steel or wrought-iron and rolling between cast-iron plates. If between wrought-iron plates add 25 per cent to w.

Rollers should be used, (under one shoe only,) where trusses *in-doors* are over eighty feet span, or *out-doors* if span is over sixty-five feet.

Example.

A truss of one hundred feet span has a reaction at each end of 95000 *pounds. The shoe-plate is* 20 *inches wide the long way of rollers, and rollers are* 1 *inch in diameter. How many rollers are required?*

Each roller will safely carry from Formula (132)

$$w = 20.750.\sqrt{1} = 15000 \text{ pounds},$$

we shall require therefore

$$\frac{95000}{15000} = 6{,}3 \text{ or say 7 rollers.}$$

Of course, where rollers are used some arrangement must be made in the cornice to allow for the movement due to expansion and contraction of the truss; or if the roof is continuous a slip-joint must be provided in the roof itself, the detail of which will depend upon the local circumstances.

CHAPTER XIII.

COLUMNS.

FIG. 270

FIG. 271

FIG. 272

FIG. 273

FIG. 274

THE Formula (3) on page 24 of Vol. I, is, of course, applicable to every kind and shape of column. The square of the radius of gyration used in this formula, can be found by any of the formulæ given in Table I; or if the shape of the column is so unusual that it is not given in Table I, the moment of inertia (i) of the cross-section of the column can be found, and this divided by the area (a) is equal to the square of the radius of gyration, see page 9 of Vol. I.

To find radius of gyration.

In Table II are given the different values of n for cast and wrought iron, steel, wood, stone and brick; also the variations in

this value for "*pin*," that is, rounded or rough bearings at ends, and for "*smooth*," that is, turned, planed or smoothed off bearings at ends. These different values have been arrived at largely by experiments, but the reason why the end bearing affects the strength of column can readily be seen in Figures 270 to 274.

Why smooth end stronger.

In Figure 270 we have a column with smooth ends between two forces crushing towards each other; as a result the column tends to bend, in this case to the right. The bending of the column will, of course, tip its ends, as these are at right angles to the (dotted) longitudinal axis of the column; as a result the further crushing is taken entirely at the edges of the ends, or at points A and B, marked with arrows in the figure.

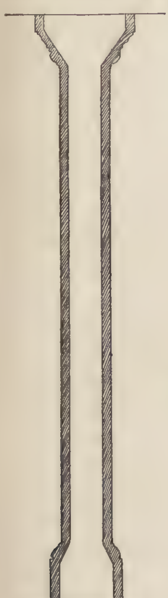


FIG. 275

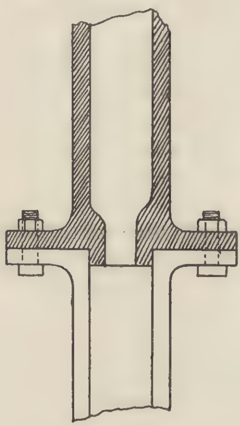


FIG. 276.

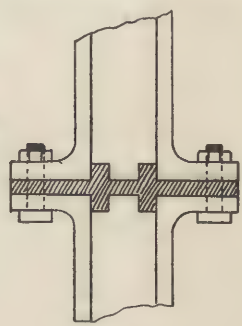


FIG. 276 A.

It will be readily seen that the least pressure at A or B tends to bring the ends back to the horizontal plane and consequently to straighten the column again. For this reason it is that columns with smooth end bearings give way the same as columns with fixed end bearings, as shown in Figure 271. If on the other hand the ends of the column are rounded, as shown in Figure 272, the effect of the crushing at A and B is to constantly increase the bending of the column, as the point of contact against the column simply slips around the circular ends, as the column bends more and more. Such

columns break as one curve instead of the triple curve shown in Figure 271. Where there is a pin bearing, as in Figure 273 the effect is, of course, the same as for rounded ends. Rough ends are considered the same as rounded ends, on account of the danger of some roughness or projection on the end of a bearing, which would greatly increase the tendency to bend, as shown at *A* and *B* in Figure 274. For this same reason *wedging* of joints of columns, should *never* be allowed.

Only those columns should be considered as having smooth ends, where the entire bearing end is perfectly smooth, and forms a true and perfect plane at right angles to the longitudinal axis of the column. In iron and steel columns the ends have to be "planed" or "turned" off, both of which are done by machinery. Planing is, as its name implies, a passing back and forth of a sharp metal planer which removes the surface iron, little by little. Turning is the same as planing but the motion of the planer is circular. Turning is done on circular columns, particularly where there are lugs or projections beyond the bearing surfaces, which would be in the way of a straight planer.

Planing and turning.

Cast-iron columns are usually made of circular cross-section and hollow. This is the cheapest cross-section there is for any column, and the metal will do more work per square inch, the thinner (within reason, of course,) the shell is made.

Shapes of cast columns.

Cast-iron columns are also frequently made square in cross-section, and hollow, which does not make a badly proportioned column. All other shapes, however, are bad, and should only be resorted to in unusual circumstances. Such shapes, for instance, are rectangular and hollow, one side (or diameter) being shorter than the other, in this case the neutral axis, when computing the square of the radius of gyration, should be taken parallel to the long side; then there are **H** or **I** shapes, **T** or **Z** shapes, etc., all bad and weak. Sometimes a column is made square or rectangular but with only three sides. The column will be greatly strengthened, if at intervals there can be cast on the fourth side a connecting bar.

All hollow castings should be drilled, as already explained, to ascertain their thickness, also near their base to allow any water to escape, which might otherwise freeze and burst the casting. Brackets,

and other projections should, by preference, be cast on the column, rather than secured to it by bolts or tap-screws. Of course, they can't be riveted to cast-iron. They should, as far as possible, be of same thickness as shell.

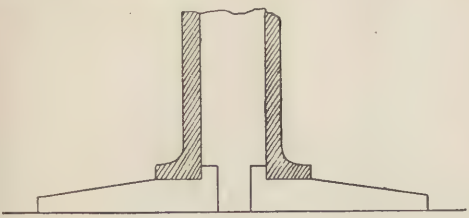


FIG. 277.

Caps and bases. Where there are heavy capitals, bases or other mouldings or ornamental work that greatly increase the thickness of shell, they had better be made separately and slipped on afterwards, and then secured by tap-screws with countersunk heads.

The bearing should always, if possible, be vertically under and over the shell, not flanged out as shown in Figure 275.

Ends of columns are usually flanged out for bolting together, in which case the angles should be well rounded. It is also well to cast on one of the columns a lug as shown on the upper, etched column in Figure 276.

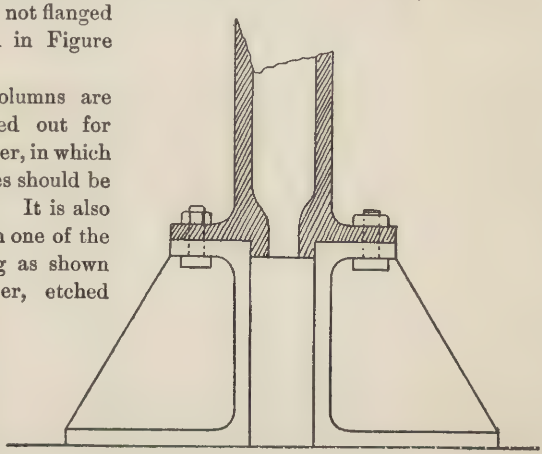


FIG. 278.

To prevent fire spreading up through columns, some building laws require solid plates at all joints; in such cases they should be made

as shown in etched part of Figure 276*a* with upper and lower lugs, and the columns bolted together through the plate.

Bottom plates. Bottom plates, which usually have to spread the weight are made, as shown in Figure 277; as they are often very large, and necessarily therefore quite thick, metal can be saved by gradually reducing the thickness towards the edges, as shown. In such plates a hole should be cast in the centre, to relieve the strain on the plate, when cooling and thereby avoid warping.

No bolts are needed where a bottom plate is used, as shown in Figure 277, as there is no possibility of tipping. Where the spread of base plate has to be very great, flanges are cast on same as shown in section in Figure 278 and in plan in Figure 279. Of course, the flanges can be more numerous. All parts of such a base should be of even thickness. If the casting is unusually large and unwieldy it can be made in two parts, as shown in Figure 279, but in such cases great care should be exercised with the foundation or substructure, to avoid one-half settling away from the other half.

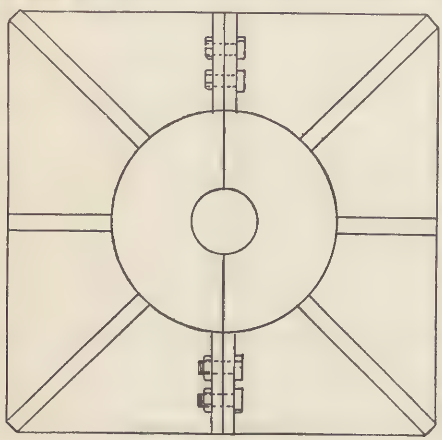


FIG. 279.

This can be done by thick granite stones, or additional iron plates under the main bottom plate.

The calculation of hollow circular columns is very tedious, particularly as one has to guess at the size and then calculate the strength, often involving several calculations for a single case.

Explanation of Tables XLV to XLVIII. Tables XLV, XLVI, XLVII and XLVIII, have therefore been prepared by the writer to take the place of these calculations, for cast-iron hollow circular columns.

Across the top of the tables, in the horizontal line, are given the lengths in feet of the columns. Down the left side in the vertical line are the *safe* loads in tons for each particular shape. The curved lines each represent a hollow circular section, and at the end of each curve is given first, the diameter of column, second the thickness of shell, third the area of cross-section. The latter enables one to pick out the cheapest section from any of the tables, by selecting any curve below and to the right of the one found — which has a smaller area of cross-section. Sometimes by reference to a subsequent table a still cheaper section can be found.

The tables have been calculated for cast-iron, according to Formula (3) ; the columns are supposed to have smoothly dressed ends and true bearings. The value used for $\left(\frac{c}{f}\right)$ was 15000 pounds. Any one desiring to use any other value, need only proportion the vertical column of safe loads in tons accordingly; thus, for 12000 pounds we should take $\frac{4}{5}$ of the loads as safe, or we could add $\frac{1}{4}$ to the load to be carried by the column, and find from the tables the diameter, etc., of column necessary to carry the increased load.

Table XLV gives columns from 3″ to 7″ diameter of different thicknesses, and from 5 feet to 12 feet long. Table XLVI gives columns from 8″ to 10″ diameter of different thicknesses, the 8″ and 9″ columns from 8 feet to 15 feet long, the 10″ columns from 10 feet to 20 feet long.

Table XLVII gives columns from 11″ to 13″ diameter, and Table XLVIII from 14″ to 16″ diameter, in both, of different thicknesses and from 10 feet to 20 feet long.

An example will best illustrate the use of table.

Example I.

What is the safe load on a hollow, circular, cast-iron column, with turned ends, 18 *feet long,* 11″ *diameter and* $1\frac{1}{2}$″ *thick?*

Example of use of tables.

Columns of 11″ diameter are given in Table XLVII to which we turn, we find the curve marked 11 — $1\frac{1}{2}$ — 44,8 cuts the vertical line 18 about two-fifth way down between the horizontal lines 150 and 155, our column will therefore safely carry 152 tons.

Had we calculated our column by Formula (3) we should have

had from Table I, section No. 8 for

$$\varrho^2 = \frac{5\frac{1}{2}^2 + 4^2}{4} = 11,56$$

and for $l^2 = (18.12)^2 = 216^2 = 46656.$

Inserting the values in Formula (3) we have the safe load

$$w = \frac{44,8.15000}{1 + \frac{46656.0,0003}{11,56}} = \frac{672000}{1 + 1,2108} = 303962 \text{ pounds,}$$

or, 151,98 or say 152 tons same as found from table.

Now, is this the cheapest section for that load and length of column. Its area is 44,8 square inches. We now pass to the next curve below and to the right, it is 13 — 1 — 37,7 or a 13″ diameter column, 1″ thick will be cheaper, as its area is only 37,7 square inches. Then, too, it is stronger, for at 18 feet long it will carry 158 tons cutting the vertical line 18 three-fifth way between 155 and 160. By passing further along, we find also that the 12″ diameter $1\frac{1}{4}$″ thick column of 42,1 square inches area will be stronger, carrying 161 $\frac{1}{2}$ tons safely at 18 feet of length.

By referring to the next Table XLVIII, we find some much cheaper sections, than any of these. For the 14″ diameter, $\frac{3}{4}$″ thick column, has only 31,2 square inches area of cross-section, but we find it carries safely only 143 tons and therefore will not answer. However the 14″ diameter 1″ thick column, has only 40,9 square inches of area and carries 185 tons, and would answer therefore, provided of course, larger diameter is not objectionable. Or, cheapest of all would be the 15″ diameter, $\frac{3}{4}$″ thick column which has only 33,6 square inches of area and carries 161 tons. The 16″ diameter, $\frac{3}{4}$″ thick column would also be economical having only 35,9 square inches of area and carrying 180 tons at 18 feet of length.

Shapes of wrought-iron columns.

In wrought-iron construction, any number of shapes of columns are used, from plain flat bars, to the most elaborate combinations of the different shapes rolled, most of which are given, with their areas, squares of radius of gyration, etc., in Tables XIX to XXV. In Figures 280 to 287 are given a few combinations, which are frequently used for columns. Besides those shown there are frequently used combinations of I-beams and channels, or of angles and plates formed in the shape of plate and box-girders stood on end.

Figure 280 shows two channels latticed together. A plate might

be used in place of latticing if the channels were placed as shown in Figure 281, but in that case the interior would not be accessible for painting.

Figure 281 is more easily riveted up than 280, but is not quite as strong.

Figure 282 shows an elevation with wrought-iron base, and Figure 283 the side view of base. Architects are apt to use cast bases with

FIG. 280.

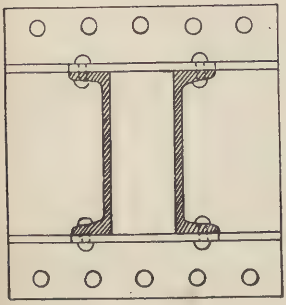


FIG. 281.

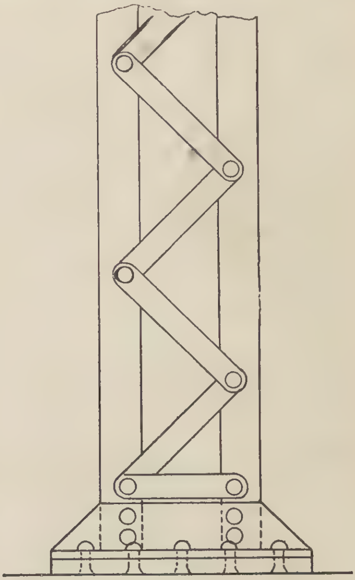


FIG. 282.

wrought-iron work, but as a rule a wrought-iron base can (and should) be designed, which will not only be better adapted to wrought-iron construction, but will be cheaper and stronger, and has the merit that it can be riveted fast to the column or other construction.

Figure 284 shows a column made of four **Z** irons, with central plate. It has the great merit of being a strong column, and though of five parts, it requires only two lines of rivets. This column adapts

itself excellently to building walls, as the horizontal wall girders, and floor girders are easily attached to it, and it readily holds the "filling-in walls" in place, without the use of anchors. Then too, it can be easily covered with fireproof blocks. The writer a number of years ago erected in New York City, a fireproof office-building, ten stories high above the sidewalk, with only twelve inch thick brick walls all the way up, by using these columns combined with horizontal girders in all the walls. At each floor level the vertical webs of the floor girders run *between* the Z irons; the vertical plates of columns butting against web plate ends both top and bottom. Joints of Z irons should *not* be at floor levels, and they should "break joints." When more than one thickness of plate is used vertically between the Z bars, the plates where joined should "break joints."

Of course, in such construction, proper precautions must be taken to prevent the building from collapsing under wind pressure. In the case above referred to, in order to avoid cross-partitions, the

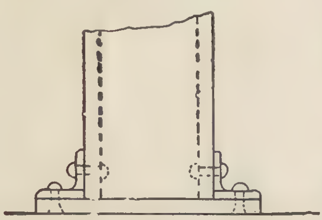


FIG. 283.

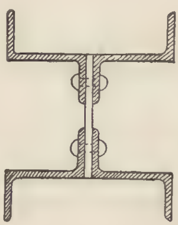


FIG. 284.

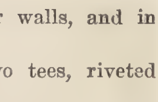


FIG. 285.

wind-bracing was all done in the front and rear walls, and in the floor levels.

Figure 285 gives a combination column of two tees, riveted together with separators.

Figure 286 is made of four angle bars latticed together, a very light, but strong column.

Figure 287 consists also of four angle bars, which adapt themselves more readily to fireproofing, and require less riveting, but the column is not nearly as strong as the previous one. In the last case separators are used in place of lattice bars.

Explanation of Table XLIX.

It would be quite impossible to give any curve tables for wrought iron construction, but Table XLIX will greatly facilitate the calculation. It will be necessary in each case to find only the ratio of the length of column (in inches) divided by the radius of gyration, or the square of the length, divided by the square of the radius of gyration, and look up the value per square inch of cross-section, according to the

condition of ends of column, and the assumed safe value for $\left(\frac{c}{f}\right)$. The table is calculated respectively for 8000, 10000 and 12000 pounds per square inch values for $\left(\frac{c}{f}\right)$. In buildings use the value 12000 for wrought-iron.

To find the ratio look up the value of the square of the radius of gyration in the tables, or if it is not given, find the moment of inertia according to rules given in Table I, or on page 10, and divide by the area, see page 9.

It should be borne in mind that where pieces are doubled or their number increased *along the same neutral axis*, the moment of inertia will be doubled or increased accordingly. But the square of the radius of gyration will remain constant, as it simply represents a

FIG. 286.

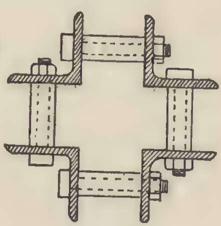


FIG. 287

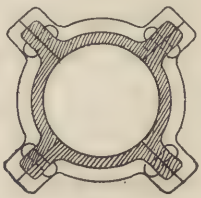


FIG. 288

ratio, and the area and moment of inertia increasing in the same amount, their ratio, which gives the square of the radius of gyration, will, of course, remain constant. In some cases, it will be easier to find the radius of gyration, instead of its square, and in such cases the second column in Table XLIX should, of course, be used.

An example will best illustrate the use of Table.

Example II.

Example of use of Table XLIX.

A flat eye-bar of wrought-iron $1\frac{1}{2}''$ x $6''$ in a truss, is liable at times to be under compressive stress, what will it safely stand? The bar is $5'$ $6''$ long from centre to centre of eyes.

From Table I, section 2, we have

$$\varrho^2 = \frac{(1\frac{1}{2})^2}{12} = 0{,}19$$

the area will be

$$a = 1\tfrac{1}{2}.6 = 9 \text{ square inches,}$$

and

$$l^2 = 66^2 = 4356$$

The ends being eye-bars, we use, of course, in Table II, the value n for "both ends pin ends," or

$$n = 0,00005$$

Inserting the values in Formula (3) we have for the safe compressive strain,

$$w = \frac{9.12000}{1 + \frac{4356.0,00005}{0,19}} = 50322$$

Now had we used Table XLIX we should have had the ratio

$$\frac{l^2}{\varrho^2} = \frac{4356}{0,19} = 22926$$

The nearest value to this in the first column of the Table is 22500 and under the heading "both ends pin ends" for a value of $\left(\frac{c}{f}\right) =$ 12000 pounds, we find 5655 which is the safe load per square inch on our bar, or the total safe strain,[1]

$$w = 9.5655 = 50895 \text{ pounds.}$$

Which closely approximates the above result.

In our case it would have been easier to use the second column of Table XLIX, we should have had

$$\varrho = \sqrt[2]{\frac{(1\frac{1}{2})^2}{12}} = \frac{1\frac{1}{2}}{\sqrt[2]{12}} = \frac{1,5}{3,464} = 0,433$$

and for the length

$$l = 66$$

therefore the ratio

$$\frac{l}{\varrho} = \frac{66}{0,433} = 152$$

The nearest value to this in the second column of Table XLIX is 150 which would give the same result as before.

Phœnix Columns. There are several patent wrought-iron columns made, of which the "Phœnix" column is undoubtedly the best.

It is made up of from four to eight segments, riveted together.

[1] (Obtained by multiplying by the area of bar.)

Each segment somewhat resembles a channel with the web bent to a segment of a circle, instead of being straight.

Figure 288 shows one of the smaller columns, made up of four segments. For heavier columns each segment is rolled thicker, as shown in the figure in outline. When it is necessary to have very heavy columns flat pieces are inserted between each flange, as shown in Figure 289. These columns can be readily covered with fire proof blocks to make a circular finish in buildings, and are largely used both for this reason, and on account of their great strength, (owing to all the metal being near the outer edge).

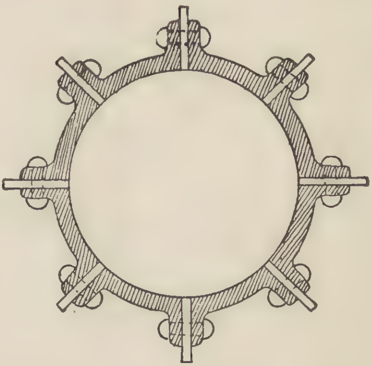


FIG. 289.

When calculating the load on a column it should be borne in mind, that if the girder or beam is *continuous* over the column, the loads will be equal to the reactions as given in Table XVII on pages 218 and 219.

Load on Column. If the girders or beams overhang columns and are built into the wall at the other end (such as gallery beams for

Gallery Beams. instance) the respective loads on the column and wall and upward pressure on wall can be found from Formulæ 116 to 119 inclusive.

If the load on the overhang is uniform its reaction would be the same as a similar amount of load concentrated at one-half the span of overhang.

If the beams or girders are inclined they can be calculated the same as already explained (in the previous chapter), when calculating transverse strains on rafters; and the amount of anchoring necessary to prevent pulling out, can readily be found by obtaining the horizontal thrust by the graphical method, as already explained.

TABLE XLV.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

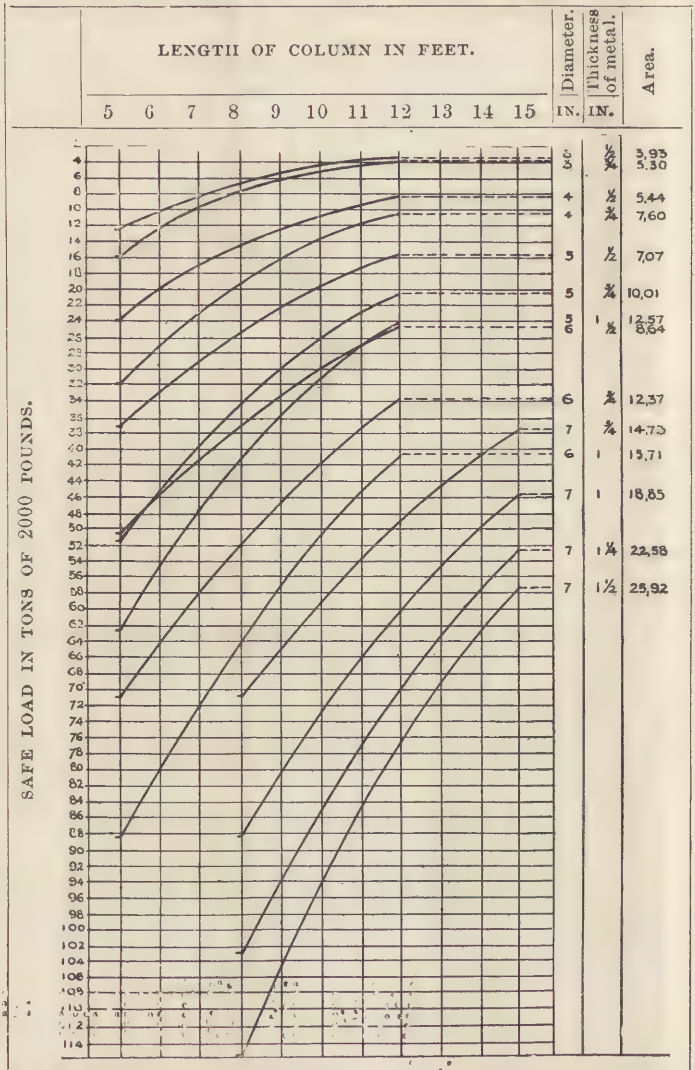
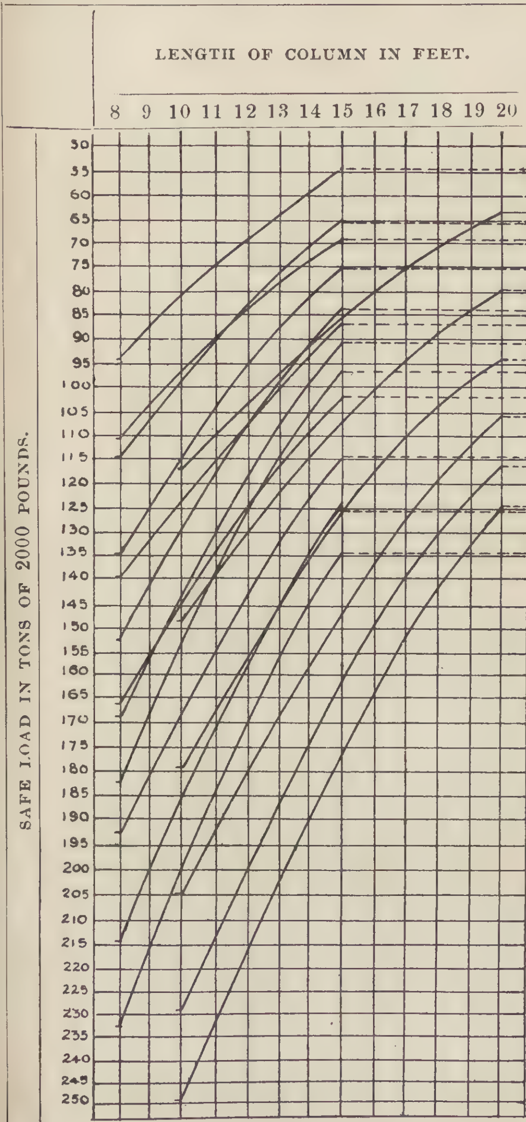


TABLE XLVI.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.



Diameter. IN.	Thickness of metal. IN.	Area. SQ. IN.
8	3/4	17,08
10	3/4	21,80
8	1	21,99
9	3/4	19,44
8	1 1/4	26,50
10	1	28,27
8	1 1/2	30,63
9	1	25,13
8	1 3/4	34,36
10	1 1/4	34,36
8	2	37,70
9	1 1/4	30,43
10	1 1/2	40,00
9	1 1/2	35,34
10	1 3/4	45,36
10	2	50,27
9	1 3/4	40,04
9	2	43,98

TABLE XLVII.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

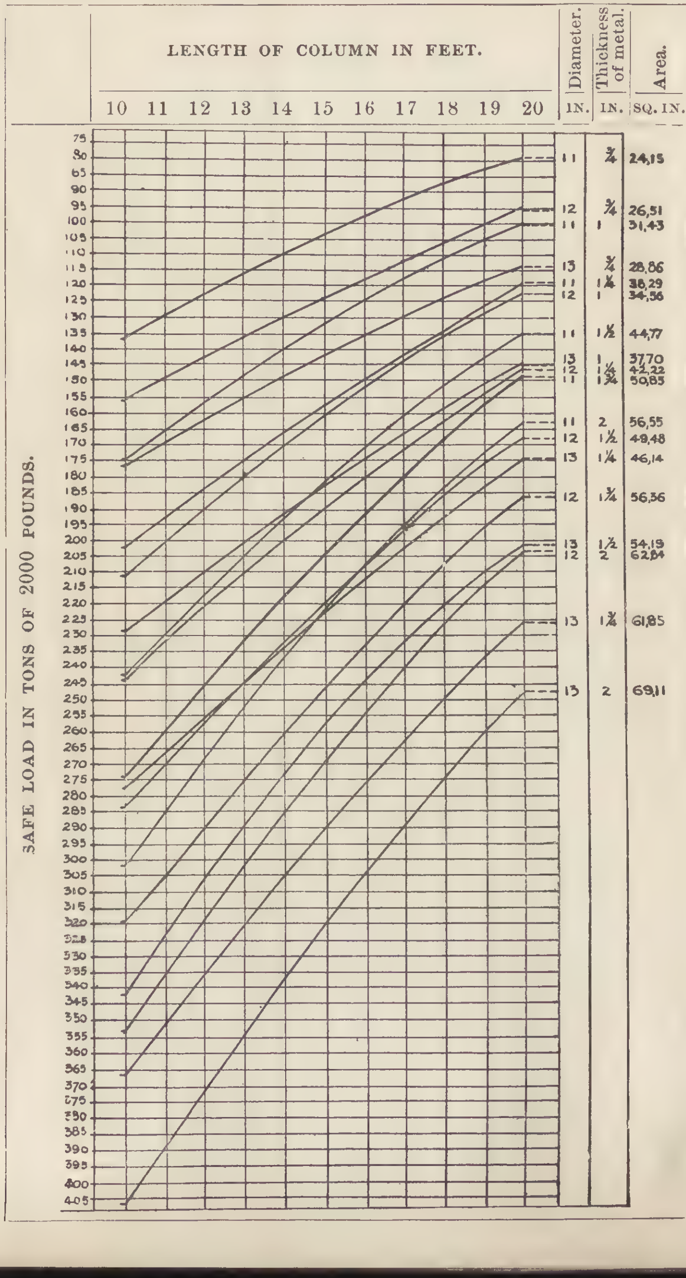


TABLE XLVIII.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

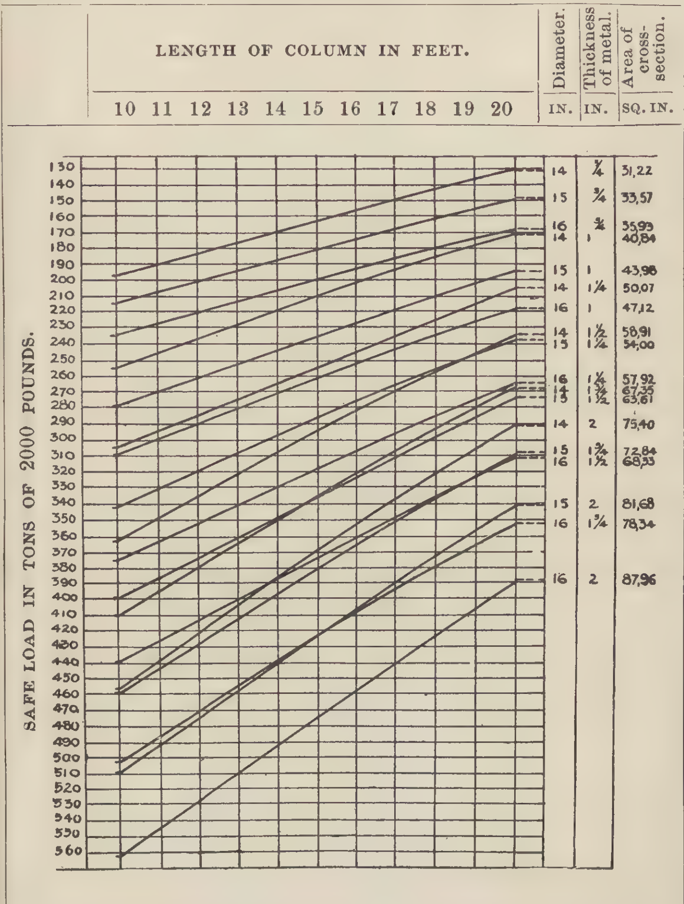


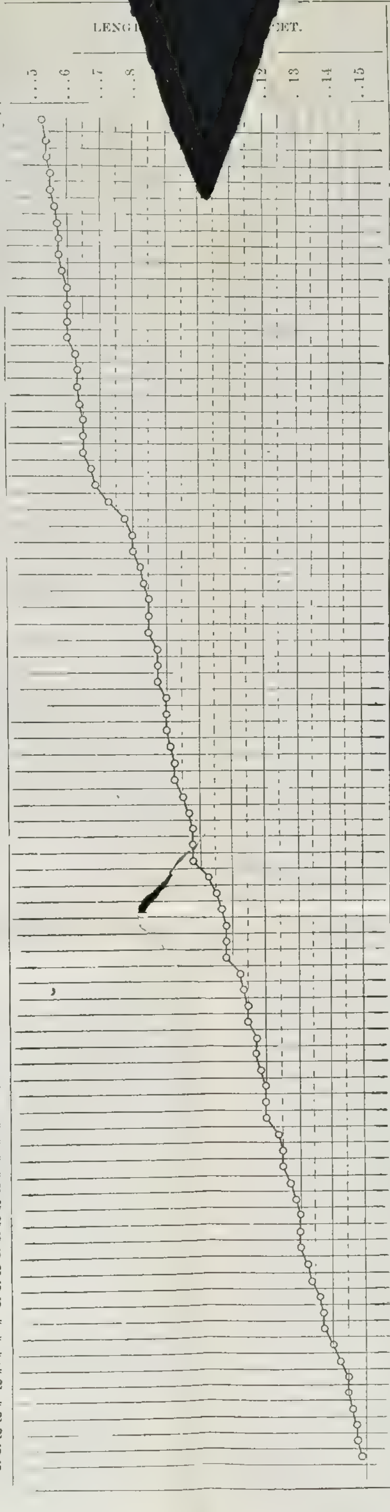
TABLE XLIX.

SAFE COMPRESSIVE LOADS PER SQUARE INCH ON WROUGHT-IRON COLUMNS.

Ratio of (in inches.)		Both ends smooth.			One end smooth one end pin end.			Both ends pin ends.		
Square of length to square of radius of gyration.	Length to radius of gyration.	For values of $\left(\frac{c}{f}\right)$ in lbs., equal.			For values of $\left(\frac{c}{f}\right)$ in lbs. equal.			For values of $\left(\frac{c}{f}\right)$ in lbs., equal.		
		8000	10000	12000	8000	10000	12000	8000	10000	12000
100	10	7989	9986	11983	7973	9966	11960	7960	9950	11940
225	15	7955	9944	11932	7940	9925	11911	7911	9888	11866
400	20	7921	9901	11881	7894	9868	11842	7843	9803	11764
625	25	7877	9846	11815	7836	9795	11755	7758	9697	11636
900	30	7821	9779	11735	7767	9708	11650	7656	9569	11483
1225	35	7762	9702	11643	7686	9607	11529	7538	9422	11307
1600	40	7692	9615	11538	7595	9493	11392	7407	9259	11111
2025	45	7614	9518	11422	7494	9367	11231	7264	9080	10897
2500	50	7529	9411	11293	7386	9232	11078	7105	8881	10657
3025	55	7437	9296	11155	7267	9084	10901	6949	8686	10423
3600	60	7339	9184	11009	7143	8928	10714	6780	8474	10169
4225	65	7236	9045	10855	7010	8762	10515	6605	8256	9907
4900	70	7127	8908	10690	6877	8596	10315	6426	8032	9638
5625	75	7015	8769	10523	6736	8420	10105	6244	7804	9365
6400	80	6896	8620	10345	6593	8241	9890	6058	7572	9086
7225	85	6777	8471	10165	6447	8059	9681	5877	7346	8815
8100	90	6653	8316	9979	6299	7874	9449	5694	7117	8541
9025	95	6527	8159	9791	6150	7687	9225	5512	6890	8269
10000	100	6400	8000	9600	6000	7500	9000	5333	6666	8000
11025	105	6271	7839	9407	5850	7312	8775	5157	6446	7736
12100	110	6140	7675	9210	5700	7125	8550	4980	6225	7470
13225	115	6010	7512	9015	5550	6937	8325	4820	6025	7230
14400	120	5880	7350	8820	5410	6762	8115	4650	5812	6975
15625	125	5750	7187	8625	5260	6575	7890	4490	5612	6735
16900	130	5620	7025	8430	5120	6400	7680	4340	5425	6510
18225	135	5500	6875	8250	4980	6225	7470	4180	5225	6270
19600	140	5370	6712	8055	4840	6050	7260	4040	5050	6060
21025	145	5240	6550	7860	4700	5875	7050	3900	4875	5850
22500	150	5120	6400	7680	4570	5712	6855	3770	4712	5655

WOOD[illegible]S.

[CALCULATED FOR 90 [illegible]OOT OF FLOOR.]

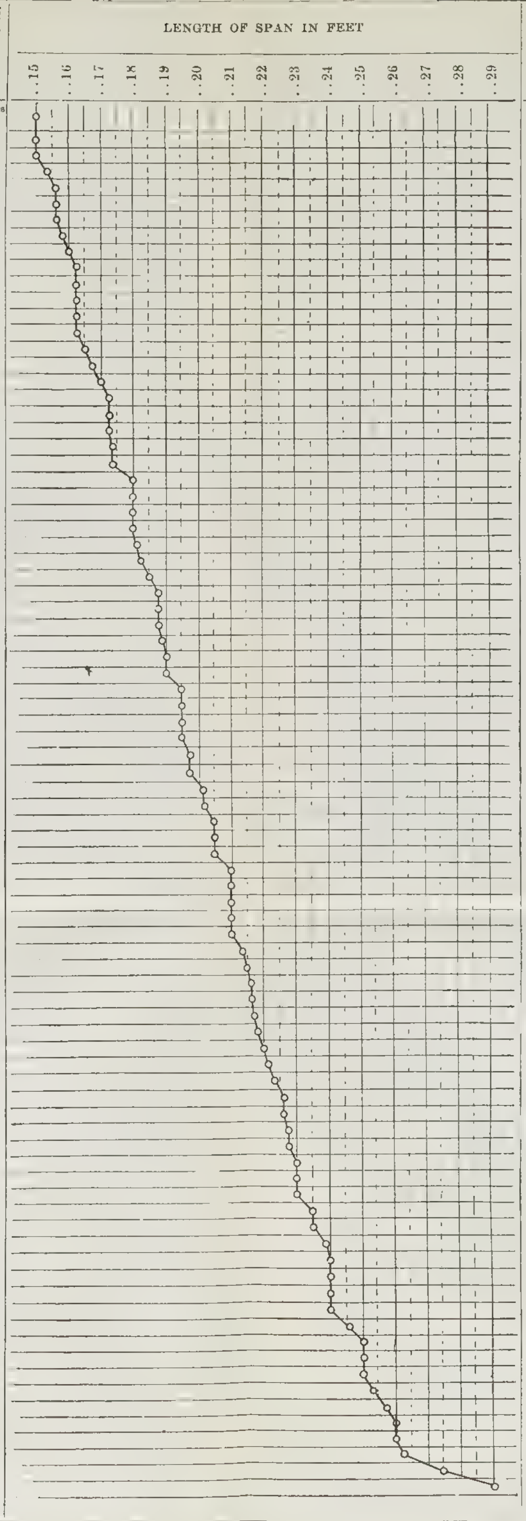


Length of span. Ft.	Ins.	MATERIAL OF BEAM.	Area of cross-section.	Thickness.	Depth.	Distance between centres.	Area of sect'n per sq. foot of floor.	Area of Section per square foot of floor for each wood. Hemlock	Spruce or W. Pine.	White Oak.	Georgia Pine.
			Inches	Inch.	Inches	Inches	Inches.	Inches.	Inches.	Inches.	Inches.
5	3	Hemlock,	12	3	4	16	9	9	..	..	..
5	4	Spruce or W. P.	12	3	4	16	9	..	9	..	..
5	5	White Oak,	12	3	4	16	9	..	..	9	..
5	6	Hemlock,	12	3	4	14	10.3	10.3	..	..	..
5	7	Spruce or W. P.	12	3	4	14	10.3	..	10.3	..	..
5	8	White Oak,	12	3	4	14	10.3	..	..	10.3	..
5	9	Hemlock,	12	3	4	12	12	12	..	..	..
5	9	Hemlock,	16	4	4	16	12	12	..	..	..
5	10	Spruce or W. P.	12	3	4	12	12	..	12	..	..
5	10	Spruce or W. P.	16	4	4	16	12	..	12	..	..
5	11	Georgia Pine,	12	3	4	16	9	..	..	..	9
6		White Oak,	12	3	4	12	12	..	..	12	..
6		White Oak,	16	4	4	16	12	..	..	12	..
6		Hemlock,	16	4	4	14	13.7	13.7	..	..	..
6	2	Spruce or W. P.	16	4	4	14	13.7	..	13.7	..	..
6	3	Georgia Pine,	12	3	4	14	10.3	..	..	..	10.3
6	3	White Oak,	16	4	4	14	13.7	..	..	13.7	..
6	4	Hemlock,	16	4	4	12	16	16	..	..	..
6	5	Spruce or W. P.	16	4	4	12	16	..	16	..	..
6	6	Georgia Pine,	12	3	4	12	12	..	..	..	12
6	6	Georgia Pine,	16	4	4	16	12	..	..	..	12
6	7	White Oak,	16	4	4	12	16	..	..	16	..
6	10	Georgia Pine,	16	4	4	14	13.7	..	..	..	13.7
7	3	Georgia Pine,	16	4	4	12	16	..	..	..	16
7	10	Hemlock,	18	3	6	16	13.5	13.5	..	..	..
8		Spruce or W. P.	18	3	6	16	13.5	..	13.5	..	..
8	1	White Oak,	18	3	6	16	13.5	..	..	13.5	..
8	2	Hemlock,	18	3	6	14	15.4	15.4	..	..	..
8	4	Spruce or W. P.	18	3	6	14	15.4	..	15.4	..	..
8	6	White Oak,	18	3	6	14	15.4	..	..	15.4	..
8	7	Hemlock,	18	3	6	12	18	18	..	..	.
8	7	Hemlock,	24	4	6	16	18	18	..	..	..
8	10	Spruce or W. P.	18	3	6	12	18	..	18	..	..
8	10	Spruce or W. P.	24	4	6	16	18	..	18	..	..
8	11	Georgia Pine,	18	3	6	16	13.5	..	..	..	13.5
9		White Oak,	18	3	6	12	18	..	..	18	..
9		White Oak,	24	4	6	16	18	..	..	18	..
9		Hemlock,	24	4	6	14	20.6	20.6	..	..	..
9	2	Spruce or W. P.	24	4	6	14	20.6	..	20.6	..	..
9	4	Georgia Pine,	18	3	6	14	15.4	..	..	..	15.4
9	4	White Oak,	24	4	6	14	20.6	..	..	20.6	..
9	6	Hemlock,	24	4	6	12	24	24	..	..	..
9	8	Spruce or W. P.	24	4	6	12	24	..	24	..	..
9	9	Georgia Pine,	18	3	6	12	18	..	..	..	18
9	9	Georgia Pine,	24	4	6	16	18	..	..	..	18
9	10	White Oak,	24	4	6	12	24	..	..	24	..
10	3	Georgia Pine,	24	4	6	14	20.6	..	..	..	20.6
10	5	Hemlock,	24	3	8	16	18	18	..	..	..
10	8	Spruce or W. P.	24	3	8	16	18	..	18	..	..
10	10	Georgia Pine,	24	4	6	12	24	..	..	..	24
10	10	White Oak,	24	3	8	16	18	..	..	18	..
10	11	Hemlock,	24	3	8	14	20.6	20.6	..	..	..
11	2	Spruce or W. P.	24	3	8	14	20.6	..	20.6	..	..
11	4	White Oak,	24	3	8	14	20.6	..	..	20.6	..
11	6	Hemlock,	24	3	8	12	24	24	..	..	..
11	6	Hemlock,	32	4	8	16	24	24	..	..	..
11	9	Spruce or W. P.	24	3	8	12	24	..	24	..	..
11	9	Spruce or W. P.	32	4	8	16	24	..	24	..	..
11	11	Georgia Pine,	24	3	8	16	18	..	..	..	18
12		White Oak,	24	3	8	12	24	..	..	24	..
12		White Oak,	32	4	8	16	24	..	.	24	..
12		Hemlock,	32	4	8	14	27.4	27.4	.	..	..
12	3	Spruce or W. P.	32	4	8	14	27.4	..	27.4	..	..
12	6	Georgia Pine,	24	3	8	14	20.6	..	..	..	20.6
12	6	White Oak,	32	4	8	14	27.4	..	..	27.4	..
12	8	Hemlock,	32	4	8	12	32	32	..	..	..
12	11	Spruce or W. P.	32	4	8	12	32	..	32	..	..
13		Georgia Pine,	24	3	8	12	24	..	..	..	24
13		Georgia Pine,	32	4	8	16	24	..	..	..	24
13		Hemlock,	30	3	10	16	22.5	22.5	..	..	..
13	2	White Oak,	32	4	8	12	32	..	..	32	..
13	4	Spruce or W. P.	30	3	10	16	22.5	..	22.5	..	..
13	7	White Oak,	30	3	10	16	22.5	..	..	22.5	..
13	8	Hemlock,	30	3	10	14	25.7	25.7	..	..	..
13	9	Georgia Pine,	32	4	8	14	27.4	..	..	..	27.4
14		Spruce or W. P.	30	3	10	14	25.7	..	25.7	..	..
14	2	White Oak,	30	3	10	14	25.7	..	..	25.7	..
14	4	Hemlock,	30	3	10	12	30	30	..	..	..
14	4	Hemlock,	40	4	10	14	34.3	34.3	..	..	..
14	6	Georgia Pine,	32	4	8	12	32	..	..	..	32
14	8	Spruce or W. P.	30	3	10	12	30	..	30	..	..
14	8	Spruce or W. P.	40	4	10	16	30	..	30	..	..
14	11	Georgia Pine,	30	3	10	16	22.5	..	..	..	22.5

Back of
Foldout
Not Imaged

TABLE XII (*Continued*).

Length of span. Ft.	Length of span. Ins.	MATERIAL OF BEAM.	Area of cross-section. Inches	Thickness. Inch.	Depth. Inches	Distance between centres. Inches	Area of sect'n per sq. foot of floor. Inches.	Area of Section per square foot of floor for each wood. Hemlock. Inches	Spruce or W. Pine. Inches.	White Oak. Inches.	Georgia Pine. Inches.
15		White Oak,	30	3	10	12	30	..	..	30	..
15		White Oak,	40	4	10	16	30	..	..	30	..
15		Hemlock,	40	4	10	14	34.3	34.3	..	..	..
15	4	Spruce or W. P.	40	4	10	14	34.3	..	34.3	..	..
15	7	Georgia Pine,	30	3	10	14	25.7	..	..	..	25.7
15	8	White Oak,	40	4	10	14	34.3	..	..	34.3	..
15	8	Hemlock,	36	3	12	16	27	27	..	..	..
15	10	Hemlock,	40	4	10	12	40	40	..	..	..
16		Spruce or W. P.	36	3	12	16	27	..	27	..	..
16	2	Spruce or W. P.	40	4	10	12	40	..	40	..	..
16	3	Georgia Pine,	30	3	10	12	30	..	..	..	30
16	3	Georgia Pine,	40	4	10	16	30	..	..	..	30
16	3	White Oak,	36	3	12	16	27	..	..	27	..
16	4	Hemlock,	36	3	12	14	30.9	30.9	..	..	..
16	5	White Oak,	40	4	10	12	40	..	..	40	..
16	9	Spruce or W. P.	36	3	12	14	30.9	..	30.9	..	..
17		White Oak,	36	3	12	14	32	..	..	32	..
17	2	Georgia Pine,	40	4	10	14	34.3	..	..	..	34.3
17	3	Hemlock,	36	3	12	12	36	36	..	..	..
17	3	Hemlock,	48	4	12	16	36	36	..	..	..
17	7	Spruce or W. P.	36	3	12	12	36	..	36	..	..
17	7	Spruce or W. P.	48	4	12	16	36	..	36	..	..
17	11	Georgia Pine,	36	3	12	16	27	..	..	..	27
18		White Oak,	36	3	12	12	36	..	..	36	..
18		White Oak,	48	4	12	16	36	..	..	36	..
18		Hemlock,	48	4	12	14	41.1	41.1	..	..	..
18	1	Georgia Pine,	40	4	10	12	40	..	..	..	40
18	3	Hemlock,	42	3	14	16	31.5	31.5	..	..	..
18	5	Spruce or W. P.	48	4	12	14	41.1	..	41.1	..	..
18	8	Georgia Pine,	36	3	12	14	30.9	..	..	..	30.9
18	8	Spruce or W. P	42	3	14	16	31.5	..	31.5	..	..
18	9	White Oak,	48	4	12	14	41.1	..	..	41.1	..
18	11	Hemlock,	48	4	12	12	48	48	..	..	..
19		White Oak,	42	3	14	16	31.5	..	..	31.5	..
19	1	Hemlock,	42	3	14	14	36	36	..	..	..
19	5	Spruce or W. P.	48	4	12	12	48	..	48	..	..
19	6	Spruce or W. P.	42	3	14	14	36	..	36	..	..
19	7	Georgia Pine,	36	3	12	12	36	..	..	..	36
19	7	Georgia Pine,	48	4	12	16	36	..	..	..	36
19	9	White Oak,	48	4	12	12	48	..	..	48	..
19	10	White Oak,	42	3	14	14	36	..	..	36	..
20	1	Hemlock,	42	3	14	12	42	42	..	..	..
20	1	Hemlock,	56	4	14	16	42	42	..	..	..
20	6	Spruce or W. P.	42	3	14	12	42	..	42	..	..
20	6	Spruce or W. P.	56	4	14	16	42	..	42	..	..
20	7	Georgia Pine,	48	4	12	14	41.1	..	..	..	41.1
20	11	Georgia Pine,	42	3	14	16	31.5	..	..	..	31.5
20	11	Hemlock,	48	3	16	16	36	36	..	..	..
21		White Oak,	42	3	14	12	42	..	..	42	..
21		White Oak,	56	4	14	16	42	..	..	42	..
21		Hemlock,	56	4	14	14	48	48	..	..	..
21	4	Spruce or W. P.	48	3	16	16	36	..	36	..	..
21	6	Spruce or W. P.	56	4	14	14	48	..	48	..	..
21	8	Georgia Pine,	48	4	12	12	48	..	..	..	48
21	9	White Oak,	48	3	16	16	36	..	..	36	..
21	10	Hemlock,	48	3	16	14	41.1	41.1	..	..	..
21	10	Georgia Pine,	42	3	14	14	36	..	..	..	36
21	11	White Oak,	56	4	14	14	48	..	..	48	..
22	2	Hemlock,	56	4	14	12	56	56	..	..	..
22	4	Spruce or W. P.	48	3	16	14	41.1	..	41.1	..	..
22	8	White Oak,	48	3	16	14	41.1	..	..	41.1	..
22	8	Spruce or W. P.	56	4	14	12	56	..	56	..	..
22	10	Georgia Pine,	42	3	14	12	42	..	..	..	42
22	10	Georgia Pine,	56	4	14	16	42	..	..	..	42
23		White Oak,	56	4	14	12	56	..	..	56	..
23		Hemlock,	48	3	16	12	48	48	..	..	..
23		Hemlock,	64	4	16	16	48	48	..	..	..
23	5	Spruce or W. P.	48	3	16	12	48	..	48	..	..
23	5	Spruce or W. P.	64	4	16	16	48	..	48	..	..
23	11	Georgia Pine,	48	3	16	16	36	..	..	..	36
24		White Oak,	48	3	16	12	48	..	..	48	..
24		White Oak,	64	4	16	16	48	..	..	48	..
24		Hemlock,	64	4	16	14	54.9	54.9	..	..	..
24		Georgia Pine,	56	4	14	14	48	..	..	..	48
24	7	Spruce or W. P.	64	4	16	14	54.9	..	54.9	..	..
24	11	Georgia Pine,	48	3	16	14	41.1	..	..	..	41.1
25		White Oak,	64	4	16	14	54.9	..	..	54.9	..
25	3	Hemlock,	64	4	16	12	64	64	..	..	..
25	4	Georgia Pine,	56	4	14	12	56	..	..	..	56
25	10	Spruce or W. P.	64	4	16	12	64	..	64	..	..
26		Georgia Pine,	48	3	16	12	48	..	..	..	48
26		Georgia Pine,	64	4	16	16	48	..	..	..	48
26	3	White Oak,	64	4	16	12	64	..	..	64	..
27	5	Georgia Pine,	64	4	16	14	54.8	..	..	..	54.8
29		Georgia Pine,	64	4	16	12	64	..	..	..	64

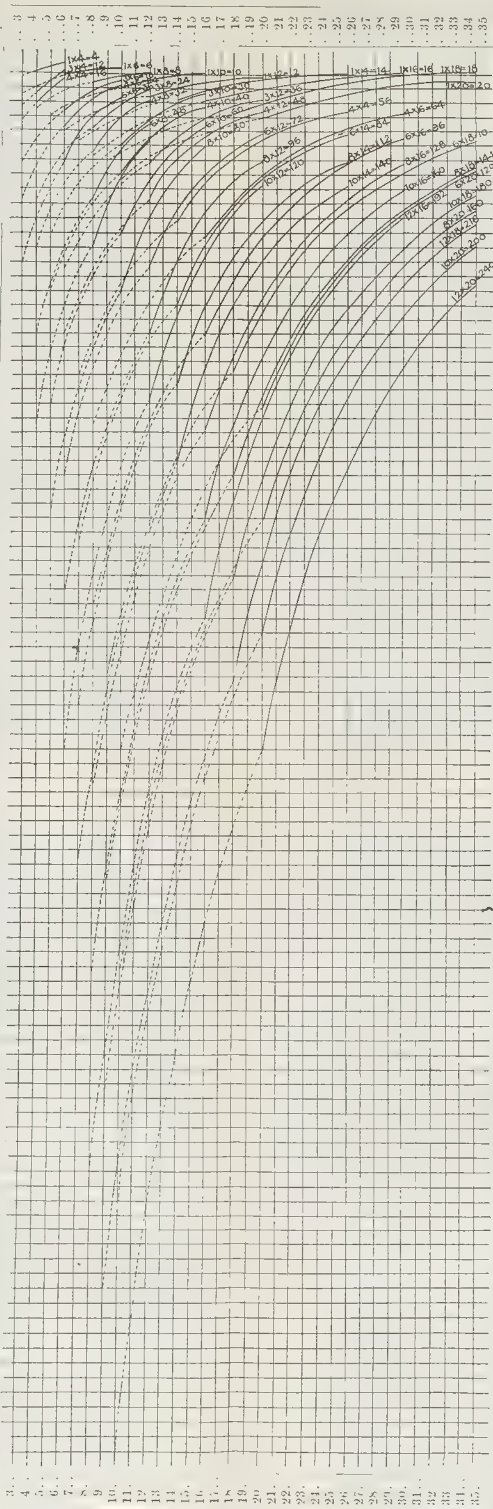


Back of
Foldout
Not Imaged

TABLE XIII.

WOODEN GIRDERS, BRACED SIDEWAYS.

LENGTH OF SPAN IN FEET.



Safe Uniform Load in Pounds for Dotted Lines.					Safe Uniform Load in Lbs. for Full Lines.			
Georgia Pine	White Oak	Spruce	White Pine	Hemlock	Georgia Pine	White Oak	Spruce or W. Pine	Hemlock
665	610	555	500	415	665	500	475	445
1330	1220	1110	1000	830	1330	1000	950	890
1995	1830	1665	1500	1245	1995	1500	1425	1335
2660	2440	2220	2000	1660	2660	2000	1900	1780
3325	3050	2775	2500	2075	3325	2500	2375	2225
3990	3660	3330	3000	2490	3990	3000	2850	2670
4665	4270	3885	3500	2905	4665	3500	3325	3115
5320	4880	4440	4000	3320	5320	4000	3800	3560
5985	5490	4995	4500	3735	5985	4500	4275	4005
6650	6100	5550	5000	4150	6650	5000	4750	4450
7315	6710	6105	5500	4565	7315	5500	5225	4895
7980	7320	6660	6000	4980	7980	6000	5700	5340
8645	7930	7215	6500	5395	8645	6500	6175	5785
9310	8540	7770	7000	5810	9310	7000	6650	6230
9975	9150	8325	7500	6225	9975	7500	7125	6675
10640	9760	8880	8000	6640	10640	8000	7600	7120
11305	10370	9435	8500	7055	11305	8500	8075	7565
11970	10980	9990	9000	7470	11970	9000	8550	8010
12635	11590	10545	9500	7885	12635	9500	9025	8455
13300	12200	11100	10000	8300	13300	10000	9500	8900
13965	12810	11655	10500	8715	13965	10500	9975	9345
14630	13420	12210	11000	9130	14630	11000	10450	9790
15295	14030	12765	11500	9545	15295	11500	10925	10235
15960	14640	13320	12000	9960	15960	12000	11400	10680
16625	15250	13875	12500	10375	16625	12500	11875	11125
17290	15860	14430	13000	10790	17290	13000	12350	11570
17955	16470	14985	13500	11205	17955	13500	12825	12015
18620	17080	15540	14000	11620	18620	14000	13300	12460
19285	17690	16095	14500	12035	19285	14500	13775	12905
19950	18300	16650	15000	12450	19950	15000	14250	13350
20615	18910	17205	15500	12865	20615	15500	14725	13795
21280	19520	17760	16000	13280	21280	16000	15200	14240
21945	20130	18315	16500	13695	21945	16500	15675	14685
22610	20740	18870	17000	14110	22610	17000	16150	15130
23275	21350	19425	17500	14525	23275	17500	16625	15575
23940	21960	19980	18000	14940	23940	18000	17100	16020
24605	22570	20535	18500	15355	24605	18500	17575	16465
25270	23180	21090	19000	15770	25270	19000	18050	16910
25935	23790	21645	19500	16185	25935	19500	18525	17355
26600	24400	22200	20000	16600	26600	20000	19000	17800
27265	25010	22755	20500	17015	27265	20500	19475	18245
27930	25620	23310	21000	17430	27930	21000	19950	18690
28595	26230	23865	21500	17845	28595	21500	20425	19135
29260	26840	24420	22000	18260	29260	22000	20900	19580
29925	27450	24975	22500	18675	29925	22500	21375	20025
30590	28060	25530	23000	19090	30590	23000	21850	20470
31255	28670	26085	23500	19505	31255	23500	22325	20915
31920	29280	26640	24000	19920	31920	24000	22800	21360
32585	29890	27195	24500	20335	32585	24500	23275	21805
33250	30500	27750	25000	20750	33250	25000	23750	22250
33915	31110	28305	25500	21165	33915	25500	24225	22695
34580	31720	28860	26000	21580	34580	26000	24700	23140
35245	32330	29415	26500	21995	35245	26500	25175	23585
35910	32940	29970	27000	22410	35910	27000	25650	24030
36575	33550	30525	27500	22825	36575	27500	26125	24475
37240	34160	31080	28000	23240	37240	28000	26600	24920
37905	34770	31635	28500	23655	37905	28500	27075	25365
38570	35380	32190	29000	24070	38570	29000	27550	25810
39235	35990	32745	29500	24485	39235	29500	28025	26255
39900	36600	33300	30000	24900	39900	30000	28500	26700
40565	37210	33855	30500	25315	40565	30500	28975	27145
41230	37820	34410	31000	25730	41230	31000	29450	27590
41895	38430	34965	31500	26145	41895	31500	29925	28035
42560	39040	35520	32000	26560	42560	32000	30400	28480
43225	39650	36075	32500	26975	43225	32500	30875	28925
43890	40260	36630	33000	27390	43890	33000	31350	29370
44555	40870	37185	33500	27805	44555	33500	31825	29815
45220	41480	37740	34000	28220	45220	34000	32300	30260
45885	42090	38295	34500	28635	45885	34500	32775	30705
46550	42700	38850	35000	29050	46550	35000	33250	31150
47215	43310	39405	35500	29465	47215	35500	33725	31595
47880	43920	39960	36000	29880	47880	36000	34200	32040
48545	44530	40515	36500	30295	48545	36500	34675	32485
49210	45140	41070	37000	30710	49210	37000	35150	32930
49875	45750	41625	37500	31125	49875	37500	35625	33375
50540	46360	42180	38000	31540	50540	38000	36100	33820
51205	46970	42735	38500	31955	51205	38500	36575	34265
51870	47580	43290	39000	32370	51870	39000	37050	34710
52535	48190	43845	39500	32785	52535	39500	37525	35155
53200	48800	44400	40000	33200	53200	40000	38000	35600
53865	49410	44955	40500	33615	53865	40500	38475	36045
54530	50020	45510	41000	34030	54530	41000	38950	36490
55195	50630	46065	41500	34445	55195	41500	39425	36935
55860	51240	46620	42000	34860	55860	42000	39900	37380
56525	51850	47175	42500	35275	56525	42500	40375	37825
57190	52460	47730	43000	35690	57190	43000	40850	38270
57855	53070	48285	43500	36105	57855	43500	41325	38715
58520	53680	48840	44000	36520	58520	44000	41800	39160
59185	54290	49395	44500	36935	59185	44500	42275	39605
59850	54900	49950	45000	37350	59850	45000	42750	40050
60515	55510	50505	45500	37765	60515	45500	43225	40495
61180	56120	51060	46000	38180	61180	46000	43700	40940
61845	56730	51615	46500	38595	61845	46500	44175	41385
62510	57340	52170	47000	39010	62510	47000	44650	41830
63175	57950	52725	47500	39425	63175	47500	45125	42275
63840	58560	53280	48000	39840	63840	48000	45600	42720
Georgia Pine	White Oak	Spruce	White Pine	Hemlock	Georgia Pine	White Oak	Spruce or W. Pine	Hemlock
Safe Uniform Load in Pounds for Dotted Lines.					Safe Uniform Load in Lbs. for Full Lines.			

LENGTH OF SPAN IN FEET.

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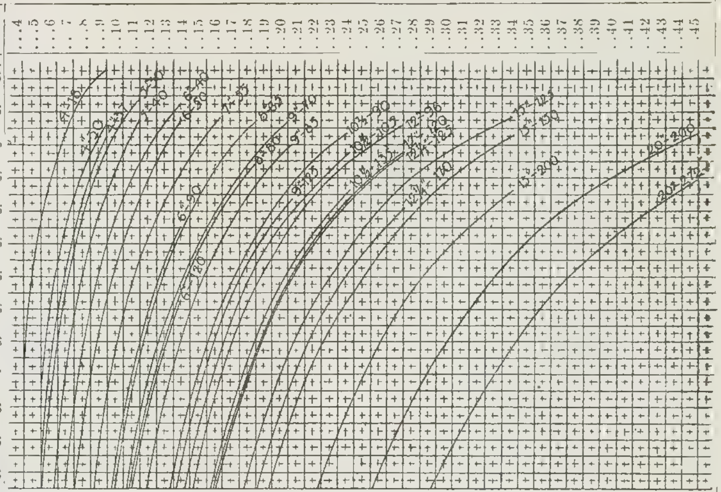
Table XIV.

IRON I BEAMS FOR FLOORS.

Distance between Centres of Beams.													
300 lbs. per sq. foot		250 lbs. per sq. foot.		200 lbs. per sq. foot		175 lbs. per sq. foot.		150 lbs. per sq. foot		125 lbs. per sq. foot		100 lbs. per sq. foot	
ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.
	10	1		1	3	1	5	1	8	2		2	6
1		1	2	1	6	1	9	2		2	5	3	
1	2	1	5	1	9	2		2	4	2	10	3	6
1	4	1	7	2		2	3	2	8	3	2	4	
1	6	1	10	2	3	2	7	3		3	7	4	6
1	8	2		2	6	2	10	3	4	4		5	
1	10	2	2	2	9	3	2	3	8	4	5	5	6
2		2	5	3		3	5	4		4	10	6	
2	2	2	7	3	3	3	9	4	4	5	2	6	6
2	4	2	10	3	6	4		4	8	5	7	7	
2	6	3		3	9	4	3	5		6		7	6
2	8	3	2	4		4	7	5	4	6	5	8	
2	10	3	5	4	3	4	10	5	8	6	10	8	6
3		3	7	4	6	5	2	6		7	2	9	
3	2	3	10	4	9	5	5	6	4	7	7	9	6
3	4	4		5		5	9	6	8	8		10	
3	6	4	2	5	3	6		7		8	5	10	6
3	8	4	5	5	6	6	3	7	4	8	10	11	
3	10	4	7	5	9	6	7	7	8	9	2	11	6
4		4	10	6		6	10	8		9	7	12	
4	2	5		6	3	7	2	8	4	10		12	6
4	4	5	2	6	6	7	5	8	8	10	5	13	
4	6	5	5	6	9	7	9	9		10	10	13	6
4	8	5	7	7		8		9	4	11	2	14	
4	10	5	10	7	3	8	3	9	8	11	7	14	6
[illegible]		6		7	6	8	7	10		12		15	

LENGTH OF SPAN OF BEAM IN FEET.



For Steel Beams, space one-quarter distance (between centres) larger than for Iron Beams; but length of span in feet must not exceed twice the depth of beam in inches, or deflection will be too great for plastering.

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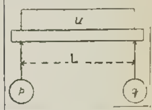
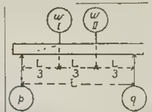
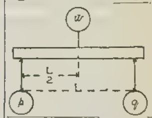
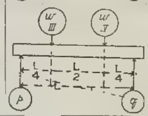
TABLE XIX.

EXPLAINING USE OF TABLES XX TO XXV.

HOW TO USE TRANSVERSE VALUE (v) FOR SECTIONS WHICH ARE UNIFORM ABOVE AND BELOW THE NEUTRAL AXIS.

MANNER OF LOADING.	TO OBTAIN SAFE LOAD IN POUNDS.	LENGTH OF SPAN NOT TO CRACK PLASTERING MUST NOT EXCEED. For Iron.	For Steel.	GREATEST ACTUAL DEFLECTION WILL BE. For Iron.	For Steel.	MANNER OF LOADING.	TO OBTAIN SAFE LOAD IN POUNDS.	LENGTH OF SPAN NOT TO CRACK PLASTERING MUST NOT EXCEED. For Iron.	For Steel.	GREATEST ACTUAL DEFLECTION WILL BE. For Iron.	For Steel.
	$u = \frac{v}{L}$	$L = 2\frac{1}{4}.d$	$L = 2.d$	$\delta = \frac{L^2}{75.d}$	$\delta = \frac{L^2}{64\frac{1}{2}.d}$		$w_{I} = w_{II} = \frac{v}{2\frac{2}{3}.L}$ or:— $w_{I} + w_{II} = \frac{v}{1\frac{1}{3}.L}$	$L = 2\frac{3}{16}.d$	$L = 1$[illegible]$.d$	$\delta = \frac{L^2}{72\frac{1}{4}.d}$	$\delta = \frac{L^2}{62\frac{1}{2}.d}$
	$w = \frac{v}{2.L}$	$L = 2\frac{2}{5}.d$	$L = 2\frac{2}{5}.d$	$\delta = \frac{L^2}{93\frac{3}{4}.d}$	$\delta = \frac{L^2}{80\frac{2}{3}.d}$		$w_{III} = w_{IIII} = \frac{v}{2.L}$ or:— $w_{III} + w_{IIII} = \frac{v}{L}$	$L = 2.d$	$L = 1\frac{3}{4}.d$	$\delta = \frac{L^2}{[illegible].d}$	$\delta = \frac{L^2}{57\frac{2}{3}.d}$

d = depth in inches; L = length in feet; δ = deflection in inches; v = transverse value, as given in Tables XX to XXV; u = uniform load in lbs.; w = centre load in lbs.; $w_{I} = w_{II}$; also $w_{III} = w_{IIII}$ = concentrated loads in lbs.

NOTE.—If the transverse values (v), given for steel, are used, test each piece carefully, as steel varies greatly in strength. For *equal* deflections of steel and iron, add only $7\frac{1}{2}$% to iron transverse values, instead of 25% as given in Tables. In calculating the transverse values, the moduli of rupture used were: for iron, 12000 pounds per square inch, and for steel, 15000 pounds per square inch.

From the "Safe Load" as obtained above, *deduct* the weight of beam in pounds, as follows:—In case of uniform load deduct *entire* weight of beam; in case of centre load, deduct *one-half* the weight of beam; in case of loads at thirds of span, deduct from *each* load $\frac{2}{3}$ the weight of beam; in case of loads at quarters of span, deduct from *each* load $\frac{1}{2}$ the weight of beam.

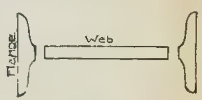
Steel sections will be slightly heavier (about 1%) than iron sections of exactly the same dimensions. In ordering steel or iron, give either the required dimensions or the required weight of section, *never both*.

The capital letters in the first column headed "Mills Rolling Shape" in each Table represent the following Rolling Mills:—

A—NEW JERSEY STEEL AND IRON COMPANY, Trenton, N. J.
B—PHŒNIX IRON COMPANY, Philadelphia, Pa.
C—PENCOYD IRON WORKS, Philadelphia, Pa.
D—POTTSVILLE IRON AND STEEL COMPANY, Pottsville, Pa.
E—UNION IRON MILLS and HOMESTEAD STEEL WORKS, Pittsburgh, Pa.
F—PASSAIC ROLLING MILL COMPANY, Passaic, N. J.

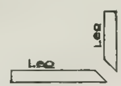
In the columns headed "Mills Rolling Shape" the first letter on each line indicates the Mill which rolls the exact shape given in the Table; the other letters give the Mills which roll an approximately similar shape.

In getting the areas of parts of sections, they were taken as shown below:—

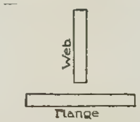


BEAM.

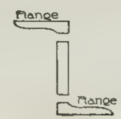
CHANNEL.



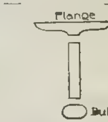
ANGLE.



TEE.



ZEE.



DECK BEAM.

See foot-note p. 205.

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Table XX.

LIST OF IRON AND STEEL I BEAMS.

(For Information as to the Use of this Table, see Table XIX.)

Mills Rolling Shape.	Depth of Beam. (d)	Weight per Yard.	Width of Flanges. (b)	Thickness of Web.	Area of each Flange.	Area of Web.	Total Area. (a)	Axis Normal to Web. Moment of Inertia. (I)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.	Axis Parallel to Web. Moment of Inertia. (I)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.
A,B	20	272	6.75	0.69	7.87	11.46	27.20	1650.3	165	60.67	1320000	1650000	46.50	13.78	1.71	110000	137800
E	20	240	7	0.60	6.68	10.64	24	1450	145	60.42	1160000	1450000	51.78	16.57	2.16	132600	165700
A,B	20	200	6	0.50	5.65	8.67	19.97	1238	123.8	61.99	990000	1238000	26.62	8.87	1.33	71000	88700
E	20	192	6.25	0.50	5.09	9.02	19.20	1146	114.6	59.68	917000	1146000	31.50	10.08	1.64	80600	100800
A.F	15$\frac{3}{16}$	150	5	0.50	4.34	6.36	15.04	523.5	69	34.80	552000	690000	15.29	6.12	1.02	49000	61200
A,F	15$\frac{1}{8}$	200	5.75	0.60	6.44	7.14	20.02	707.1	93.5	35.32	748000	935000	27.46	9.55	1.37	76000	95500
A	15$\frac{1}{8}$	125	5	0.42	[illegible].37	5.62	12.36	434.5	57.5	35.15	460000	575000	11.64	4.66	.94	37000	46600
D	15	250	5.875	0.875	7.22	10.56	25	813	108	32.52	864000	1080000	40.84	13.90	1.62	111200	139000
E	15	240	5.81	0.93	6.29	11.42	24	750	100	31.24	800000	1000000	29.90	10.29	1.25	82300	103000
D,B,C,E....	15	200	5.56	0.625	[illegible].06	7.88	20	694	92.5	34.70	740000	925000	33.79	12.12	1.69	97000	121000
E	15	195	5.33	0.77	4.84	9.82	19.50	614	81.5	31.47	655000	819000	20	7.50	1.02	60000	75000
D,B,E	15	150	5	0.47	4.47	6.06	15	528	70.4	35.20	563000	704000	18.34	7.34	1.22	58700	73400
C	15	145	5.125	0.44	4.48	5.59	14.55	521.2	69.5	35.76	556000	695000	16.91	6.60	1.17	52800	66000
D,B,E	15	125	4.875	0.44	3.35	5.80	12.50	430	57.3	34.40	458000	573000	13.13	5.34	1.05	42700	53400
A	12$\frac{5}{16}$	170	5.50	0.60	5.47	5.83	16.77	391.2	63.5	23.32	508000	635000	25.41	9.24	1.52	74000	92400
F	12¼	170	5.25	0.66	5.56	5.88	17	385	63	22.60	504000	630000	20.90	7.96	1.23	63700	80000
A,F	12¼	125	4.79	0.47	3.78	4.77	12.33	288	47	23.35	376000	470000	11.54	4.82	.93	38500	48200
E	12	180	5.09	0.96	4.26	9.48	18	340	56.7	18.92	454000	567000	15.50	6.09	.86	48700	61000
B,C,D	12	170	5.50	0.59	5.77	[illegible].46	17	381.9	63.7	22.46	510000	637000	24.08	8.76	1.42	70100	87600
B,D,E	12	125	4.75	0.49	3.80	4.90	12.50	282.6	47.1	22.60	377000	471000	12.98	5.46	1.04	43700	54600
A,C	12	120	5.50	0.39	3.87	3.99	11.73	281.3	46.9	23.98	375000	469000	16.76	6.10	1.43	49000	61000
D	12	100	4.44	0.44	2.71	4.58	10	218	36.3	21.80	290000	363000	8.74	4	.87	32000	40000
A,B,E	12	96	5.25	0.31	[illegible].09	3.28	9.46	229.2	38.[illegible]	24.22	306000	382000	11.66	4.44	1.23	35500	44400
A,B,C,D,E,F,	10½	135	5	0.47	4.84	3.68	13.36	233.7	44.5	17.49	356000	445000	15.80	[illegible].32	1.18	50500	63200
A,B,C,D,F...	10½	105	4.50	0.375	3.67	3.10	10.14	185.6	35.3	17.77	282000	353000	9.43	4.19	.90	33500	41900
E	10½	95	4.54	0.41	2.96	3.58	[illegible].50	165	31.4	17.39	251000	314000	8.01	3.53	.85	28200	35300
A,B,C,D,F...	10½	90	4.50	0.31	3.11	2.68	8.90	164	31.2	18.42	250000	312000	8.09	3.59	.91	28700	35900
E	10	135	4.77	0.77	3.67	6.16	13.50	187	37.5	13.91	300000	375000	11.30	4.74	.83	37900	47400
C	10	112	4.625	0.50	3.61	3.95	11.17	173.6	34.7	15.52	278000	347000	10.64	4.60	.96	36800	46000
D	10	105	4.625	0.50	3.24	4.02	10.50	161	32.2	15.35	258000	322000	11.08	4.79	1.05	38300	48000
C,D,E	10	90	4.375	0.34	3.14	2.76	9.04	148.3	29.7	16.40	237000	297000	8.09	3.69	.90	29500	36900
B,E	9	150	5.375	0.60	5.59	3.82	15	189.1	42	12.63	336000	420000	23.16	8.62	1.54	69000	86000
E	9	135	4.94	0.75	4.22	5.06	13.50	159	35.3	11.70	282000	353000	14	5.67	1.02	45400	56700
A	9	125	4.50	0.57	4.41	3.51	12.33	150.8	33.5	12.23	268000	335000	11.23	4.99	.91	40000	49900
E	9	99	4.33	0.58	[illegible].85	4.20	9.90	117	26	11.83	208000	260000	7.14	3.30	.72	26400	33000
C,D	9	90	4.375	0.41	3.07	2.93	9.07	118.8	26.4	13.10	211000	264000	8.44	3.86	.92	30900	38600
A,B,D,F.....	9	85	4.50	0.375	2.93	[illegible].64	8.50	111.9	24.9	13.16	199000	249000	7.35	[illegible].27	.86	26000	32700
A,B,C,D,E,F,	9	70	4	0.30	2.40	2.20	7	93.9	20.9	13.41	167000	209000	4.92	2.46	.70	19700	24600
E	8	105	4.29	0.79	2.74	5.02	10.50	90.4	22.[illegible]	8.64	181000	226000	6.96	3.25	.67	26000	32500
A,B,C,D,F...	[illegible]	80	4.50	0.375	2.82	2.39	8.03	83.9	21	10.44	168000	210000	7.55	3.35	.93	26800	33500
A,B,C,D,E,F,	8	65	4	0.30	2.21	1.95	[illegible].37	67.4	16.9	10.58	135000	169000	4.55	2.27	.71	18200	22700
E	7	75	3.91	0.53	2.29	2.92	7.50	54.3	15.5	7.24	124000	155000	4.87	2.50	.66	20000	25000
B	7	69	4	0.375	2.47	1.96	6.90	55.7	15.[illegible]	8.08	127000	159000	5.42	2.71	.785	21680	27100
C,D	7	65	3.81	0.44	2.08	2.42	6.58	49.8	14.2	7.56	113800	142000	4.15	2.18	.62	17400	21800
F	7	60	3.50	0.40	1.88	2.24	6	45	12.[illegible]	7.50	103200	129000	3.15	1.80	.53	14400	18000
A,B,D,E	7	55	3.75	0.30	1.90	1.70	5.50	44.3	12.7	8.05	101000	127000	3.90	2.08	.71	16600	20800
C	7	52	3.61	0.234	1.92	1.30	5.14	43.1	12.3	8.35	98500	123000	3.43	1.90	.67	15200	19000
A	6	120	5.25	0.625	4.78	2.28	11.84	64.9	21.6	5.48	172000	216000	18.59	7.08	1.57	56600	70800
A,F	6	90	5	0.50	3.31	2.08	8.70	49.8	16.6	5.72	132000	166000	10.78	4.32	1.24	34500	43200
E	6	54	3.46	0.46	1.61	2.18	5.40	28.4	9.5	5.29	76000	95000	2.51	1.45	.46	11600	14500
A,B,C,D,F...	6	50	3.50	0.30	1.77	1.37	4.91	29	9.6	5.90	76800	96000	2.74	1.57	.56	12500	15700
A,B,C,D,E,F,	6	40	3	0.25	1.42	1.17	4.01	23.5	7.8	5.86	62400	78000	1.61	1.07	.40	8500	10700
A,D,E,F.....	5	40	3	0.31	1.36	1.18	3.90	15.4	6.1	3.95	48800	61000	1.68	1.12	.43	9000	11200
B	5	36	3	0.30	1.20	1.20	3.60	14.9	5.96	4.14	47700	59600	1.74	1.16	.483	9260	11580
C	5	34	2.844	0.31	1.06	1.26	3.38	13.4	5.4	3.96	42800	53500	1.21	.85	.36	6800	8500
A,B,C,D,E,F,	5	30	2.75	0.25	.99	1.01	2.99	12.1	4.8	4.04	38400	48000	1.04	.76	.35	6000	7600
A,F	4	37	3	0.31	1.40	.86	3.66	9.2	4.6	2.51	36800	46000	1.74	1.16	.48	9500	11600
A,B,D,E,F ..	4	30	2.75	0.25	1.08	.75	2.91	7.5	3.75	2.57	30000	37500	1.11	.81	.38	6500	8100
C	4	28	2.75	0.25	1.07	.76	2.90	7.7	3.84	2.65	30700	38400	1.17	.85	.40	6800	8500
D,E	4	24	2.25	0.31	.71	.98	2.40	5.6	2.80	2.33	22400	28000	.58	.52	.22	4160	5200
A,B,C,D,F ..	4	18	2	0.19	.58	.61	1.77	4.5	2.25	2.54	18000	22500	.31	.31	.175	2500	3100
E	3	27	2.52	0.39	.93	.84	2.70	3.54	2.36	1.32	18900	23600	.84	.67	.31	5360	6700
C	3	23	2.50	0.25	.86	.53	2.25	3.29	2.16	1.46	17300	21600	.77	.62	.35	4960	6200
E	3	21	2.32	0.19	.85	.40	2.10	3.09	2.06	1.46	16500	20600	.55	.47	.30	3760	4700
C	3	17	2.25	0.156	.68	.35	1.71	2.66	1.77	1.56	14200	17700	.48	.43	.28	3440	4300
A	1¼	5¼	1.50	0.125	.19	14	.52	.135	.21	.26	....	2160	.069	.092	.013	....	920

Back of
Foldout
Not Imaged

Table XXI.

LIST OF IRON AND STEEL CHANNELS.

(For Information as to Use of this Table, see Table XIX.)

Mills Rolling Shape.	Depth of Beam. (d)	Weight per Yard.	Width of Flanges. (b)	Thickness of Web.	Area of each Flange.	Area of Web.	Total Area. (a)	Axis Normal to Web. Moment of Inertia. (i)	Axis Normal to Web. Moment of Resistance. (r)	Axis Normal to Web. Square of Radius of Gyration. (ρ^2)	Axis Normal to Web. Transverse Value (v) in lbs. For Iron	Axis Normal to Web. Transverse Value (v) in lbs. For Steel	Axis Parallel to Web. Moment of Inertia. (i)	Axis Parallel to Web. Moment of Resistance. (r)	Axis Parallel to Web. Square of Radius of Gyration. (ρ^2)	Axis Parallel to Web. Distance of Neutral Axis from Back of Web.	Axis Parallel to Web. Transverse Value (v) in lbs. For Iron	Axis Parallel to Web. Transverse Value (v) in lbs. For Steel
F..........	$15\frac{3}{16}$	180	4.17	0.82	3.62	10.76	18.00	524.0	70.00	29.20	560000	700000	21.60	6.93	1.21	1.06	55600	69500
F..........	$15\frac{3}{8}$	150	3.97	0.62	3.38	8.24	15.00	467.5	62.30	31.30	498400	623000	19.70	6.84	1.32	1.09	54720	68400
A	15	230	5.016	1.016	5.05	12.90	23.00	660.7	88.09	28.73	705000	881000	37.56	10.05	1.63	1.28	80500	100500
B,A,C,D	15	200	4.38	1.00	3.41	13.18	20.00	554.6	73.94	27.73	591500	739400	23.61	7.15	1.18	1.08	57200	71500
A,B,C,D	15	190	4.75	0.75	4.75	9.50	19.00	586.0	78.13	30.84	625000	781300	32.25	9.24	1.70	1.26	74000	92400
D,A,B,C,E ..	15	175	4.75	0.75	3.85	9.80	17.50	527.0	70.26	30.03	562080	702600	31.41	8.92	1.80	1.23	71360	89200
B,A,C,D,E...	15	150	4.00	0.625	3.38	8.24	15.00	449.1	59.88	29.94	479000	598800	18.27	6.09	1.22	1.00	48720	60900
A,E	15	120	4.00	0.50	2.62	6.76	12.00	376.0	50.13	31.33	401000	501300	14.47	4.74	1.21	0.95	38000	47400
A..........	$12\frac{1}{4}$	178	4.31	1.00	3.83	10.14	17.80	339.4	55.41	19.07	443300	554100	20.65	6.55	1.16	1.16	52500	65500
A,F	$12\frac{1}{4}$	140	4.00	0.69	3.51	6.98	14.00	291.6	47.64	20.83	381000	476400	17.87	6.20	1.28	1.12	49600	62000
F,A	$12\frac{1}{4}$	100	3.28	0.46	2.60	4.80	10.00	214.3	35.00	21.40	280000	350000	8.93	3.68	0.88	0.86	29440	36800
F,A	$12\frac{1}{4}$	80	2.95	0.39	2.03	3.94	8.00	168.9	27.70	21.10	221600	277000	5.44	2.39	0.67	0.73	19120	23900
A	$12\frac{1}{4}$	70	3.00	0.33	1.69	3.62	7.00	153.2	25.03	21.89	200000	250300	5.04	2.24	0.72	0.755	18000	22400
B,C,D,E	12	150	3.50	1.00	2.25	10.50	15.00	235.7	39.29	15.72	314200	392900	8.44	3.13	0.56	0.80	25040	31300
D,B,C,E	12	90	3.00	0.44	2.20	4.60	9.00	181.5	30.25	20.16	242000	302500	7.11	3.29	0.79	0.84	26320	32900
C,E.........	12	60	2.61	0.28	1.44	3.06	5.94	123.7	20.62	20.79	164960	206200	3.22	1.62	0.55	0.62	12960	16200
A,F	$10\frac{1}{2}$	105	3.19	0.80	1.50	7.50	10.50	129.4	24.64	12.32	197000	246400	4.96	1.98	0.47	0.69	16000	19800
F,A	$10\frac{1}{2}$	75	2.84	0.42	1.80	3.90	7.50	116.0	22.10	15.50	176800	221000	5.20	2.55	0.71	0.80	20400	25500
A,F	$10\frac{1}{2}$	60	2.75	0.375	1.20	3.60	6.00	88.4	16.84	14.73	134700	168400	3.84	1.81	0.64	0.63	14500	18100
F	$10\frac{1}{2}$	55	2.59	0.30	1.30	2.90	5.50	87.0	16.60	15.90	132800	166000	3.20	1.70	0.58	0.70	13600	17000
D	10	129	3.50	1.06	1.72	9.44	12.88	140.0	28.00	10.82	224000	280000	7.79	2.93	0.61	0.84	23440	29300
B	10	111	3.00	0.875	1.83	7.44	11.10	128.6	25.72	11.59	205760	257200	5.26	2.35	0.47	0.76	18800	23500
A,B,C,D,E ..	10	92	2.94	0.75	1.17	6.86	9.20	100.5	20.10	10.92	161000	201000	3.02	1.31	0.33	0.626	10500	13100
B,A,C,D,E ..	10	75	2.63	0.50	1.63	4.24	7.50	97.4	19.47	12.98	155760	194700	3.51	1.78	0.47	0.66	14240	17800
B,A,C,D,E ..	10	57	2.25	0.375	1.21	3.28	5.70	73.2	14.63	12.84	117040	146300	1.97	1.15	0.35	0.53	9200	11500
A,D,E	10	48	2.50	0.31	0.97	2.86	4.80	64.0	12.80	13.33	102400	128000	2.20	1.14	0.46	0.565	9100	11400
A	9	109	3.56	0.875	2.15	6.60	10.90	108.7	24.16	9.97	193300	241600	7.30	2.75	0.67	0.91	22000	27500
E,A,B,C.....	9	90	2.83	0.705	1.59	5.82	9.00	89.1	19.80	9.92	158400	198000	4.25	2.02	0.47	0.73	16160	20200
A,B,C,D,E,F.	9	70	3.125	0.44	1.85	3.30	7.00	82.1	18.24	11.73	146000	182400	5.35	2.35	0.76	0.85	18800	23500
A,B,C,D,F...	9	50	2.50	0.33	1.21	2.58	5.00	58.8	13.07	11.76	104600	130700	2.53	1.35	0.51	0.63	10800	13500
D,C.........	9	37	2.19	0.25	0.84	2.02	3.70	42.0	9.33	11.42	74640	93300	1.52	0.99	0.41	0.55	7920	9900
E...........	8	84	2.75	0.75	1.75	4.90	8.40	64.5	16.14	7.67	129100	161400	4.00	1.98	0.48	0.73	15840	19800
A,C,E	8	74	2.86	0.625	1.62	4.16	7.40	60.1	15.03	8.12	120200	150300	3.53	1.69	0.48	0.755	13500	16900
A,C,D,E	8	62	2.56	0.56	1.14	3.92	6.20	48.4	12.10	7.81	97000	121000	1.94	0.98	0.31	0.584	7840	9800
A,B,C,D,E ..	8	45	2.50	0.26	1.38	1.74	4.50	44.5	11.12	9.89	89000	111200	2.54	1.46	0.56	0.76	11700	14600
C,B,D	8	30	2.00	0.20	0.67	1.62	2.96	28.2	7.06	9.55	56480	70600	1.06	0.71	0.36	0.50	5680	7100
E,C.........	7	60	2.55	0.55	1.42	3.16	6.00	37.9	10.83	6.30	86640	108300	2.85	1.53	0.48	0.68	12240	15300
A,B,C,D,E ..	7	46	2.29	0.49	0.78	3.04	4.60	25.6	7.31	5.57	58500	73100	1.11	0.62	0.24	0.51	4960	6200
A,B,C,D,E ..	7	36	2.50	0.25	1.06	1.48	3.60	27.1	7.74	7.53	62000	77400	1.96	1.10	0.54	0.715	8800	11000
B,A,C,D	7	25	2.00	0.22	0.57	1.36	2.50	17.6	5.03	7.05	40240	50300	0.75	0.49	0.30	0.470	3920	4900
A	6	66	2.85	0.75	1.51	3.58	6.60	28.0	9.33	4.24	74600	93300	2.95	1.43	0.45	0.788	11440	14300
A,B,C,F.....	6	54	2.60	0.625	1.15	3.10	5.40	23.4	7.80	4.33	62400	78000	1.84	0.95	0.34	0.662	7600	9500
A,C,D,E,F ..	6	45	2.50	0.40	1.30	1.90	4.50	21.7	7.23	4.82	58000	72300	2.12	1.20	0.47	0.725	9600	12000
A,B,C,D,E,F.	6	33	2.25	0.28	0.95	1.40	3.30	17.2	5.73	5.21	46000	57300	1.30	0.80	0.39	0.630	6400	8000
B,A,C,D,E,F.	6	22	1.75	0.17	0.65	0.90	2.20	10.4	3.47	4.74	27760	34700	0.62	0.46	0.28	0.400	3680	4600
E,C.........	5	42	2.23	0.545	1.03	2.14	4.20	13.4	5.34	3.17	42720	53400	1.50	0.94	0.36	0.640	7520	9400
A,C,D,E	5	34	1.92	0.50	0.65	2.10	3.40	10.3	4.12	3.03	33000	41200	0.63	0.44	0.19	0.493	3520	4400
D,A,C,E	5	26	1.88	0.25	0.80	1.00	2.60	9.5	3.80	3.72	30400	38000	0.87	0.68	0.34	0.610	5440	6800
B,D,F.......	5	17	1.75	0.19	0.44	0.82	1.70	6.4	2.54	3.74	20320	25400	0.43	0.37	0.25	0.470	2928	3660
D,C,E.......	4	31.5	2.06	0.44	0.91	1.33	3.15	7.0	3.50	2.16	28000	35000	1.14	0.83	0.36	0.680	6640	8300
B,A,C,D,E ..	4	24	2.00	0.31	0.71	0.98	2.40	5.5	2.77	2.30	22160	27700	0.79	0.56	0.33	0.600	4512	5640
A,B,D	4	16.5	1.50	0.20	0.50	0.65	1.65	3.9	1.95	2.36	15600	19500	0.32	0.31	0.19	0.460	2480	3100
F...........	4	13.5	1.44	0.17	0.42	0.51	1.35	3.25	1.62	3.41	12960	16200	0.23	0.22	0.17	0.380	1760	2200
A	3	25	1.83	0.53	0.67	1.16	2.50	2.70	1.80	1.08	14400	18000	0.47	0.37	0.19	0.569	2960	3700
B,A,C,E.....	3	18	1.63	0.375	0.48	0.84	1.80	2.26	1.51	1.26	12080	15100	0.36	0.33	0.20	0.530	2616	3270
A,B,C,E.....	3	15	1.50	0.20	0.53	0.44	1.50	2.00	1.33	1.33	10640	13300	0.29	0.29	0.19	0.510	2320	2900
C...........	$2\frac{1}{4}$	11.3	1.38	0.25	0.34	0.45	1.13	0.80	0.71	0.72	5680	7100	0.21	0.24	0.185	0.460	1920	2400
C...........	2	8.75	1.09	0.27	0.27	0.34	0.88	0.48	0.48	0.55	3840	4800	0.08	0.11	0.096	0.370	880	1100
E	$1\frac{3}{4}$	3.9	0.63	0.13	0.11	0.17	0.39	0.16	0.19	0.41	1520	1900	0.014	0.032	0.036	0.200	256	360

Back of
Foldout
Not Imaged

Table XXII.

LIST OF IRON AND STEEL EVEN-LEGGED ANGLES.

(For Information as to Use of this Table, see Table XIX.)

Mills Rolling Shape.	Size of Angle.	Weight per Yard.	Actual Length of Legs.	Thickness.	Area of each Leg.	Total Area. (a)	Axis Parallel to One Side.						Axis at 45°.	
							Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Distance of Centre of Gravity from Base.	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.	Moment of Inertia. (i)	Square of Radius of Gyration. (ρ^2)
C,E.........	6 ×6	110	6.562	1	5.50	11.	35.46	7.54	3.24	1.860	60350	75400	15	1.37
A,B,C,D,E...	6 ×6	97.3	6.375	.875	4.865	9.73	31.91	7.01	3.28	1.820	56100	70100	13.10	1 34
A,B,C,D,E,F.	6 ×6	57.5	6	.500	2.875	5.75	19.91	4.61	3.46	1.685	36900	46100	7.75	1.35
B,C	6 ×6	50.3	6	.438	2.515	5.03	17.22	3.90	3.42	1.580	31200	39000	6.77	1.35
C...........	5 ×5	90	5.562	1	4.50	9	19.64	4.97	2.19	1.610	39760	49700	8.67	.96
B,C,F	5 ×5	62	5.282	.688	3.10	6.20	14.70	3.94	2.37	1.550	31550	39400	6.07	.98
B,C	5 ×5	37	5.	.406	1.85	3.70	9.35	2.64	2.52	1.460	21100	26400	3.77	1.02
A...........	4½×4½	61.9	4.812	.750	3.095	6.19	11.20	3.28	1.80	1.396	26250	32800	4.88	.772
A...........	4½×4½	37.5	4.500	.438	1.875	3.75	7.20	2.24	1.92	1.286	18000	22400	2.65	.707
A,C,E,F.....	4 ×4	54.4	4.375	.750	2.72	5.44	7.66	2.47	1.40	1.271	19800	24700	3.45	.634
B,A,C,E,F...	4 ×4	51.6	4.313	.688	2.58	5.16	7.18	2.32	1.39	1.220	18550	23200	3.01	.624
A,B,C,D,E,F.	4 ×4	28.6	4	.375	1.43	2.86	4.36	1.52	1.52	1.138	12200	15200	1.86	.650
E...........	3½×3½	51	3.875	.750	2.55	5.10	6.38	2.40	1.21	1.220	19200	24000	2.40	.470
A,B,C,E,F...	3½×3½	43.4	3.813	.688	2.17	4.34	4.68	1.74	1.08	1.122	13920	17400	2.04	.470
A,B,C,D,E,F.	3½×3½	24·8	3.500	.375	1.24	2.48	2.86	1.15	1.15	1.013	9200	11500	1.20	.484
B...........	3½×3½	20.5	3.500	.313	1.025	2.05	2.30	.90	1.12	.930	7200	9000	.95	162
A,C,E,F.....	3 ×3	36.5	3.438	.688	1.825	3.65	2.77	1.13	.76	.996	9040	11300	1.05	338
B,A,C,D,E,F.	3 ×3	28.1	3.250	.500	1.405	2.81	2.23	.96	.79	.930	7680	9600	.95	.336
A,B,C.......	3 ×3	14.4	3	.250	.72	1.44	1.24	.58	.86	.842	4640	5800	.52	.361
A,B,C,D,E...	2¾×2¾	27.7	3	.563	1.385	2.77	1.83	.79	.66	.887	6320	7900	.61	.300
A,B,C,E.....	2¾×2¾	16.2	2.750	.313	.81	1.62	1.15	.59	.71	.802	4720	5900	.50	.309
C,B.........	2¾×2¾	13.1	2.750	.250	.655	1.31	.95	.48	.72	.780	3840	4800	.39	.303
B,E,F	2½×2½	23.6	2.781	.500	1.18	2.36	1.22	.61	.52	.770	4880	6100	.52	.221
A,B,C,D,E,F.	2½×2½	22.5	2.750	.500	1.125	2.25	1.28	.66	.57	.806	5280	6600	.51	.227
A,B,C,D,E,F.	2½×2½	11.9	2.500	.250	.595	1.19	.70	.39	.59	.717	3120	3900	.30	.252
B...........	2½×2½	10.5	2.500	.219	.525	1.05	.62	.34	.59	.700	2720	3400	.25	.240
B,E,F.......	2¼×2¼	18.3	2.500	.438	.915	1.83	.82	.46	.45	.740	3680	4600	.35	.194
A,B,C,D,E,F.	2¼×2¼	17.8	2.438	.438	.89	1.78	.78	.45	.44	.720	3600	4500	.35	.197
A,B,C,D,E,F.	2¼×2¼	10.6	2.250	.250	.53	1.06	50	.31	.47	.654	2480	3100	.22	.208
B...........	2¼×2¼	8	2.250	.188	.40	.80	.40	.26	.50	.690	2080	2600	.17	.212
F...........	2 ×2	19.2	2.250	.500	.96	1.92	.78	.51	.38	.730	4080	5100	.28	.150
A,B,C,D,E,F.	2 ×2	13.6	2.156	.375	.68	1.36	.45	.30	.33	.634	2400	3000	.21	.154
A,B,C,D,E,F.	2 ×2	9.4	2	.219	.47	.94	.31	.22	.33	.580	1765	2200	.13	.158
C,B.........	2 ×2	7.1	2	.188	.355	.71	.27	.19	.38	.570	1520	1900	.11	.160
E,F.........	1¾×1¾	15	2	.438	.75	1.50	.48	.35	.31	.640	2800	3500	.18	.120
A,B,C,D,E,F.	1¾×1¾	10	1.875	.313	.50	1	.27	.20	.27	.550	1600	2000	.12	.120
A,B,C,D,E,F.	1¾×1¾	6.21	1.750	.188	31	.62	.18	.15	.29	.507	1200	1500	.08	.129
C,E,F.......	1½×1½	9.80	1.688	.375	.49	.98	.19	.17	.19	.510	1360	1700	.09	.096
A,C,E,F.....	1½×1½	8.40	1.625	.313	.42	.84	.16	.14	.19	.487	1120	1400	.07	.083
B,A,C,E,F...	1½×1½	7.10	1.594	.250	.355	.71	.14	.12	.19	.450	960	1200	.06	.081
A,B,C,D,E,F.	1½×1½	5.27	1.500	.188	.265	.53	.11	.10	.21	.444	800	1000	.05	.094
B,D.........	1½×1½	4.40	1.500	.156	.22	.44	.09	.085	.21	.440	680	850	.04	.081
F...........	1¼×1¼	7.50	1.438	.313	.375	.75	.123	.126	.152	.460	1010	1260	.05	.070
A,C,E,F.....	1¼×1¼	5.63	1.375	.250	.28	.56	.077	.079	.138	.404	632	790	.04	.071
A,B,C,D,E,F.	1¼×1¼	3	1.250	.125	.15	.30	.044	.050	.147	.358	400	500	.02	.067
A,C,F.......	1 ×1	4.40	1.125	.250	.22	.44	.037	.047	.084	.340	376	470	.02	.045
B,A,C,D,E,F.	1 ×1	3.60	1.063	.185	.18	.36	.030	.040	.084	.310	320	400	.01	.040
A,B,C,D,E,F.	1 ×1	2.30	1	.125	.115	.23	.022	.031	.096	.296	250	310	.01	.043
A,F.........	⅞× ⅞	2.93	.938	.188	.145	.29	.019	.029	.066	.286	232	290	..	..
A,F.........	⅞× ⅞	2.03	.875	.125	.100	20	.014	.023	.070	.264	184	230	..	..
A,F.........	¾× ¾	2.46	.813	.188	.125	.25	.012	.021	.048	.254	168	210	..	..
A,F.........	¾× ¾	1.72	.750	.125	.085	.17	.009	.017	.053	.233	136	170	..	..

Back of
Foldout
Not Imaged

Table XXIII.

LIST OF IRON AND STEEL UNEVEN-LEGGED ANGLES.

(For Information as to Use of this Table, see Table XIX.)

Mills Rolling Shape.	Size of Angle.	Weight per Yard.	Thickness of Legs.	Actual Length of Long Leg.	Actual Length of Short Leg.	Area of Long Leg.	Area of Short Leg.	Total Area. (a)	Axis Parallel to Short Leg. Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Distance of Centre of Gravity from Base.	Transverse Value (r) in lbs. For Iron.	For Steel.	Axis Parallel to Long Leg. Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Distance of Centre of Gravity from Base.	Transverse Value (r) in lbs. For Iron.	For Steel.	Axis Paral'l to A-B. Moment of Inertia. (i)	Square of Radius of Gyration. (ρ^2)
C..........	7 × 3½	95	1	7.38	3.88	6.50	3	9.50	45.37	9.72	4.80	2.71	77760	97200	7.53	2.61	.77	.96	20880	26100	6.70	0.71
C..........	7 × 3½	61.7	.625	7	3.50	4.17	2	6.17	30.25	6.82	4.88	2.57	54560	68200	5.28	1.96	.85	.82	15680	19600	4.45	0.72
C..........	6½ × 4	95	1	7.06	4.56	6	3.50	9.50	38.66	8.25	4.08	2.38	66000	82500	11.	3.21	1.17	1.13	25680	32100	8.35	0.86
B,C........	6½ × 4	74.8	.75	6.84	4.34	4.70	2.78	7.48	31.74	6.87	4.21	2.23	54960	68700	8.88	2.70	1.19	1.03	21600	27000	6.75	0.90
B,C........	6½ × 4	40.7	.406	6.50	4	2.54	1.53	4.07	17.61	4.07	4.33	2.18	32560	40700	5.38	1.75	1.32	.91	14000	17500	3.60	0.88
C..........	6 × 4	90	1	6.56	4.56	5.50	3.50	9	30.75	7	3.42	2.17	56000	70000	10.75	3.17	1.19	1.17	25360	31700	7.46	0.83
A,B,C,D,E,F.	6 × 4	69.4	.75	6.31	4.31	4.22	2.72	6.94	24.52	5.75	3.53	2.05	46000	57500	8.68	2.68	1.25	1.08	21450	26800	5.72	0.83
A,B,C,D,E,F.	6 × 4	41.8	.438	6	4	2.53	1.65	4.18	15.46	3.83	3.70	1.96	30650	38300	5.60	1.84	1.31	.96	14720	18400	3.55	0.85
B..........	6 × 4	36.5	.375	6	4	2.20	1.45	3.65	13.60	3.40	3.72	2	27200	34000	5	1.65	1.37	.97	13200	16500	2.89	0.79
C..........	6 × 3½	85	1	6.56	4.06	5.50	3	8.50	29.24	6.79	3.46	2.26	54300	67900	7.21	2.36	.85	1.01	18880	23600	5.75	0.66
B,C........	6 × 3½	56.2	.625	6.25	3.75	3.59	2.03	5.62	20.08	4.92	3.57	2 17	43360	49200	5.18	1.81	.92	.90	14480	18100	3.78	0.67
B,C........	6 × 3½	33.8	.375	6	3.50	2.17	1.21	3.38	12.59	3.24	3.72	2.11	25900	32400	3.38	1.26	1	.82	10100	12600	2.11	0.62
C..........	5½ × 3½	52.3	.625	5.75	3.75	3.27	1.96	5.23	15.73	4.09	2 99	1.91	32700	40900	4.96	1.75	.94	.91	14000	17500	3.35	0.64
C..........	5½ × 3½	32.3	.375	5.50	3.50	1.99	1.24	3.23	10.12	2.75	3.13	1.82	22000	27500	3.27	1.22	1.10	.82	9760	12200	2.14	0.66
C..........	5 × 4	80	1	5.63	4.63	4.50	3.50	8	18.17	4.68	2.28	1.75	37450	46800	10.17	3	1.28	1.25	24000	30000	6.10	0.74
B,C,E.......	5 × 4	53.1	.625	5.25	4.25	2.97	2.34	5.31	12.76	3.49	2.40	1.60	27900	34900	7.27	2.31	1.37	1.11	18480	23100	3.93	0.74
B,C,E.......	5 × 4	31.9	.375	5	4	1.79	1.40	3.19	8.06	2.33	2.53	1.55	18600	23300	4.59	1.55	1.44	1.04	12400	15500	2.20	0.69
A,B,C,E,F...	5 × 3½	58.1	.75	5.38	3.88	3.47	2.34	5.81	13.90	3.82	2.39	1.74	30550	38200	5.40	1.86	.93	.99	14880	18600	3.72	0.64
A,B,C,D,E,F.	5 × 3½	30.5	.375	5	3.50	1.80	1.25	3.05	7.78	2.30	2.55	1.61	18400	23000	3.19	1.21	1.05	.86	9680	12100	1.96	0.64
B..........	5 × 3½	27.5	.313	5	3.50	1.65	1.10	2.75	7.13	2.03	2.59	1.50	16250	20300	2.92	1.07	1.06	.79	8560	10700	1 94	0.71
C,E,F......	5 × 3	54.4	.75	5.38	3.38	3.47	1.97	5.44	13.15	3.72	2.40	1.84	29750	37200	3.51	1.38	.64	.84	11040	13800	2.58	0.48
B,C,D,E,F...	5 × 3	47.1	.625	5.28	3.28	2.98	1.73	4.71	11.46	3.32	2.43	1.83	26550	33200	3.09	1.25	.66	.81	10000	12500	2.11	0.45
B,C,D,E,F...	5 × 3	25	.343	5	3	1.60	.90	2.50	6.04	1.88	2.56	1.80	15050	18800	1.75	.77	.72	.74	6160	7700	.97	0.41
F..........	4½ × 3	52.5	.75	4.88	3.38	3.19	2.06	5.25	12.20	3.81	2.10	1.68	30500	38100	4.52	1.85	.77	.94	14800	18500	2.54	0.48
A,B,C,D,F...	4½ × 3	43	.625	4.75	3.25	2.62	1.68	4.30	8.44	2.66	1.96	1.58	21300	26600	2.90	1.20	.67	.83	9600	12000	2.45	0.48
A,B,C,D,F...	4½ × 3	26.7	.375	4.50	3	1.62	1.05	2.67	5.49	1.83	2.05	1.49	14650	18300	1.98	.88	.74	.74	7040	8800	1.25	0.47
E,F........	4 × 3½	54.9	.75	4.38	3.88	2.93	2.56	5.49	9.14	3.08	1.66	1.41	24640	30800	6.65	2.50	1.21	1.16	20000	25000	2.80	0.57
B,C,D,E,F...	4 × 3½	39.7	.563	4.19	3.69	2.13	1.84	3.97	5.89	1.99	1.49	1.23	15900	19900	4.21	1.56	1.06	.99	12480	15600	2.29	0.58
B,C,D,E,F...	4 × 3½	26.5	.375	4	3.50	1.42	1.23	2.65	4.14	1.43	1.56	1.12	11450	14300	2.98	1.14	1.10	.89	9120	11400	1.61	0.61
E,F........	4 × 3	51	.75	4.38	3.38	2.93	2.17	5.10	8.70	3	1.69	1.49	24000	30000	4.38	1.80	.86	.99	14400	18000	2.05	0.40
B,A,C,D,E,F.	4 × 3	36.9	.563	4.25	3.25	2.13	1.56	3 69	5.67	1.97	1 54	1.36	15760	19700	2.73	1.14	.74	.86	9120	11400	1.56	0.42
A,B,C,D,E,F.	4 × 3	20.9	.313	4	3	1.20	.89	2.09	3.37	1.23	1.61	1.26	9850	12300	1.64	.73	.78	.76	5856	7320	.77	0.37
E..........	3½ × 3	47.4	.75	3.88	3.38	2.56	2.18	4.74	6.07	2.40	1.28	1.29	19200	24000	4.21	1.80	.88	1.04	14400	18000	1.86	0.39
B,A,C,D,E...	3½ × 3	34.1	.563	3.75	3.25	1.90	1.51	3.41	3.90	1.48	1.14	1.12	11850	14800	2.58	1.08	.76	.88	8640	10800	1.35	0.40
A,B,C,D,E...	3½ × 3	15.6	.25	3.50	3	.84	.72	1.56	1.91	.74	1.23	1.04	6200	7740	1.30	.59	.83	.79	4720	5900	.61	0.39
B,C........	3½ × 2½	28.1	.50	3.75	2.75	1.66	1.15	2.81	3.43	1.35	1.21	1.22	10800	13500	1.47	.72	.52	.72	5760	7200	.80	0.28
B,C........	3½ × 2½	15	.25	3.50	2.50	.88	.62	1.50	1.89	.79	1.25	1.12	6320	7900	.81	.43	.53	.62	3440	4300	.43	0.29
A..........	3½ × 1½	11.9	.25	3.50	1.50	.84	.35	1.19	1.50	.69	1.26	1.32	5500	6900	.17	.144	.14	.32	1150	1440	.16	0.134
E..........	3¼ × 2	25.5	.50	3.50	2.25	1.60	.95	2.55	2.93	1.30	1.14	1.24	10400	13000	.91	.560	.36	.61	4480	5600	.80	0.31
E..........	3¼ × 2	12.6	.25	3.25	2	.78	.48	1.26	1.36	.63	1.08	1.10	5040	6300	.40	.260	.32	.48	2080	2600	.41	0.32
A,B,C,D,E...	3 × 2½	27.7	.563	3.31	2.81	1.52	1.25	2.77	2.34	1.02	.84	1.02	8160	10200	1.45	.710	.52	.77	5680	7100	.91	0.33
A,B,C,D,E...	3 × 2½	13.1	.25	3	2.50	.72	.59	1.31	1.17	.56	.89	.91	4480	5600	.74	.402	.57	.66	3220	4020	.36	0.28
A,C,E,F.....	3 × 2	22.5	.50	3.28	2.28	1.38	.87	2.25	1.93	.87	.86	1.08	6960	8700	.62	.365	.28	.58	2920	3650	.47	0.21
B,A,C,D,E,F.	3 × 2	17.8	.375	3.13	2.13	1.07	.71	1.78	1.54	.73	.86	1.02	5840	7300	.56	.350	.31	.53	2800	3500	.39	0.22
A,B,C,D,E,F.	3 × 2	10.4	.219	3	2	.63	.41	1.04	.97	.48	.93	.98	3840	4800	.35	.230	.34	.48	1840	2300	.24	0.23
C,E........	2½ × 2	20	.50	2.75	2.25	1.13	.87	2	1.09	.58	.55	.87	4640	5800	.63	.386	.31	.62	3100	3860	.37	0.18
C,E........	2½ × 2	10.6	.25	2.50	2	.59	.47	1.06	.71	.41	.66	.78	3280	4100	.37	.253	.35	.54	2030	2530	.20	0.18
C,F........	2¼ × 1½	12.6	.375	2.44	1.69	.77	.49	1.26	.50	.31	.40	.82	2480	3100	.21	.168	.16	.44	1350	1680	.15	0.12
C,B,F......	2¼ × 1½	6.7	.188	2.25	1.50	.41	.26	.67	.34	.23	.50	.76	1840	2300	.13	.115	.18	.38	920	1150	.08	0.12
E..........	2 × 1⅜	12	.375	2.13	1.50	.72	.48	1.20	.50	.36	.42	.76	2880	3600	.20	.190	.17	.44	1520	1900	.11	0.095
E..........	2 × 1⅜	7.8	.250	2.00	1.38	.47	.31	.78	.31	.23	.45	.69	1840	2300	.12	.120	.15	.37	960	1200	.08	0.106
C..........	2 × 1¼	9.2	.375	2.19	1.44	.60	.32	.92	.32	.21	.35	.70	1680	2100	.10	.089	.17	.32	712	890	.08	0.084
C..........	2 × 1¼	5.7	.188	2	1.25	.36	.21	.57	.23	.16	.40	.68	1280	1600	.07	.075	.15	.31	600	750	.05	0.096

Back of
Foldout
Not Imaged

Table XXIV.

LIST OF IRON AND STEEL TEES.

(For Information as to the Use of this Table, see Table XIX.)

Mills Rolling Shape.	Width of Flange (b)	Depth over all. (d)	Weight per Yard.	Thickness of Flange.	Area of Flange.	Area of Web.	Total Area. (a)	Axis Normal to Web.						Axis Parallel to Web.				
								Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Distance of Centre of Gravity from Base.	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel	Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel
F	6	4	47.5	0.500	3	1.75	4.75	6.25	2.07	1.32	.99	16560	20700	9.04	3.01	1.99	24080	30100
C	5	4	44.1	0.500	2.50	1.91	4.41	6.24	2.08	1.42	1.08	16640	20800	5.25	2.10	1.19	16800	21000
C	5	3½	48.44	0.500	2.50	2.35	4.85	5.37	2.17	1.10	1.05	17360	21700	5.31	2.12	1.08	16960	21200
E,F	5	3	39	0.600	2.80	1.10	3.90	2.50	1.10	.64	73	8800	11000	5.70	2.30	1.46	18400	23000
B	5	2¾	35	0.500	2.53	.97	3.50	2.21	1.11	.62	.77	8880	11100	5.24	2.10	1.49	16800	21000
A,C,E,F.....	5	2½	35	0.500	2.50	1	3.50	1.50	.80	.43	.61	6400	8000	5.23	2.09	1.49	16720	20900
E,C,F	5	2½	30.8	0.500	2.07	1.01	3.08	1.40	.71	.44	.58	5680	7100	4.60	1.80	1.46	14400	18000
B	5	2⅜	29	0.375	1.88	1.02	2.90	1.39	.81	.48	.66	6480	8100	3.94	1.58	1.37	12640	15800
E,C	4½	3½	45	0.560	2.25	2.25	4.50	5.20	2.18	1.14	1.13	17440	21800	3.90	1.70	.86	13600	17000
B	4½	3	25	0.313	1.41	1.09	2.50	1.94	86	.77	.76	6880	8600	2.39	1.06	.96	8480	10600
E	4	5	42	0.625	2	2.20	4.20	10.50	3.05	2.46	1.57	24400	30500	2.70	1.40	.64	11200	14000
E,C	4	4½	40.5	0.625	2	2.05	4.05	7.80	2.48	1.93	1.37	19840	24800	2.70	1.40	.67	11200	14000
A,C,E,F	4	4	37.5	0.500	2	1.75	3.75	5.56	1 97	1.48	1.18	15760	19700	2.62	1.31	.70	10480	13100
C	4	3½	41.8	0.438	2.25	1.93	4.18	4.65	1.94	1.10	1.09	15520	19400	3.23	1.61	.77	12880	16100
E,C,D	4	3	27.8	0.470	1.75	1.03	2.78	2.10	.96	.76	.80	7680	9600	2.30	1.10	.81	8800	11000
E	4	2½	22.5	0.438	1.50	.75	2.25	1.10	.60	.49	.62	4800	6000	2.00	1	.86	8000	10000
E,B,C,F	4	2	19.5	0.375	1.37	.58	1.95	.54	.35	.28	.46	2800	3500	1.80	.91	.92	7280	9100
E	3½	4	33.8	0.530	1.70	1.68	3.38	5.15	1.87	1.51	1.24	14960	18700	1.80	1	.52	8000	10000
A,C,E,F	3½	3½	32.5	0.500	1.75	1.50	3.25	3.64	1.49	1.12	1.06	11920	14900	1.82	1.04	.56	8320	10400
A,B,D,E,F ..	3½	3½	28.7	0.438	1.53	1.34	2.87	3.26	1.32	1.14	1.03	10560	13200	1.53	.87	.53	6960	8700
E,D	3½	3	27.8	0.530	1.64	1.14	2.78	2.14	1	.77	.85	8000	10000	1.60	.93	.59	7440	9300
E.F	3	4	36.8	0.625	1.69	1.99	3.68	5.55	2.10	1.54	1.35	16800	21000	1.30	.87	.36	6960	8700
E	3	3½	35.3	0.690	1.82	1.71	3.53	3.93	1.67	1.12	1.15	13360	16700	1.40	.92	.38	7360	9200
C	3	3½	28.25	0.500	1.41	1.42	2.83	3.12	1.25	1.10	1.10	10000	12500	1.06	.71	.37	5680	7100
A,C,E,F	3	3	27.5	0.500	1.50	1.25	2.75	2.21	1.07	.80	.93	8560	10700	1.15	.77	.42	6160	7700
A,B,C,D,E,F,	3	3	21.1	0.375	1.13	.98	2.11	1.76	.83	.83	.89	6670	8340	.97	.65	.46	5200	6500
C,E	3	3	19.3	0.313	1.03	.90	1.93	1.59	.76	.82	.84	6080	7600	.75	.50	.38	4000	5000
C	3	2½	23.8	0.438	1.20	1.18	2.38	1.38	.85	.58	.82	6800	8500	.94	.63	.40	5040	6300
C,E	3	2½	20.6	0.438	1.21	.85	2.06	1.08	.60	.52	.70	4800	6000	.89	.60	.44	4800	6000
C,E	3	2½	17.6	0.375	1.02	.74	1.76	.94	.52	.53	.69	4160	5200	.74	.49	.42	3920	4900
A,F	3	2	17.3	0.375	1.12	.61	1.73	.54	.37	.31	.54	2960	3700	.85	.567	.49	4540	5670
A	3	2	14.6	0.313	.94	.52	1.46	.47	.32	.32	.52	2590	3240	.68	.453	.47	3630	4530
F	3	1½	15.1	0.375	1.13	.38	1.51	.22	.20	.12	.40	1600	2000	.57	.455	.38	3640	4550
C	3	1½	11.2	0.250	.75	.37	1.12	.19	.17	.17	.37	1360	1700	.56	.372	.50	2976	3720
C,E	2¾	2	21	0.344	.90	1.20	2.10	.83	.66	.40	.75	5280	6600	.63	.460	.30	3680	4600
C,E	2¾	1¾	18.75	0.344	.90	.98	1.88	.56	.51	.30	.66	4080	5100	.62	.450	.31	3600	4500
E	2½	3	19.5	0.440	.97	.98	1.95	1.66	.81	.86	.96	6480	8100	.50	.400	.26	3200	4000
B,E	2½	2¾	18	0.375	.94	.86	1.80	1.26	.66	.71	.84	5280	6600	.50	.400	.28	3200	4000
C,E	2½	2½	19.5	0.407	1.09	.86	1.95	1.12	.75	.61	.75	6000	7500	.58	.460	.30	3680	4600
A,B,C,E,F...	2½	2½	17.3	0.375	.93	.80	1.73	.98	.56	.57	.76	4500	5630	.49	.390	.28	3120	3900
A,F	2½	2½	14.7	0.313	.78	.69	1.47	.85	.48	.58	.74	3860	4830	.40	.320	.27	2560	3200
E,C	2½	1¼	9	0.313	.65	.25	.90	.09	.10	.10	.30	800	1000	.33	.260	.37	2080	2600
A,C,E	2¼	2¼	11.9	0.281	.63	.56	1.19	.56	.35	.47	.66	2820	3520	.26	.231	.22	1850	2310
F	2¼	1¼	8.1	0.250	.562	.25	.81	.09	.097	.01	.32	776	970	.24	.214	.30	1712	2140
A	2¼	1⅛	7.4	0.250	.56	.22	.78	.06	.072	.077	.29	580	720	.18	.160	.23	1280	1600
C	2¼	9/16	6.5	0.250	.56	.09	.65	.01	.026	.014	.18	2080	2600	.24	.213	.37	1704	2130
B	2⅛	1 3/16	6.5	0.188	.39	.26	.65	.86	1.060	.130	.37	8480	10600	.15	.140	.023	1120	1400
A,C,F	2	2	11.5	0.313	.63	.52	1.15	.41	.302	.352	.61	2420	3020	.21	.210	.182	1680	2100
A,B,D,E,F...	2	2	9.4	0.250	.50	.44	.94	.35	.247	.372	.59	1980	2470	.16	.160	.170	1280	1600
A,C,E	2	1½	9	0.281	.56	.34	.90	.17	.170	.190	.50	1360	1700	.18	.180	.200	1440	1800
A,C	2	1	6.5	0.250	.50	.19	.69	.04	.054	.058	.26	430	540	.14	.140	.203	1120	1400
C	2	9/16	5.88	0.250	.50	.09	.59	.01	.026	.017	.17	2080	2600	.17	.170	.292	1360	1700
F	1¾	1¾	8.20	0.250	.438	.38	.82	.235	.192	.292	.52	1536	1920	.11	.125	.137	1000	1250
C,F	1¾	1¾	7.10	0.250	.39	.32	.71	.210	.168	.292	.50	1344	1680	.10	.114	.137	912	1140
A,C,D,E,F..	1½	1½	6.88	0.250	.38	.31	.69	.130	.125	.188	.46	1000	1250	.07	.093	.101	745	930
A	1½	1	5.60	0.250	.37	.19	.56	.040	.055	.071	.28	440	550	.07	.093	.125	745	930
A,E	1¼	1¼	5.46	0.250	.31	.25	.56	.076	.089	.136	.40	710	890	.042	.067	.075	535	670
A,C,E,F....	1¼	1¼	4.86	0.219	.27	.22	.49	.065	.075	.133	.38	600	750	.034	.054	.070	430	540
A,C,E.......	1	1	3.30	0.188	.18	.15	.33	.030	.044	.091	.315	350	440	.016	.032	.048	255	320
A	1	1	2.76	0.156	.15	.13	.28	.024	.034	.085	.295	270	340	.012	.024	.043	190	240
F,E	1	1	2.30	0.125	.12	.11	.23	.022	.031	.096	.290	248	310	.011	.022	.005	176	220

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Table XXV.

LIST OF IRON AND STEEL DECKS, HALF-DECKS, AND ZEES.

(For Information as to Use of this Table, see Table XIX.)

									Axis Normal to Web.						Axis Parallel to Web.						Axis Paral'l to A B.	
Mills Rolling Shape.	Depth over all. (d)	Weight per Yard.	Width of Flanges.	Thickness of Web.	Area of Flange.	Area of Bulb.	Area of Web.	Total Area. (a)	Moment of inertia. (i)	Moment of resistance (r)	Square of radius of gyration. (ρ^2)	Distance of centre of gravity from long flange.	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.	Moment of inertia. (i)	Moment of resistance. (r)	Square of radius of gyration (ρ^2)	Distance of centre of gravity from outside of web.	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.	Moment of inertia. (i)	Square of radius of gyration. (ρ^2)
Deck Beams.																						
C........	12	104	5.75	0.406	3.59	2.89	3.90	10.4	221.98	32.80	21.34	5.24	262400	328000	9.33	3.18	0.90	....	25440	31800	....	....
B........	11½	95	5	0.438	3.05	2.05	4.40	9.5	168.75	23.35	17.72	4.27	186800	233500	5.17	2.07	0.55	....	16560	20700	....	....
C........	11	91	5.5	0.375	3.26	2.52	3.28	9.1	164.09	25.99	18.06	4.68	207900	259900	7.64	2.78	0.85	....	22240	27800	....	....
B........	10	85	5	0.438	3.05	2.05	3.40	8.5	151.53	24.32	17.80	3.77	194600	243200	5.16	2.06	0.61	....	16480	20600	....	....
C........	10	80	5.25	0.375	2.87	2.19	2.96	8	118.22	20.64	14.75	4.27	165100	206400	6.13	2.31	0.76	....	18480	23100	....	....
B,E......	9	93	5	0.593	2.50	2.30	4.50	9.3	101.08	20.39	10.96	4.03	163100	203900	4.41	1.76	0.48	....	14080	17600	....	....
C,B,E....	9	72	5	0.375	2.50	2.06	2.61	7.2	84.77	17	11.83	4	136000	170000	4.92	1.97	0.69	....	15760	19700	....	....
E,B......	8	84	4	0.750	2	1.80	4.60	8.4	63.30	14.10	7.51	3.50	112800	141000	2.96	1.48	0.35	....	11800	14800	....	....
C,A,B,E..	8	61	4.625	0.344	2.17	1.85	2.09	6.1	57.66	12.81	9.42	3.50	102500	128100	3.63	1.56	0.59	....	12480	15600	....	....
B,E......	7	75	5	0.438	2.65	2.40	2.45	7.5	53.13	12.80	7.24	2.85	102400	128000	5.34	2.14	0.72	....	17120	21400	....	....
C,A,B,E..	7	52	4.25	0.344	1.86	1.55	1.80	5.2	34.40	9.05	6.60	3.20	72400	90500	2.59	1.22	0.50	....	9760	12200	....	....
B........	6	54	4.5	0.438	2	1.40	2	5·4	25.16	6.97	4.75	2.39	55760	69700	3.08	1.32	0.58	....	10560	13200	....	....
C,B......	6	42	3.75	0.313	1.52	1.28	1.38	4.2	21.95	6.55	5.24	2.65	52400	65500	1.64	.88	0.40	....	7040	8800	....	....
C,B......	5	34	3.25	0.313	1.22	1.04	1.11	3.4	12.04	4.29	3.57	2.22	34320	42900	.98	.61	0.29	....	4880	6100	....	....
Half-Deck Beams.																						
B........	9	80	3.5	0.625	2.20	1	4.80	8	75.84	14.98	9.49	3.93	119800	149800	5.76	2.09	0.72	0.75	16720	20900	6.26	0.77
B........	9	66	3.5	0.500	1.80	1	3.80	6.6	63.96	12.54	9.67	3.90	100300	125400	4.66	1.67	0.71	0.71	13360	16700	4.90	0.74
B........	8	75	3.5	0.640	2.34	1.10	4.06	7.5	56.75	12.51	7.56	3.47	100100	125100	5.76	2.12	0.76	0.78	16960	21200	6.75	0.90
B........	8	60	3.5	0.484	1.74	1.10	3.16	6	45.84	10	7.62	3.42	80000	100000	4.42	1.60	0.74	0.73	12800	16000	5.34	0.89
B........	7½	75	3.5	0.670	2.45	1	4.05	7.5	49.78	11.58	6.66	3.20	92640	115800	6.03	2.27	0.79	0.84	18160	22700	5.68	0.76
B........	7½	60	3.5	0.515	1.86	1	3.14	6	40.55	9.37	6.76	3.16	75000	93700	4.58	1.68	0.76	0.77	13440	16800	4.50	0.75
B........	7	60	3	0.578	1.80	.95	3.25	6	34.90	8.63	5.81	3.07	69000	86300	3.35	1.44	0.56	0.68	11520	14400	3.46	0.58
B........	7	48	3	0.453	1.40	.93	2.47	4.8	28.12	7.05	5.86	3.02	56400	70500	2.58	1.09	0.53	0.65	8720	10900	2.72	0.56
B........	6	55	3	0.610	1.88	.79	2.83	5.5	22.79	6.49	4.16	2.48	51900	64900	3.35	1.49	0.61	0.75	11920	14900	2.87	0.52
B........	6	45	3	0.484	1.46	.79	2.25	4.5	18.95	5.35	4.20	2.46	42800	53500	2.62	1.14	0.58	0.70	9120	11400	2.28	0.50
B........	5	40	2.5	0.515	1.34	.69	1.97	4	12.11	4.31	3.03	2.19	34500	43100	1.69	.90	0.42	0.64	7200	9000	1.58	0.39
B........	5	30	2.5	0.360	.93	.69	1.38	3	9.20	3.25	3.06	2.17	26000	32500	1.16	.60	0.38	0.57	4800	6000	1.14	0.38
Bars.																						
					Long flan.	Short flan																
B........	10	66	3 —3	0.453	1.30	1.30	4	6.6	87.37	17.47	13.25	5	139800	174700	6.37	2.12	.98	....	16960	21200	....	....
B........	10	57	3 —3	0.375	1.12	1.12	3.46	5.7	76.88	15.37	13.47	5	123000	153700	5.60	1.87	.98	....	14960	18700	....	....
B........	6	48	3½—3	0.422	1.44	1.23	2.13	4.8	25.78	8.32	5.38	2.90	66600	83200	7.52	2.21	1.56	....	17680	22100	....	....
B........	6	42	3½—3	0.344	1.25	1.05	1.90	4.2	22.30	7.15	5.29	2.88	57200	71500	6.77	2.00	1.61	....	16000	20000	....	....
B........	5	41	3 —3	0.406	1.20	1.20	1.70	4.1	15.61	6.25	3.80	2.50	50000	62500	5.85	1.95	1.42	....	15600	19500	....	....
B...... ..	5	36	3 —3	0.344	1.04	1.04	1.52	3.6	13.50	5.40	3.76	2.50	43200	54000	5.20	1.73	1.44	....	13840	17300	....	....
B........	4	37	3 —2½	0.422	1.27	1.06	1.37	3.7	8.88	4.27	2.40	1.92	34200	42700	5.10	1.76	1.37	....	14080	17600	....	....
B...... .	4	33	3 —2½	0.375	1.13	.94	1.23	3.3	7.80	3.71	2.37	1.896	29700	37100	4.28	1.48	1.30	....	11840	14800	....	....

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TABLE XXVI.

ANALYSIS OF CAST IRONS.

	Combined Carbon.	Graphitic Carbon.	Silicon.
Greatest softness	0,15	3,1	2,5
" hardness	—	—	under 0,8
" general strength	0,50	2,8	1,42
" stiffness	—	—	1,0
" tensile strength	—	—	1,8
" crushing strength	over 1,0	under 2,6	about 0,8

TABLE XXVII.

DENSITIES AND WEIGHTS OF CAST IRONS.

MATERIAL.	Density.	Weight per cubic foot in lbs.
Dark-grey foundry-iron	6,80	425
Grey foundry-iron	7,20	450
Mottled foundry-iron	7,35	458
White iron	7,50	474

TABLE XXIX.

Classification of Structural Irons and Steels.

GENERIC TERM	IRON.								
HOW OBTAINED	CAST *or obtained from a fluid mass.*						WROUGHT. *or welded from a pasty mass.*		
DISTINGUISHING QUALITY	Fusible, but non malleable.		Fusible and slightly malleable.	Fusible and malleable.			malleable but practically non-fusible.		
SPECIES	CAST IRON.		CAST STEEL.	WROUGHT STEEL.			WROUGHT IRON.		
VARIETIES	Ordinary Castings.		Ordinary. Castings.	Crucible—Open Hearth—Bessemer *(Steel ingots cast from fluid mass.)*			Puddling Balls. *(made from pasty mass of refined pig iron.)*		
DEVELOPMENT		Malleable Cast Iron. *(made by annealing in oxides.)*		Steel Bloom *(made from ingot by rolling or hammering, when hot.)*			Muck Bar or Iron Bloom *(made by rolling or hammering puddling ball when hot.)*		Case Hardened Iron. *(made by annealing in bone dust.)*
FINAL STATE	Cast Iron.	Malleable Cast Iron.	Cast Steel.	Forged Steel. *(made by hammering when hot.)*	Rolled Steel *(made by rolling when hot.)*	Welded Steel and Iron *(made by rolling steel and iron together, when hot.)*	Forged Iron *(made by hammering when hot.)*	Rolled Iron. *(made by rolling when hot.)*	Case Hardened Iron.

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TABLE XXVIII.

CLASSIFICATION OF IRONS AND STEELS.

Name.	Percentage of Carbon.	Properties.
1. Malleable iron.	0,25	Is not sensibly hardened by sudden cooling.
2. Steely iron.	0,35	Can be slightly hardened by quenching.
3. Steel.	0,50	Gives sparks with a flint when hardened.
4. Steel.	1,00 to 1,50	Limits for steel of maximum hardness and tenacity.
5. Steel.	1,75	Superior limit of welding steel.
6. Steel.	1,80	Very hard cast steel, forging with great difficulty.
7. Steel.	1,90	Not malleable hot.
8. Cast-iron.	2,00	Lower limits of cast-iron, cannot be hammered.
9. Cast-iron.	6,00	Highest carburetted compound obtainable.

TABLE XXXII.

ULTIMATE BREAKING STRENGTH OF MATERIALS UNDER DIFFERENT KINDS OF STRAINS.

Material.	If Dead Load (Static).	Intermittent Loads (off-and-on continuously.)			
		If in one direction only.		If in opposite directions.	
		Without shock.	Rolling (dynamic).	Without shock.	Rolling (dynamic).
Wrought Iron and Steel.	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
Wrought Copper and Brass, also Slate, Timber, Masonry, etc.	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
Cast metals : Iron, Copper, Brass, Lead, etc.	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

By combining this table with the safe stresses given in Tables IV and V, that is, taking whatever part of the safe-stress there given for dead loads, that the nature of the load demands, we can obtain the safe-stress under any manner of loading. Where stresses in opposite directions take place, the material will yield in the direction of the weakest stress.

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TABLE XXX.

Amount of Extension and Contraction, in inches, of Cast and Wrought Iron Bars, 100 ft. long, under different strains.

Strain, per square inch, in pounds.	CAST IRON.		WROUGHT IRON.
	Extension, under tension.	Contraction, under compression.	Extension, under tension or contraction, under compression.
1000	0,08308	0,09155	0,0444
2000	0,17150	0,18404	0,0889
3000	0,26528	0,27747	0,1333
4000	0,36442	0,37185	0,1778
5000	0,46890	0,46715	0,2222
6000	0,57874	0,56341	0,2667
7000	0,69392	0,66061	0,3111
8000	0,81446	0,75875	0,3556
9000	0,94036	0,85782	0,4000
10000	1,07160	0,95784	0,4444
11000	1,20820	1,05880	0,4889
12000	1,35014	1,16070	0,5333
13000	1,49744	1,26354	0,5778
14000	1,65010	1,36733	0,6222
15000	1,80810	1,47205	0,6667
16000	......	1,57871	0,7111
17000	......	1,68432	0,7556
18000	......	1,79186	0,8000
19000	......	1,90035	0,8444
20000	......	2,00978	0,8889
21000	......	2,11994	0,9333
22000	......	2,23145	0,9778
23000	......	2,34370	1,0222
24000	......	2,45690	1,0667
25000	......	2,57102	1,1111

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TABLE XXXI.

Length of Cast or Wrought Iron Bars, in feet, that will stretch or contract exactly one inch under different strains.

Strain, in pounds, per square inch.	CAST IRON.		WROUGHT IRON.
	Length, in feet, to extend one inch.	Length, in feet, to shorten one inch.	Length, in feet, to either extend or shorten one inch.
1000	1204	1094	2250
2000	583	543	1125
3000	377	360	750
4000	274	269	562
5000	213	214	450
6000	173	177	375
7000	144	151	321
8000	123	132	281
9000	106	117	250
10000	93	104	225
11000	83	94	204
12000	74	86	187
13000	67	79	173
14000	61	73	161
15000	55	68	150
16000	....	63	141
17000	....	59	132
18000	....	56	125
19000	....	53	118
20000	....	50	112
21000	....	47	107
22000	....	45	102
23000	....	43	98
24000	..	41	94
25000	....	40	90

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TABLE XXXIII. PROPORTIONS FOR SCREW THREADS, NUTS, and BOLT HEADS.

Diameter of Screw.	Threads per Inch.	Diameter at root of Thread.	Width of Flat.	Nuts: Short Diameter Rough.	Nuts: Short Diameter Finish.	Nuts: Long Diameter Rough.	Nuts: Long Diameter Rough.	Nuts: Thickness Rough.	Nuts: Thickness Finish.	Bolt Heads: Short Diameter Rough.	Bolt Heads: Short Diameter Finish.	Bolt Heads: Long Diameter Rough.	Bolt Heads: Long Diameter Rough.	Bolt Heads: Thickness Rough.	Bolt Heads: Thickness Finish.
1/4	20	,185	,0062	1/2	7/16	37/64	7/10	1/4	3/16	1/2	7/16	37/64	7/10	1/4	3/16
5/16	18	,240	,0074	19/32	17/32	11/16	10/12	5/16	1/4	19/32	17/32	11/16	10/12	19/64	1/4
3/8	16	,294	,0078	11/16	5/8	51/64	63/64	3/8	5/16	11/16	5/8	51/64	63/64	11/32	5/16
7/16	14	,344	,0089	25/32	23/32	9/10	1 7/64	7/16	3/8	25/32	23/32	9/10	1 7/64	25/64	3/8
1/2	13	,400	,0096	7/8	13/16	1	1 15/64	1/2	7/16	7/8	13/16	1	1 15/64	7/16	7/16
9/16	12	,454	,0104	31/32	29/32	1 1/8	1 23/64	9/16	1/2	31/32	29/32	1 1/8	1 23/64	31/64	1/2
5/8	11	,507	,0113	1 1/16	1	1 7/32	1 1/2	5/8	9/16	1 1/16	1	1 7/32	1 1/2	17/32	9/16
3/4	10	,620	,0125	1 1/4	1 3/16	1 7/16	1 49/64	3/4	11/16	1 1/4	1 3/16	1 7/16	1 49/64	5/8	11/16
7/8	9	,731	,0138	1 7/16	1 3/8	1 21/32	2 1/32	7/8	13/16	1 7/16	1 3/8	1 21/32	2 1/32	23/32	13/16
1	8	,837	,0156	1 5/8	1 9/16	1 7/8	2 19/64	1	15/16	1 5/8	1 9/16	1 7/8	2 19/64	13/16	15/16
1 1/8	7	,940	,0178	1 13/16	1 3/4	2 3/32	2 9/16	1 1/8	1 1/16	1 13/16	1 3/4	2 3/32	2 9/16	29/32	1 1/16
1 1/4	7	1,065	,0178	2	1 15/16	2 5/16	2 53/64	1 1/4	1 3/16	2	1 15/16	2 5/16	2 53/64	1	1 3/16
1 3/8	6	1,160	,0208	2 3/16	2 1/8	2 17/32	3 3/32	1 3/8	1 5/16	2 3/16	2 1/8	2 17/32	3 3/32	1 3/32	1 5/16
1 1/2	6	1,284	,0208	2 3/8	2 5/16	2 3/4	3 23/64	1 1/2	1 7/16	2 3/8	2 5/16	2 3/4	3 23/64	1 3/16	1 7/16
1 5/8	5 1/2	1,389	,0227	2 9/16	2 1/2	2 31/32	3 5/8	1 5/8	1 9/16	2 9/16	2 1/2	2 31/32	3 5/8	1 9/32	1 9/16
1 3/4	5	1,491	,0250	2 3/4	2 11/16	3 3/16	3 57/64	1 3/4	1 11/16	2 3/4	2 11/16	3 3/16	3 57/64	1 3/8	1 11/16
1 7/8	5	1,616	,0250	2 15/16	2 7/8	3 13/32	4 5/32	1 7/8	1 13/16	2 15/16	2 7/8	3 13/32	4 5/32	1 15/32	1 13/16
2	4 1/2	1,712	,0277	3 1/8	3 1/16	3 5/8	4 27/64	2	1 15/16	3 1/8	3 1/16	3 5/8	4 27/64	1 9/16	1 15/16
2 1/4	4 1/2	1,962	,0277	3 1/2	3 7/16	4 1/16	4 61/64	2 1/4	2 3/16	3 1/2	3 7/16	4 1/16	4 61/64	1 3/4	2 3/16
2 1/2	4	2,176	,0312	3 7/8	3 13/16	4 1/2	5 31/64	2 1/2	2 7/16	3 7/8	3 13/16	4 1/2	5 31/64	1 15/16	2 7/16
2 3/4	4	2,426	,0312	4 1/4	4 3/16	4 29/32	6	2 3/4	2 11/16	4 1/4	4 3/16	4 29/32	6	2 1/8	2 11/16
3	3 1/2	2,629	,0357	4 5/8	4 9/16	5 3/8	6 17/32	3	2 15/16	4 5/8	4 9/16	5 3/8	6 17/32	2 5/16	2 15/16
3 1/4	3 1/2	2.879	,0357	5	4 15/16	5 13/16	7 1/16	3 1/4	3 3/16	5	4 15/16	5 13/16	7 1/16	2 1/2	3 3/16
3 1/2	3 1/4	3,100	,0384	5 3/8	5 5/16	6 7/64	7 39/64	3 1/2	3 7/16	5 3/8	5 5/16	6 7/64	7 39/64	2 11/16	3 7/16
3 3/4	3	3,317	,0413	5 3/4	5 11/16	6 21/32	8 1/8	3 3/4	3 13/16	5 3/4	5 11/16	6 21/32	8 1/8	2 7/8	3 13/16
4	3	3,567	,0413	6 1/8	6 1/16	7 3/32	8 41/64	4	3 15/16	6 1/8	6 1/16	7 3/32	8 41/64	3 1/16	3 15/16
4 1/4	2 7/8	3,798	,0435	6 1/2	6 7/16	7 9/16	9 3/16	4 1/4	4 3/16	6 1/2	6 7/16	7 9/16	9 3/16	3 1/4	4 3/16
4 1/2	2 3/4	4,028	,0454	6 7/8	6 13/16	7 31/32	9 3/4	4 1/2	4 7/16	6 7/8	6 13/16	7 31/32	9 3/4	3 7/16	4 7/16
4 3/4	2 5/8	4,256	,0476	7 1/4	7 3/16	8 13/32	10 1/4	4 3/4	4 11/16	7 1/4	7 3/16	8 13/32	10 1/4	3 5/8	4 11/16
5	2 1/2	4,480	,0500	7 5/8	7 9/16	8 27/32	10 49/64	5	4 15/16	7 5/8	7 9/16	8 27/32	10 49/64	3 13/16	4 15/16
5 1/4	2 1/2	4,730	,0500	8	7 15/16	9 9/32	11 23/64	5 1/4	5 3/16	8	7 15/16	9 9/32	11 23/64	4	5 3/16
5 1/2	2 3/8	4,953	,0526	8 3/8	8 5/16	9 23/32	11 7/8	5 1/2	5 7/16	8 3/8	8 5/16	9 23/32	11 7/8	4 3/16	5 7/16
5 3/4	2 3/8	5,203	,0526	8 3/4	8 11/16	10 5/32	12 3/8	5 3/4	5 11/16	8 3/4	8 11/16	10 5/32	12 3/8	4 3/8	5 11/16
6	2 1/4	5,423	,0555	9 1/8	9 1/16	10 19/32	12 15/16	6	5 15/16	9 1/8	9 1/16	10 19/32	12 15/16	4 9/16	5 15/16

Rough Nut=one and one-half diameter of bolt $+ \frac{1}{8}$.

Finished Nut=one and one-half diameter of bolt $+ \frac{1}{16}$.

Rough Nut=diameter of bolt.

Finished Nut=diameter of bolt$-\frac{1}{16}$.

Rough Head=one and one-half diameter of bolt $+ \frac{1}{8}$.

Finished Head=one and one-half diameter of bolt $+ \frac{1}{16}$.

Rough head=one-half distance between parallel sides of head.

Finished head=diameter of bolt$-\frac{1}{16}$.

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TABLE XXXIV.

Recent Tests of Irons and Steels.

Material.	Remarks.	Per Cent. of Carbon	Per Cent. of Silicon.	Per Cent. of Manganese.	Modulus of Elasticity, in pounds inch, for Tension.	Modulus of Elasticity, for Compression.	Modulus of Elasticity, for Cross-Breaking.	Tension, in pounds, per square inch. Elastic Limit.	Tension, Ultimate Stress, t.	Compression, in pounds, per square inch. Elastic Limit.	Compression, Ultimate Stress, c.	Ultimate Modulus of Rupture, per square inch, k.	Ultimate Shearing Stress, per square inch. g=across grain. g_1=along grain	Per Cent. of Elongation.	Per Cent. of Contraction of Area.	Number of tests averaged	Authority.
Wrought Iron.	Different English Irons.				28560000	28985000		28500	56000	27600				7.01 to 17,87	5,9 to 51,4	12	English Steel Committee
	" German " .	TESTED IN SIX DIFFERENT DIRECTIONS ACROSS AND ALONG THE GRAIN.											g=34 to 45000 g_1=15 to 22000				Bauschinger
	American Tie Rod 3¾"d..							22865	38085					2,4	2,2	2	Watertown Arsenal
	" " ⅝"d.							32716	54416					21,0	42,4	3	
Wrought Steel.	10" Channel							44650	73675	36235	41390			23,6	43,3	2	
	½" Rolled Plate	0,14			32000000	38250000		42000	63000	39500			g=48600	21,8	49,2		Bauschinger
	" "	0,51			31450000	32500000		48400	80000	46000			g=57000	14,3	25,1		
	" "	0,78			33550000	32450000		53200	92000	53000			g=59000	11,4	19,1	12	
	" "	0,96			30900000	32700000		69000	118000	71000			g=83000	6,6	10,0		
Cast, Crucible and Bessemer English Steels	Mean of 27 samples				29300000	30000000		47600	88800	47000				0,89	1,30		English Steel Committee
	Lowest (Bessemer)				28400000	29100000		35840	74000	33600				to	to	27	
	Highest (Cast)				29900000	31500000		60480	122600	60480				13,61	48,7		
American Steel.	Tubes for Rifled Cannon							32 to 52000	58 to 91000	46 to 54000	83 to 104000			4.7 to 30	11,8 to 59,3	206	Watertown Arsenal
	8" Rifled Cannon.							30 to 67000	74 to 115000					5,8 to 31	0 to 57	52	
Steel Wire.	Am. tinned wr. for Can'n								160 to 181000						8,4 to 21,8	13	
	Foreign wire " "								264 to 329680						12,2 to 23,5	9	
Cast Steels.	Cast Steel, Open-Hearth	0,55	0,13	0,96	& 0,029%	of phosphorus	32000000	35 to 48000	60 to 97440	39 to 45000	65 to 74800	134340		2,0 to 18,0	1,5 to 30,6	23	
	Averaged from 14 tests	0,33	0,23	0,62					74000					23	41	5	Foster
		0,50	0.40	0,66					96000					9	12	3	
		0,77	0,46	0,67					85000					1,5	2	3	
		0,96	0,62	0,64					78000					1,33	1,6	3	
	American Steel				29120000		24400000	29120	47264	28672		24125		0,4 to 29,9		11	Abbott
Steel.	Bessemer Plates	TESTED IN SIX DIFFERENT DIRECTIONS ACROSS AND ALONG THE GRAIN.											g=53 to 66000 g_1=47 to 56000				Bauschinger
Malleable Cast Iron.	Square ¾"	American						1635	36176					3,8	5,8	2	Ricketts
	Rectangular 1"×⅜"	"						2979	34272					2,4	6,6	1	
	Circular ½" diameter	"						2285	38976					3,5	10,0	1	
	" ¾" "	"						1456	31032					1,3	3,9	2	
	German							9856	36736					2,5	8,2		Martens
Cast Iron.	American, average								17 to 30740		48 to 54800					44	Watertown Arsenal
	" ⅛" to 1" thick	All undressed (with skin on,)					16453000					44680				7	
	" 1" thick	the dressed ones (planed,) were					15483000					40640				1	
	" 1" to 2" thick	found to be slightly stronger					15296000					37932				5	
	" 2" thick	and results more regular.					13767000					36600				2	

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TABLE XXXV.

BEARING VALUES FOR IRON AND STEEL RIVETS AND PINS.

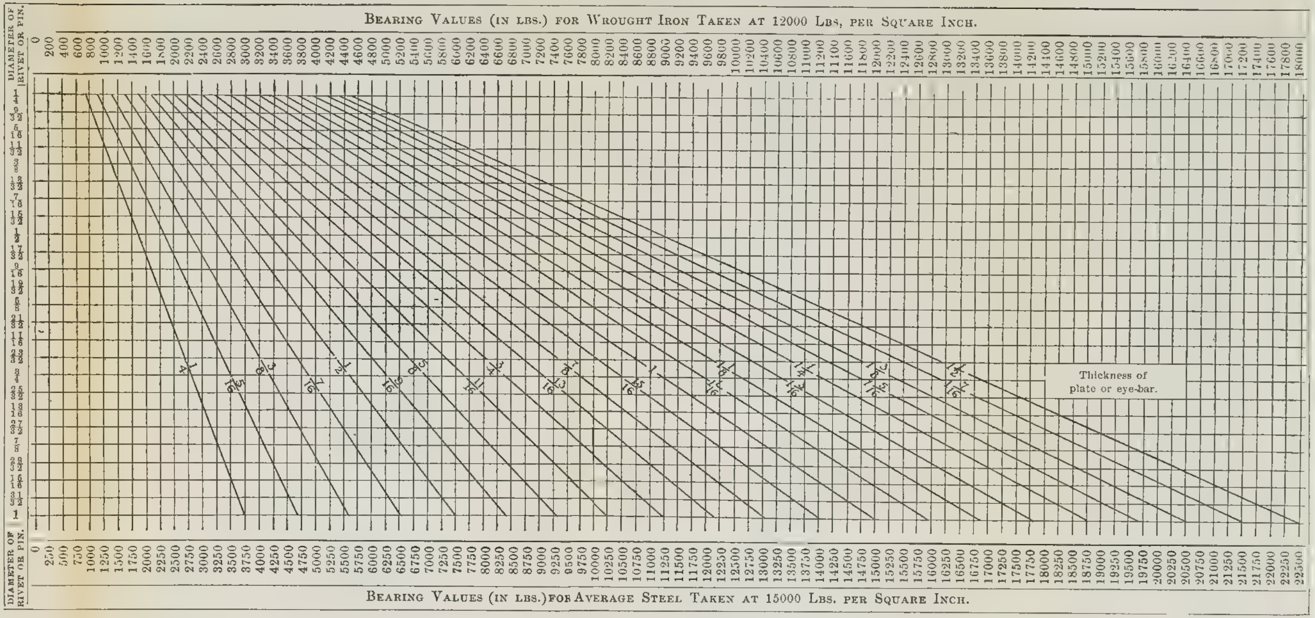

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TABLE XXXVI.

Bearing Values for Iron and Steel Pins and Bolts.

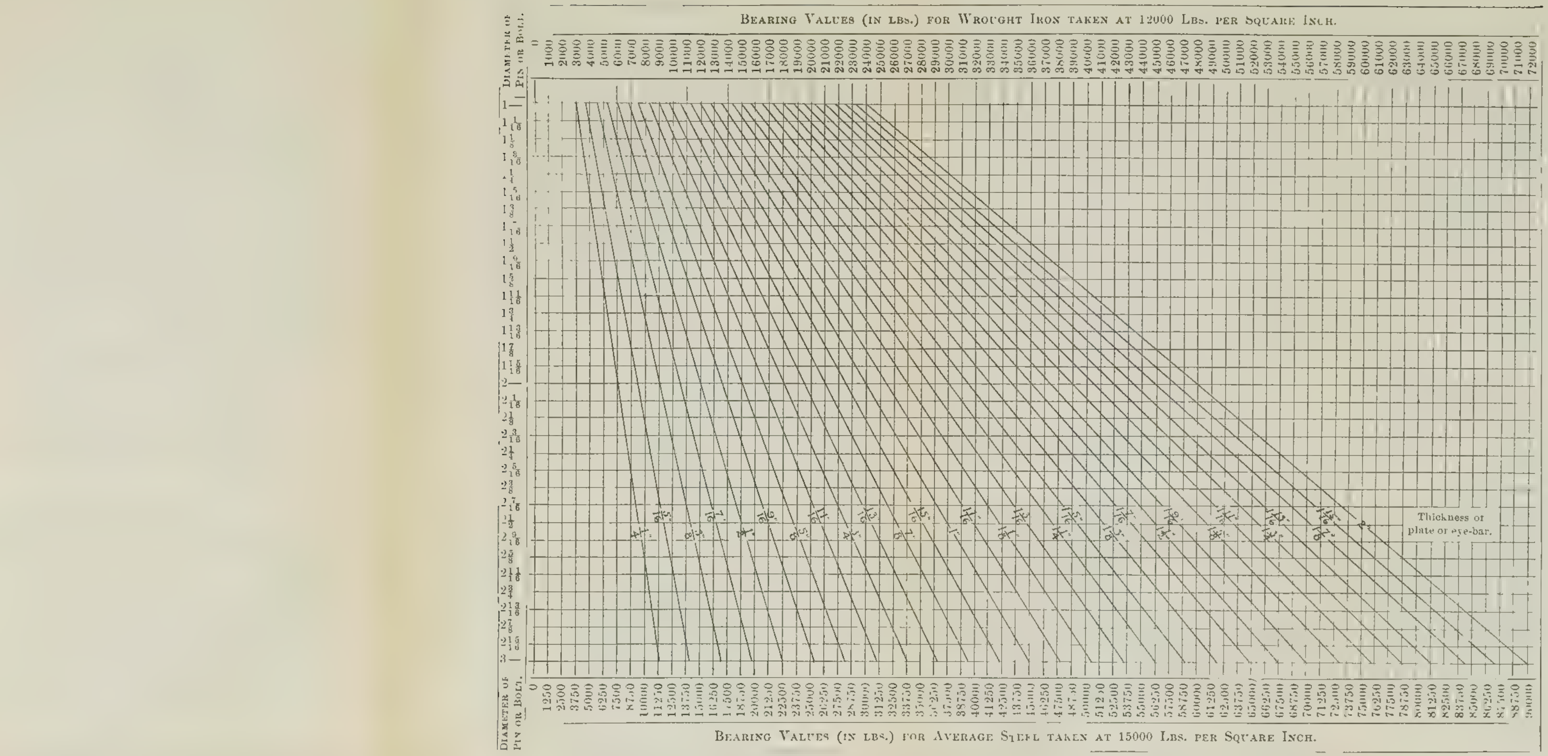

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TABLE XXXVII.

Bearing Value for Iron and Steel Pins and Bolts.

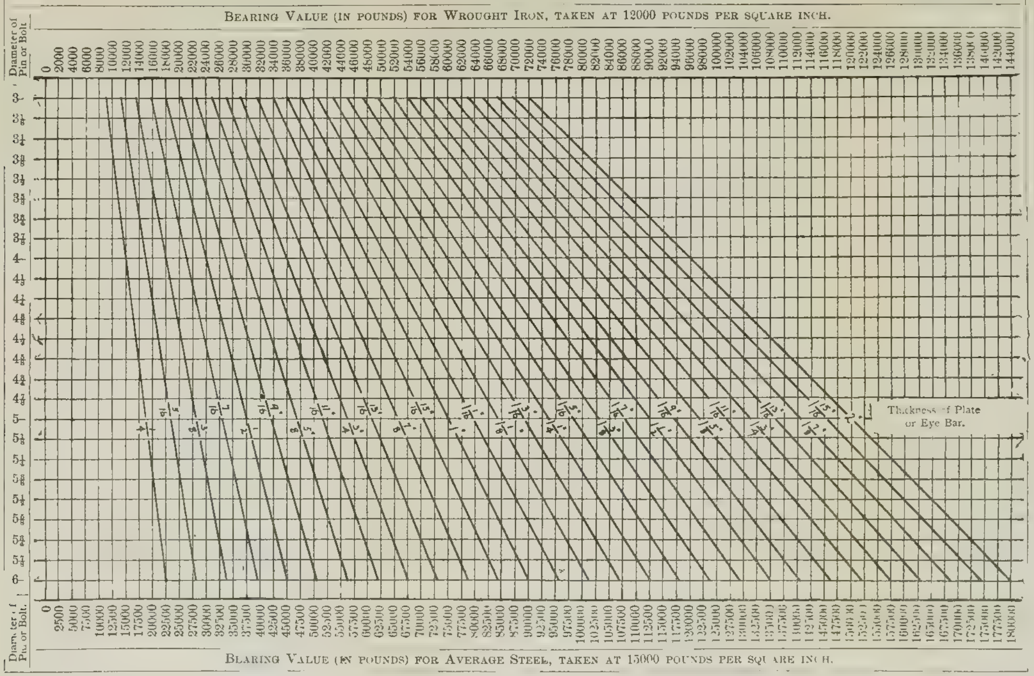

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TABLE XXXVIII.

SHEARING, BENDING AND TENSIONAL VALUES.

FOR IRON AND STEEL RIVETS, PINS AND RODS OF ONE-FOURTH TO ONE INCH IN DIAMETER.

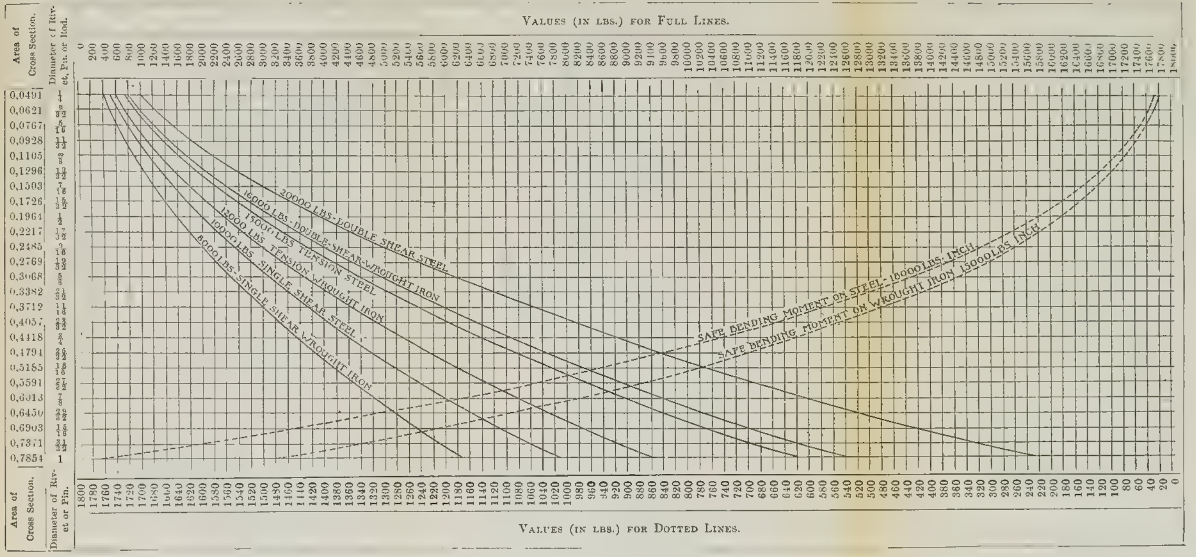

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TABLE XXXIX.

Shearing, Bending and Tensional Values for Iron and Steel Pins and Rods 1 to 3 Inches in Diameter.

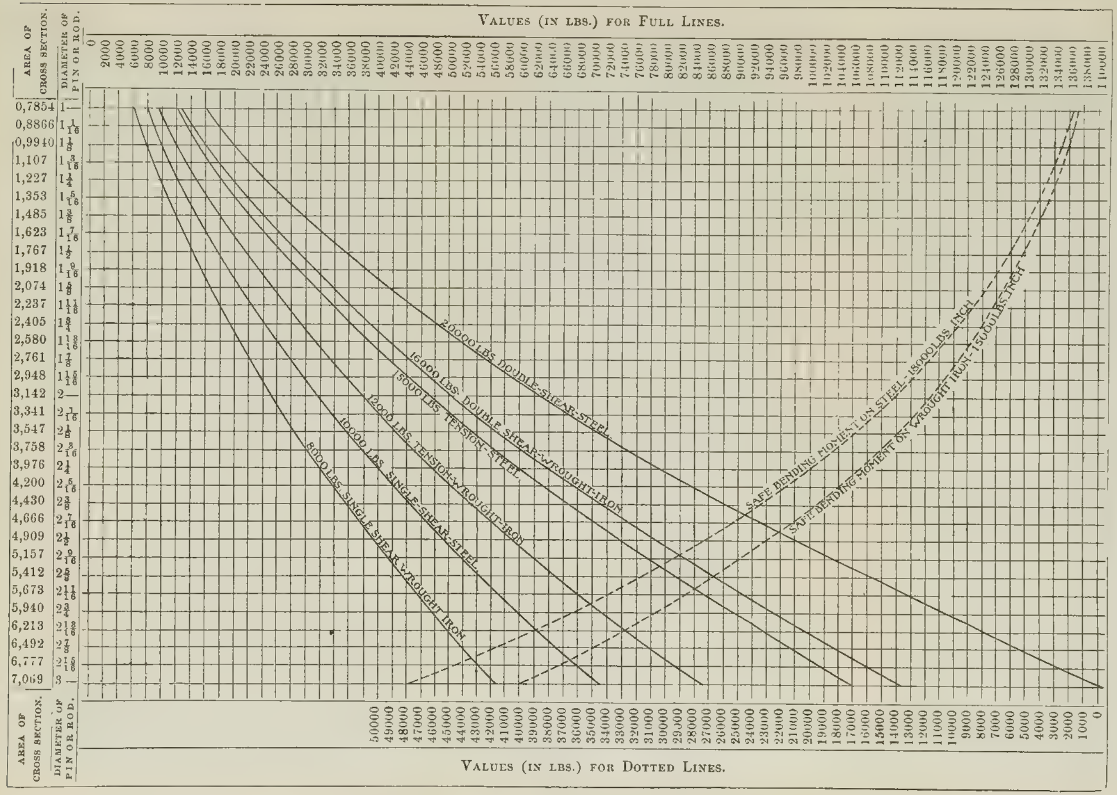

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TABLE XL.

SHEARING, BENDING AND TENSIONAL VALUES FOR IRON AND STEEL PINS AND RODS 3 TO 6 INCHES IN DIAMETER.

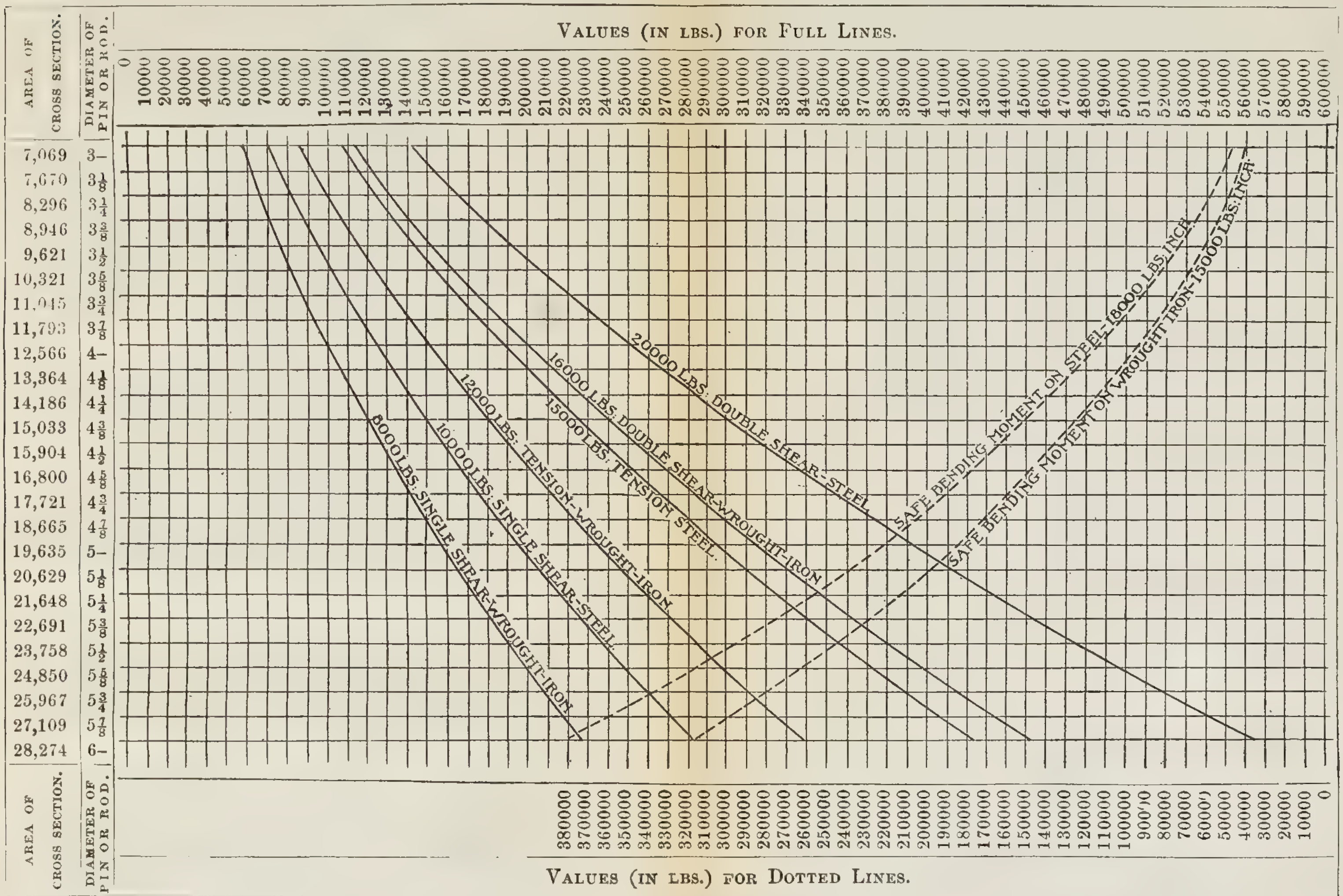

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TABLE XLI.

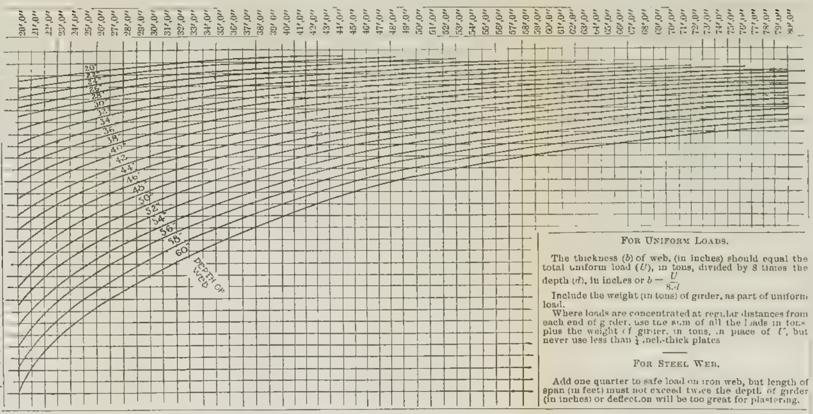
WROUGHT-IRON RIVETED GIRDERS. STRENGTH OF WEBS.

SAFE UNIFORM LOAD (U) IN TONS OF 2000 POUNDS.

THICKNESS OF WEB (IN INCHES).

$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$
3,0	2.5	2,0	1,75	1,5	1 25	1,0
6,0	5 0	4 0	3.50	3.0	2,50	2,0
9,0	7.5	6.0	6 25	4,5	3,75	3,0
12 0	10 0	8,0	7,00	6,0	5 00	4,0
15,0	12,5	10 0	8,75	7,5	6,25	5,0
18,0	15 0	12,0	11 50	9.0	7.50	6,0
21 0	17,5	14,0	12,25	10,5	8,75	7,0
24,0	20 0	16,0	14 00	12 0	10.00	8,0
27.0	22 5	18,0	15 75	13,5	11,25	9,0
30,0	25 0	20,0	17,50	15,0	12,50	10,0
33,0	27 5	22,0	19 25	16,5	13,75	11,0
36 0	30 0	24 0	21 00	18,0	15,00	12,0
39,0	32 5	26 0	22 75	19,5	16,25	13,0
42 0	35,0	28 0	24,50	21,0	17,50	14,0
45 0	37,5	30,0	26,25	22,5	18,75	15,0
48,0	40,0	32,0	28 00	24,0	20,00	16,0
51 0	42,5	34.0	29,75	25 5	21,25	17,0
54 0	45 0	36,0	31,50	27 0	22,50	18,0
57.0	47.5	38,0	33,25	28.5	23,75	19 0
60 0	50,0	40,0	35 00	30,0	25.00	20,0
63.0	52,5	42,0	36.75	31,5	26,25	21,0
66 0	55 0	44,0	38.50	33 0	27.50	22,0
69.0	57,5	46 0	40 25	34,5	28,75	23,0
72 0	60 0	48,0	42,00	36,0	30 00	24,0
75 0	62 5	50 0	43,75	37,5	31 25	25,0
78 0	65 0	52 0	45,50	39,0	32 50	26,0
81 0	67 5	54 0	47 25	40,5	33 75	27,0
84 0	70,0	56.0	49 00	42 0	35 00	28,0
87,0	72,5	58.0	50 75	43 5	36 25	29,0
90,0	75,0	60 0	52,50	45 0	37,50	30,0

LENGTH OF SPAN IN FEET.



FOR UNIFORM LOADS.

The thickness (b) of web, (in inches) should equal the total uniform load (U), in tons, divided by 8 times the depth (d), in inches or $b = \frac{U}{8.d}$

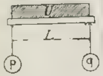
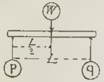
Include the weight (in tons) of girder, as part of uniform load.

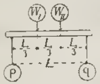
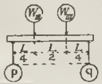
Where loads are concentrated at regular distances from each end of girder, use the sum of all the loads in tons plus the weight of girder, in tons, in place of U, but never use less than $\frac{1}{4}$ inch-thick plates

FOR STEEL WEB.

Add one quarter to safe load on iron web, but length of span (in feet) must not exceed twice the depth of girder (in inches) or deflection will be too great for plastering.

FOR DIFFERENT MANNERS OF LOADING.

Manner of Loading	To Obtain Safe Load in Tons		Length of Span not to crack Plastering must not exceed		Greatest Actual Deflection will be	
	For Iron.	For Steel.	For Iron.	For Steel.	For Iron.	For Steel.
	Use U as given in Tables.	Use $\frac{5}{4}.U$	$L = 2\frac{1}{4}.d$	$L = 2.d$	$\delta = \frac{L^2}{75.d}$	$\delta = \frac{3.L^2}{200.d}$
	$W = \frac{U}{2}$	$W = \frac{5.U}{8}$	$L = 2\frac{1}{5}.d$	$L = 2\frac{2}{5}.d$	$\delta = \frac{4.L^2}{375.d}$	$\delta = \frac{3.L^2}{242.d}$

Manner of Loading	To Obtain Safe Load in Tons		Length of Span not to crack Plastering must not exceed		Greatest Actual Deflection will be	
	For Iron.	For Steel.	For Iron.	For Steel.	For Iron.	For Steel.
	$W_{\text{I}} = W_{\text{II}} = \frac{3.U}{8}$ or $W_{\text{I}} + W_{\text{II}} = \frac{3.U}{4}$	$W_{\text{I}} = W_{\text{II}} = \frac{15.U}{32}$ or $W_{\text{I}} + W_{\text{II}} = \frac{15.U}{16}$	$L = 2\frac{3}{16}.d$	$L = 1\frac{7}{8}.d$	$\delta = \frac{L^2}{73.d}$	$\delta = \frac{2\,L^2}{125.d}$
	$W_{\text{III}} = W_{\text{IIII}} = \frac{U}{2}$ or $W_{\text{III}} + W_{\text{IIII}} = U$	$W_{\text{III}} = W_{\text{IIII}} = \frac{5.U}{8}$ or $W_{\text{III}} + W_{\text{IIII}} = \frac{5.U}{4}$	$L = 2.d$	$L = 1\frac{3}{4}.d$	$\delta = \frac{3.L^2}{200.d}$	$\delta = \frac{3.L^2}{175.d}$

d = DEPTH IN INCHES; L = LENGTH IN FEET; δ = DEFLECTION IN INCHES; U = UNIFORM LOADS IN TONS AS GIVEN IN TABLES XLII AND XLIII; W = CENTRE LOAD IN TONS; $W_{\text{I}} = W_{\text{II}}$ also $W_{\text{III}} = W_{\text{IIII}}$ = CONCENTRATED LOADS IN TONS.

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TABLE XLII.

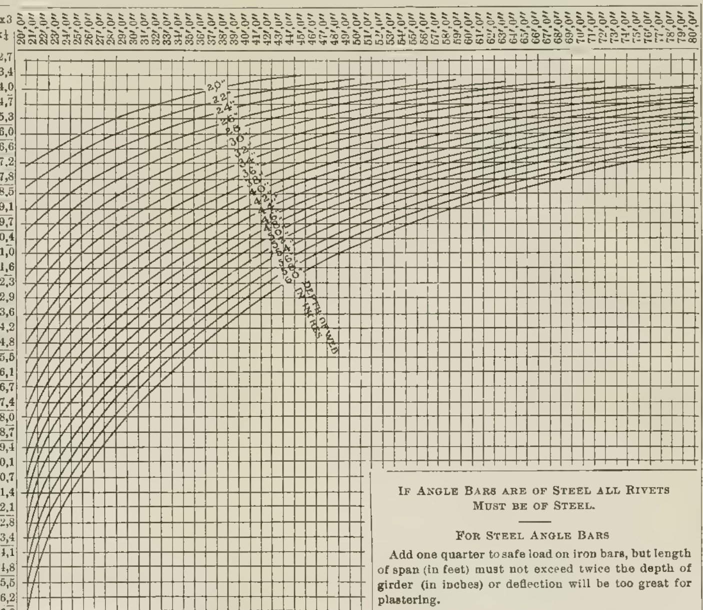
Wrought Iron Riveted Girders.—Strength of the Angle Bars.

Safe Uniform Load (U) in Tons of 2000 Lbs.

Size of Angle Bars (in In.)

6x6 x⅞	6x6 x½	4½ x4½ x½	4x4 x½	3x3 x½	3x3 x¼
14,2	9,9	8,0	7,3	5,0	2,7
18,9	12,6	10,0	9,0	6,3	3,4
23,6	15,3	12,0	10,8	7,6	4,0
28,2	18,0	14,0	12,5	8,9	4,7
32,9	20,7	16,0	14,3	10,2	5,3
37,6	23,4	18,0	16,0	11,5	6,0
42,2	26,1	20,0	17,8	12,8	6,6
46,9	28,8	22,0	19,5	14,1	7,2
51,6	31,5	24,0	21,2	15,4	7,8
56,2	34,3	26,0	23,0	16,6	8,5
60,9	37,1	28,0	24,7	17,8	9,1
65,6	39,9	30,0	26,5	19,0	9,7
70,2	42,7	32.0	28,2	20,2	10,4
74,9	45,5	34,0	29,9	21,4	11,0
79,6	48,3	36,0	31,7	22,6	11,6
84,2	51,1	38,0	33,4	23,8	12,3
88,9	53,8	40,0	35,1	25,0	12,9
93,6	56,6	42,0	36,9	26,2	13,6
98,2	59,4	44,0	38,6	27,4	14,2
102,9	62,2	46,0	40,4	28,6	14,8
107,6	65,0	48,0	42,1	29,8	15,5
112,2	67,8	50,0	43,9	31,0	16,1
116,9	70,6	52,0	45,6	32,2	16,7
121,6	73,4	54,0	47,3	33,4	17,4
126,2	76 1	56,0	49,1	34,6	18,0
130,9	78,9	58,0	50,8	35,8	18,7
135,6	81,7	60,0	52,6	37,0	19,4
140,2	84,5	62,0	54,3	38,2	20,1
144,9	87,2	64,0	56,0	39,4	20,7
149,6	90,0	66,0	57,8	40,6	21,4
154,2	92,8	68,0	59,5	41,8	22,1
158,9	95,6	70,0	61,3	43,0	22,8
163,6	98,3	72,0	63,0	44,2	23,4
168,2	101,1	74,0	64,7	45,4	24,1
172,9	103,9	76,0	66,5	46,6	24,8
177,6	106,7	78,0	68,2	47,8	25,5
182,2	109,4	80,0	70,0	49,0	26,2
186,9	112,2	82,0	71,7	50,2	26,8

Length of Span in Feet.



If Angle Bars are of Steel all Rivets Must be of Steel.

For Steel Angle Bars

Add one quarter to safe load on iron bars, but length of span (in feet) must not exceed twice the depth of girder (in inches) or deflection will be too great for plastering.

For Different Manners of Loading, Deflection, Etc., See Table XLI.

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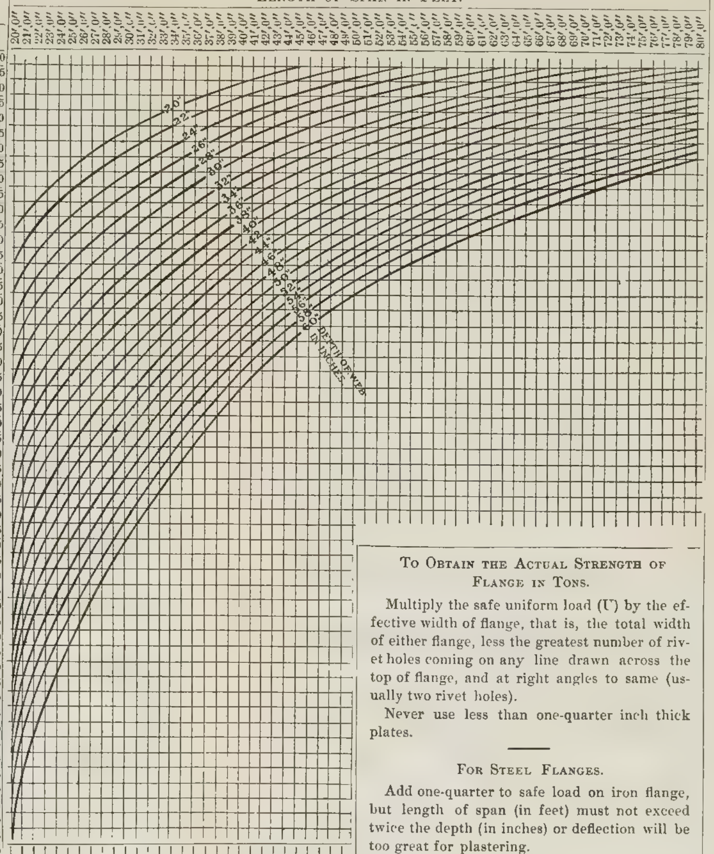
TABLE XLIII.

Wrought Iron Riveted Girders.—Strength of Flanges.

Safe Uniform Load (U) in Tons of 2000 Lbs. for Each Inch of Effective Width of Flange.

Thickness of Flange in Inches.										
2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$
3,20	2,80	2,40	2,00	1,60	1,40	1,20	1,00	0,80	0,60	0,40
3,60	3,15	2,70	2,25	1,80	1,58	1,35	1,13	0,90	0,68	0,45
4,00	3,50	3,00	2,50	2,00	1,75	1,50	1,25	1,00	0,75	0,50
4,40	3,85	3,30	2,75	2,20	1,93	1,65	1,38	1,10	0,83	0,55
4,80	4,20	3,60	3,00	2,40	2,10	1,80	1,50	1,20	0,90	0,60
5,20	4,55	3,90	3,25	2,60	2,28	1,95	1,63	1,30	0,98	0,65
5,60	4,90	4,20	3,50	2,80	2,45	2,10	1,75	1,40	1,05	0,70
6,00	5,25	4,50	3,75	3,00	2,63	2,25	1,88	1,50	1,13	0,75
6,40	5,60	4,80	4,00	3,20	2,80	2,40	2,00	1,60	1,20	0,80
6,80	5,95	5,10	4,25	3.40	2,98	2,55	2,13	1,70	1,28	0,85
7,20	6,30	5,40	4,50	3,60	3,15	2,70	2,25	1,80	1,35	0,90
7,60	6,65	5,70	4.75	3,80	3,33	2,85	2,38	1.90	1,43	0,95
8,00	7,00	6,00	5,00	4,00	3,50	3,00	2,50	2,00	1,50	1,00
8,40	7,35	6,30	5,25	4,20	3,68	3,15	2,63	2,10	1,58	1,05
8,80	7,70	6,60	5,50	4,40	3,85	3,30	2,75	2,20	1,65	1,10
9,20	8,05	6,90	5,75	4,60	4,03	3,45	2,88	2,30	1,73	1,15
9,60	8,40	7,20	6,00	4,80	4,20	3,60	3,00	2,40	1,80	1,20
10,00	8,75	7,50	6,25	5,00	4,38	3,75	3,13	2,50	1,88	1,25
10,40	9,10	7,80	6,50	5,20	4,55	3,90	3,25	2,60	1,95	1,30
10,80	9,45	8,10	6,75	5,40	4,73	4,05	3,38	2,70	2,03	1,35
11,20	9,80	8,40	7,00	5,60	4,90	4,20	3.50	2,80	2,10	1,40
11,60	10,15	8,70	7,25	5,80	5,08	4,35	3,63	2,90	2,18	1,45
12,00	10,50	9,00	7,50	6,00	5,25	4,50	3,75	3,00	2,25	1,50
12,40	10,85	9,30	7,75	6,20	5,43	4,65	3,88	3,10	2,33	1,55
12,80	11,20	9,60	8,00	6,40	5,60	4,80	4,00	3,20	2,40	1,60
13,20	11,55	9,90	8,25	6,60	5,78	4,95	4.13	3,30	2,48	1,65
13,60	11,90	10,20	8,50	6,80	5.95	5,10	4,25	3,40	2,55	1,70
14,00	12,25	10,50	8,75	7,00	6,13	5,25	4,38	3.50	2,63	1,75
14,40	12,60	10,80	9,00	7,20	6,30	5,40	4,50	3,60	2,70	1,80
14,80	12,95	11,10	9.25	7.40	6,38	5,55	4,63	3,70	2,78	1,85
15,20	13,30	11,40	9,50	7,60	6,65	5,70	4,75	3,80	2,85	1,90
15,60	13,65	11,70	9,75	7,80	6,83	5,85	4,88	3,90	2,93	1,95
16,00	14,00	12,00	10,00	8,00	7,00	6,00	5,00	4,00	3,00	2,00
16,40	14,35	12,30	10,25	8,20	7,18	6,15	5,13	4,10	3,08	2,05
16,80	14,70	12,60	10,50	8,40	7,35	6,30	5,25	4,20	3,15	2,10
17,20	15,05	12,90	10,75	8,60	7,53	6,45	5,38	4,30	3,23	2,15
17,60	15,40	13,20	11,00	8,80	7,70	6,60	5,50	4,40	3,30	2,20
18,00	15,75	13,50	11,25	9,00	7,88	6,75	5,63	4,50	3,38	2,25
18,40	16,10	13,80	11,50	9,20	8,05	6,90	5,75	4,60	3,45	2,30
18,80	16,45	14,10	11,75	9,40	8,23	7,05	5,88	4,70	3.53	2,35
19,20	16,80	14,40	12,00	9,60	8,40	7,20	6,00	4,80	3,60	2,40
19,60	17,15	14,70	12,25	9,80	8,58	7,35	6,13	4,90	3,68	2,45
20,00	17.50	15,00	12,50	10,00	8,75	7,50	6,25	5,00	3,75	2,50
20,40	17,85	15.30	12,75	10,20	8,93	7,65	6,38	5,10	3,83	2,55
20,80	18,20	15.60	13,00	10,40	9,10	7,80	6,50	5,20	3,90	2,60
21,20	18,55	15,90	13,25	10,60	9,28	7,95	6,63	5,30	3,98	2,65
21,60	18,90	16,20	13,50	10,80	9,45	8,10	6,75	5,40	4,05	2,70
22,00	19,25	16,50	13,75	11,00	9,63	8,25	6,88	5,50	4,13	2,75
22.40	19,60	16,80	14,00	11,20	9,80	8,40	7,00	5 60	4,20	2,80
22.80	19,95	17,10	14,25	11,40	9,98	8,55	7,13	5,70	4,28	2,85
23,20	20,30	17,40	14,50	11,60	10,15	8,70	7,25	5,80	4,35	2,90
23,60	20,65	17,70	14,75	11,80	10,33	8,85	7,38	5,90	4,43	2,95
24 00	21,00	18 00	15,00	12,00	10,50	9,00	7,50	6,00	4,50	3,00

Length of Span in Feet.



To Obtain the Actual Strength of Flange in Tons.

Multiply the safe uniform load (U) by the effective width of flange, that is, the total width of either flange, less the greatest number of rivet holes coming on any line drawn across the top of flange, and at right angles to same (usually two rivet holes).

Never use less than one-quarter inch thick plates.

For Steel Flanges.

Add one-quarter to safe load on iron flange, but length of span (in feet) must not exceed twice the depth (in inches) or deflection will be too great for plastering.

For Different Manners of Loading, Deflection, Etc., See Table XLI.

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